Astronomy. — *"On the possibility of statistical equilibrium of the universe"*. By Prof. W. DE SITTER.

(Communicated at the meeting of Nov. 27, 1920).

EINSTEIN has on several occasions expressed the opinion that the existence of a finite amount of matter in the universe must necessarily lead to the adoption of a finite three-dimensional space. In his inaugural address at Leiden [1]) he says:

"Wir können aber auf Grund der relativistischen Gravitations-"gleichungen behaupten, dass eine Abweichung vom euklidischen "Verhalten bei Räumen von kosmischer Grössenordnung dann vor-"handen sein muss, wenn eine noch so kleine positive mittlere "Dichte der Materie in der Welt existiert. In diesem Falle muss die "Welt notwendig räumlich geschlossen und von endlicher Grösse "sein, wobei ihre Grösse durch den Wert jener mittleren Dichte "bestimmt wird."

It appears to me that this statement cannot be accepted unreservedly. The gravitational field-equations are:

$$G_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu}\,(G - 2\lambda) = -\varkappa T_{\mu\nu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If we suppose all matter to be at rest and free from any strain or internal forces, then the tensor $T_{\mu\nu}$ has the value

$$T_{44} = g_{44}\,\varrho, \text{ all other } T_{\mu\nu} = 0, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

ϱ being the density in natural measure. We can put $\varrho = \varrho_0 + \varrho_1$, where the average value of ϱ_1 is zero; ϱ_0 is then the average density. If we neglect ϱ_1 the equations (1) are satisfied by the $g_{\mu\nu}$ implied by the line-element:

$$ds^2 = -dr^2 - R^2 \sin^2\frac{r}{R}\,[d\psi^2 + \sin^2\psi\, d\theta^2] + c^2\, dt^2, \quad . \quad . \quad (3A)$$

if we take

$$\varkappa\varrho_0 = \frac{2}{R^2}, \quad \lambda = \frac{1}{R^2}, \text{ (EINSTEIN)} \quad . \quad . \quad . \quad . \quad . \quad (4A)$$

or by those of the line-element:

$$ds^2 = -dr^2 - R^2 \sin^2\frac{r}{R}\,[d\psi^2 + \sin^2\psi\, d\theta^2] + \cos^2\frac{r}{R}\,c^2dt^2, \quad . \quad (3B)$$

with

[1]) Aether und Relativitätstheorie. Berlin, Julius Springer, 1920, p. 13.

$$\varrho_0 = 0, \quad \lambda = \frac{3}{R^2} \text{ (DE SITTER)} \quad . \quad . \quad . \quad . \quad . \quad (4B)$$

For $R = \infty$ both (A) and (B) degenerate into:

$$ds^2 = -dr^2 - r^2\,[d\psi^2 + \sin^2\psi\, d\theta^2] + c^2dt^2, \quad . \quad . \quad . \quad (3C)$$

with

$$\varrho_0 = 0, \quad \lambda = 0 \text{ (NEWTON)} \quad . \quad . \quad . \quad . \quad . \quad (4C)$$

It thus appears that EINSTEIN's solution (A), in which three-dimensional space is finite and closed, is the only one which admits of a finite average density ϱ_0. But this is only true, if the tensor $T_{\mu\nu}$ has the value (2), i. e. if the matter is at rest and in equilibrium. If the matter is either in motion, or subjected to stresses or pressures, the value (2) cannot be used; the equations (3) and (4) no longer represent the exact solution, and we can have finite values of ϱ_0 also in the systems (B) and (C) [1]). EINSTEIN's assertion can thus only be maintained if we make the additional hypothesis that for the whole universe, or for regions of very large, or "cosmical", size, we can still use the value (2) of the tensor $T_{\mu\nu}$, i. e. if for such regions we assume the matter to be in statistical equilibrium.

This result can also be expressed thus: If the system (A) is the true one, then it is possible for the universe, or for large portions of it, to be in statistical equilibrium. If either (B) or (C) is the true system, then this is not possible. Now the possibility of statistical equilibrium of large portions of the universe is, to my mind at least, by no means self-evident, or even probable. The idea of evolution in a determined sense appears to me to be rather opposed to the actual existence, if not to the possibility, of equilibrium.

The systems (A) and (B), involving the introduction of the constant λ, originated from the wish to make the three-dimensional world finite [2]). At the present time the choice between the systems (A),

[1]) Similarly in the system (A) the value of ρ_0 differs from that given by $(4A)$. See also: DE SITTER, On EINSTEIN's theory of gravitation and its astronomical consequences, Monthly Notices of the R. A. S. Vol. LXXVII, pp. 6—7, 18 and 20—23.

[2]) If we assume the three dimensional line-element to be

$$d\sigma^2 = dr^2 + R^2 \sin^2\frac{r}{R}\,[d\psi^2 + \sin^2\psi\, d\theta^2], \quad . \quad . \quad . \quad . \quad (5)$$

and $g_{i4} = 0$, then no other solutions than (A) and (B) exist. Of the two possible three-dimensional spaces of constant curvature having the line-element (5) we must choose the so called elliptical space. The analogy with two-dimensional geometry suggests the spherical space, but this analogy is misleading. The elliptical space is really the one of which our ordinary euclidian geometry is the limiting case for $R = \infty$. In our common geometry a plane has a line (and not a point) at

(*B*) and (*C*) is purely a matter of taste. There is no physical criterion as yet available to decide between them. It is true that the systems (*B*) and (*C*) do not satisfy MACH's postulate that inertia must be traceable to a material source. But this postulate is a purely metaphysical one, and has no physical foundation whatever. It appears to me to be the last remnant of the desire for a purely *mechanical* interpretation of nature, which logically and historically is based on the belief in forces at a distance, and the impossibility of which has been so clearly demonstrated by EINSTEIN in his Leiden address.

The three systems differ however in their physical consequences at large distances, and an experimental discrimination between them may be possible in the future. The decision between (*B*) on the one, and (*A*) and (*C*) on the other hand may be brought about by the study of systematic radial motions of spiral nebulae [1]). The distinction between (*A*) and (*C*) is more difficult, since they both have $g_{44} = 1$, and differ only in the g_{ij} with i and j different from 4, the values of which at great distances it is not so easy to ascertain. The decision between these two systems must, I fear, for a long time be left to personal predilection.

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infinity; two straight lines have only one (and not two) point of intersection, which may be situated at infinity; if we go to infinity along one branch of a hyperbola, we return along the other branch on the other (and not on the same) side of the asymptote. All these are properties of the elliptical as contrasted with the spherical space. The spherical is only a quite unnecessary reduplication of the elliptical one.

[1]) See DE SITTER, l. c. pp. 27—28. At that time (1917) the radial velocities of only three spirals were known, of which one was negative; the mean being + 600 *km/sec.* Now the radial velocities of 25 spirals are known (see Mount Wilson Publications, Nr. 161, p. 19) of which only three are negative, the mean being + 560 *km/sec* (or + 677 *km/sec* if the four brightest are omitted). The system (*B*) requires a (spurious) positive radial velocity for distant objects.

Physics. — *"The Mechanism of the Automatic Current Interrupter".* By Prof. J. K. A. WERTHEIM SALOMONSON.

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The mechanism of the automatic current interrupter as represented by HELMHOLTZ's tuningfork interrupter, by NEEFF-WAGNER's hammer-break, and by the ordinary electric bell, has not yet been explained in an entirely satisfactory way. Lord RAYLEIGH was the first to give an explanation, without, however, entering into details. Later on its mechanism was studied by LIPPMANN, DVORAK, GUILLET, BOUASSE and others although no new points of view were opened. In this paper I intend to submit a few considerations on this subject, principally based on a research into the attraction by the electromagnet on the armature during the working of the apparatus. As an indicator for the attraction I used the number of lines of force passing through the armature at each moment. These were measured by an oscillographic method. This might have been done by the new ABRAHAM-rheograph, but as I did not possess this instrument I employed DEGUISNE's method, described in the Physikalische Zeitschrift 1910, p. 513. The results of this method were compared with those obtained by a new method, which I shall describe in an appendix to this paper.

The interrupter used in my experiments has a horizontal horseshoe magnet. The cores turned from a solid bar of swedish iron completely bored and slit lengthways, have a length of 5 cm and a diameter of 1 cm. They are screwed at a distance of 3.2 cm from each other into a yoke of 1.4 cm^2 transverse section, and are each wound with 200 turns of well insulated copper wire of 1.2 ohm resistance each. The armature measured $1.2 \times 0.75 \times 4.4$ cm. It is screwed to a strong steel spring of 0.12×1.0 cm. with a free length of 1.3 cm. Into the other end of the armature a brass bar 0.4 cm. in diameter and 5 cm. in length was fixed, on which, if desired, a small copper weight could be screwed. It was generally used without weight and then made about 47 complete vibrations per second, the platinum contact being so adjusted as to make and break the current during one half of the periodic time. The arma-