W. H. Mills [1] proved the following

**Theorem.** There exists a real number \( A \), such that \([A^x] \), where \([R] \) denotes the greatest integer less than or equal to \( R \), is a prime number for every positive integer \( x \).

In order to prove this statement he makes use of the inequality, deduced by A. E. Ingham [2]

\[ p_{n+1} - p_n < K \, p_n^{5/12}, \]

where \( p_n \) denotes the \( n \)th prime number and \( K \) is a positive integer.

We generalise the theorem mentioned above as follows:

**Theorem.** For each integer \( c \geq 3 \) a real number \( A \) depending only on \( c \) can be found such that \([A^x] \) is a prime number for all positive integers \( x \).

The proof runs as follows. The integers \( a = (c - 1) \, w - 1 \) and \( b = cw - 1 \) satisfy the relations

\[ (a, b) = 1; \, a < b - 1; \, a/b \geq 5/8; \, ca + 1 \equiv 0 \pmod{b}. \]

Then

\[ p_{n+1} - p_n < K \, p_n^{a/b}, \]

where \( K \) has the same value as before.

The lemma used by Mills can be established here in the following form:

If \( a, b \) and \( c \) are positive integers satisfying (1) and (2), and if \( N \) is an integer with

\[ N > K^b, \]

then there exists a prime \( p \) such that

\[ N^c < p < (N+1)^c - 1. \]

**Proof.** If \( p_n \) is the greatest prime less than \( N^c \), then we have, using (1), (2) and (3)

\[ p_n < N^c < p_{n+1} < p_n + K \, p_n^{a/b} < N^c + K \, N^{a/b} < N^c + N^{(a+1)/b} < (N+1)^c - 1. \]

\[ 1 \] We are indebted to Prof. J. G. van der Corput for a generalisation of the Theorem as it was originally stated by us.
If now $P_0$ is a prime greater than $K^b$, then we are able to construct a sequence of primes $P_0, P_1, \ldots$ such that

$$(4) \quad P_n^c < P_{n+1} < (P_n + 1)^c - 1 \quad (n = 0, 1, 2, \ldots).$$

In the same way, as it has been done by Mills for the case $a = 5, b = 8, c = 3$, we can prove by (4), that the sequence $P_n^c$ is a bounded increasing sequence, and that therefore $P_n^c$ tends to a limit, say $A$.

The boundedness of $P_n^c$ follows from the inequality $P_n^c < (P_n + 1)^c - 1$, where the sequence $(P_n + 1)^c - 1$ is a decreasing sequence, which also follows from (4).

So we have

$$P_n^c < A < (P_n + 1)^c - 1$$

or

$$P_n < A^c < P_n + 1.$$

This means that $[A^c] \, (x = 1, 2, \ldots)$ is a prime-representing function.

For two different values of $c$, say $c$ and $\gamma$, one of the corresponding sequences $P_n(c)$ and $p_n(\gamma)$ cannot be a subsequence of the other. For instance, if we assume, that the sequence $p_n(\gamma)$ is a subsequence of $P_n(c)$, then there would be a subsequence $Q_n(c)$ of $P_n(c)$, such that $Q_n^c$ and $Q_n^{\gamma^c}$ would tend to the same limit, and this is impossible on account of $c \neq \gamma$.

Bandung, February 7, 1950.

LITERATURE