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Royal Academy of Sciences. Amsterdam.

PROCEEDINGS OF THE MEETING of Saturday May 28th 1898.

(Translated from: Verslag van de gewone vergadering der Wis- en Natuurkundige Afdeeling van Zaterdag 28 Mei 1898 Dl. VII).

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The following papers were read:

Mathematics. — „On the cyclographic space representation of Joachimsthal's circles.” By Prof. P. H. SCHOUTE.

1. In his „Cyklographie” Dr. W. FIEDLER has developed a theory, in which any circle of the plane is represented in space by one of two points of the normal, erected in its centre on the plane, and having on either side a distance from this centre equal to the radius. The ambiguity of this representation can be useful in the distinction of the two senses, in which a point can move along the circle. This is not necessary here.

According to the FIEDLERIAN representation the right cone, whose vertex is a point P of the plane of the circles, whose axis is the normal in P on this plane, and whose vertex angle is a right one, corresponds to the net of the circles passing through P . Likewise the pencil of the circles passing through P and Q has as image a rectangular hyperbola situated in the plane bisecting PQ orthogo-

nally; of this hyperbola the line common to its plane and the plane of the circles is the imaginary axis, the intersection of this line with PQ is the centre, and the normal in this point to the plane of the circles is the real axis. Or, put more generally: the circles of a pencil are represented by a rectangular hyperbola, whose real or imaginary axis is normal to the plane of the circles according to the points common to the circles being real or imaginary, and which breaks up into two straight lines, if these points coincide. And the circles of a general net are represented by a rectangular hyperboloid with one or two sheets, according to the circle cutting the circles of the net orthogonally being real or imaginary.

In different cases the FIEDLERIAN theory can give a clear and concise idea of the position of ranges of circles. So a. o. the circles having double contact with a given ellipse ϵ . If this ellipse ϵ be represented by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$, the ellipse $\frac{x^2}{a^2 - b^2} + \frac{z^2}{b^2} = 1, y = 0$ and the hyperbola $-\frac{y^2}{a^2 - b^2} + \frac{z^2}{a^2} = 1, x = 0$ correspond to the two ranges of circles having double contact with ϵ . As is easily explained these curves are transformed into the focal conics of ϵ by inverting the sign of z^2 .

In the following lines we study the surface that forms the image of the twofold infinite system of JOACHIMSTHAL's circles of ϵ .

2. Through any point P of the plane of ϵ can be drawn four normals to ϵ . The footpoints A, B, C, D of these normals may be called „conormal”. If the point diametrically opposite to A on ϵ is indicated by A' , the known theorem of JOACHIMSTHAL says that A', B, C, D are concyclic, if A, B, C, D are conormal.

This non-reversible theorem has been completed by LAGUERRE in remarking that the circle $A'B'C'D$ meets the tangent $t_{a'}$ in A' to ϵ for the second time in the projection $O_{a'}$ of the centre O of ϵ on $t_{a'}$. In other words:

If P describes the normal n_a in A to ϵ , the corresponding circles $A'B'C'D$ form a pencil, as all these circles pass through A' and $O_{a'}$. This pencil being represented by a rectangular hyperbola, the image in question is the locus of a simply infinite number of rectangular hyperbolae. However, before we proceed to the deduction of this surface, we investigate somewhat more closely the correspondence between the points P of the normal n_a and the centres M on the line $l_{a'}$ bisecting orthogonally the segment $A'O_{a'}$.

3. The relation between the points P and M on n_a and $l_{a'}$ is a (1,1) correspondence, i. e. these points describe projective ranges. If P

is at infinity on n_a , this is also the case with M on $l_{a'}$. So the point at infinity common to n_a and $l_{a'}$ corresponds to itself, i. e. the projective ranges are in perspective. The centre of perspective T_a is immediately found as the point common to the joints $P_1 M_1$ and $P_2 M_2$ (fig 1) of the pairs of corresponding points (P_1, M_1) and (P_2, M_2) . This point being found, it is possible to indicate the centre M of the circle of JOACHIMSTHAL corresponding to any point P of n_a .

Analytically the obtained results are given back as follows. The coordinates of A being $a \cos \varphi, b \sin \varphi$, the equations of the right lines $n_a, l_{a'}, P_1 M_1, P_2 M_2$ are successively:

$$n_a \dots \frac{ax}{\cos \varphi} - \frac{by}{\sin \varphi} = c^2, \quad l_{a'} \dots \frac{-2ax}{\cos \varphi} + \frac{2by}{\sin \varphi} = c^2,$$

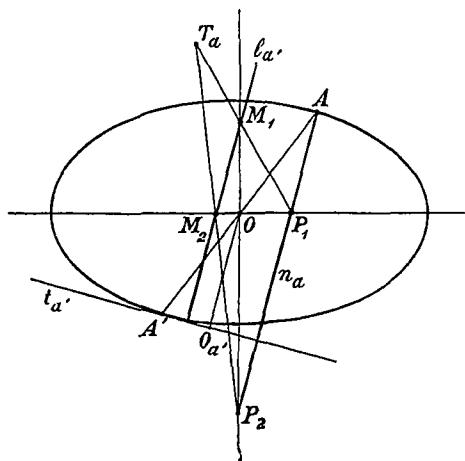
$$P_1 M_1 \dots \frac{ax}{\cos \varphi} + \frac{2by}{\sin \varphi} = c^2, \quad P_2 M_2 \dots \frac{-2ax}{\cos \varphi} - \frac{by}{\sin \varphi} = c^2.$$

So the centre of perspective T_a has the coordinates $-\frac{c^2}{a} \cos \varphi$,

$\frac{c^2}{b} \sin \varphi$ and this point describes the ellipse ε' , the four vertices of which are the four real cusps of the evolute of ε . And the joints $A T_a$ and $A' T_a$ likewise envelope ellipses, etc.

4. The simple relation between the lines n_a and $l_{a'}$ proves immediately that $l_{a'}$ is normal to the ellipse $\frac{4x^2}{a^2} + \frac{4y^2}{b^2} = 1$ and that through any point M pass four of these normals $l_{a'}$. In other words, any normal to the plane of the circles meets four of the rectangular hyperbolae and so contains eight points of the locus. As the point at infinity common to all these normals does not lie on one of the rectangular hyperbolae, the locus is a surface of the eighth order. We confirm this result by the deduction of its equation.

The cones that form the images of all the circles through A' and all the circles through Oa' are represented by



(4)

$$\left. \begin{aligned} (x - a \cos \varphi)^2 + (y - b \sin \varphi)^2 &= z^2 \\ \left(x - \frac{ab^2 \cos \varphi}{b^2 \cos^2 \varphi + a^2 \sin^2 \varphi} \right)^2 + \left(y - \frac{a^2 b \sin \varphi}{b^2 \cos^2 \varphi + a^2 \sin^2 \varphi} \right)^2 &= z^2 \end{aligned} \right\} .$$

So these equations represent together the rectangular hyperbola projecting itself in $l_{a'}$. Putting for simplicity's sake u^2 for $x^2 + y^2 - z^2$, these equations can be reduced to

$$\left. \begin{aligned} u^2 + b^2 + c^2 \cos^2 \varphi - 2ax \cos \varphi &= 2by \sin \varphi \\ u^2 (a^2 - c^2 \cos^2 \varphi) - 2ab^2 x \cos \varphi + a^2 b^2 &= 2a^2 by \sin \varphi \end{aligned} \right\} .$$

So elimination of $\sin \varphi$ gives for $\cos \varphi$ the relation

$$\cos \varphi [(u^2 + a^2) \cos \varphi - 2ax] = 0 ,$$

which breaks up into

$$\cos \varphi = 0 , \quad \cos \varphi = \frac{2ax}{u^2 + a^2} .$$

As φ is variable, the first condition cannot serve here. It corresponds in fact to this, that the two cones with the vertices A' and $O_{a'}$ coincide instead of determining together a rectangular hyperbola, when A' is one of the extremities of the minor axis of ε ; whilst a similar treatment, in which the parts of $\cos \varphi$ and $\sin \varphi$ are inverted, leads to the relation $\sin \varphi = 0$, corresponding in the same manner to the coincidence of these cones, if A' is one of the extremities of the major axis of ε . Substitution of the other value of $\cos \varphi$ in the first of the second pair of cone equations gives the result in the form

$$\frac{4a^2 x^2}{(u^2 + a^2)^2} + \frac{4b^2 y^2}{(u^2 + b^2)^2} = 1 (1),$$

which really represents a surface of the eighth order.

Inversely this simple equation shows that the surface represented by it may be generated by rectangular hyperbolae, by considering it as the result of the elimination of ψ between

$$\left. \begin{aligned} 2ax &= (u^2 + a^2) \cos \psi \\ 2by &= (u^2 + b^2) \sin \psi \end{aligned} \right\} ,$$

which represent for any constant value of ψ a rectangular hyperbola lying in the plane

$$\frac{2ax}{\cos \psi} - \frac{2by}{\sin \psi} = c^2 .$$

For $\psi = \varphi \pm 180^\circ$ this equation transforms itself into that of $l_{a'}$.

5. We signalize here some particularities of the found surface.

a. The intersection of the cone $x^2 + y^2 - z^2 = 0$ with the plane at infinity is a fourfold curve of the surface. For the substitution of

$$x = p + z \cos \lambda, \quad y = q + z \sin \lambda$$

into the equation 1) yields an equation of the fourth degree for z . It is reduced to a cubic equation under the condition

$$(p \cos \lambda + q \sin \lambda)^2 = a^2 \cos^2 \lambda + b^2 \sin^2 \lambda$$

and to a quadratic equation for

$$p \cos \lambda + q \sin \lambda = 0.$$

So the four tangent planes in the point $x = z \cos \lambda, y = z \sin \lambda$ at infinity are represented by

$$x \cos \lambda + y \sin \lambda - z = \pm \sqrt{a^2 \cos^2 \lambda + b^2 \sin^2 \lambda},$$

$$x \cos \lambda + y \sin \lambda - z = 0,$$

the last of these counting twice. The deduction of the envelopes of these planes for various values of λ shows that the surface 1) is touched at infinity by the developable surface or torse circumscribed to the tangential pencil of quadrics, to the four quadrics flattened to conics of which belong the fourfold conic of 1) and the ellipse ε ; moreover it is osculated at infinity by the cone $x^2 + y^2 - z^2 = 0$. So this cone intersects the surface 1) in a curve in space of the sixteenth order to which the fourfold conic of 1) belongs six times.

The completing curve in space lies on the cylinder $\frac{4x^2}{a^2} + \frac{4y^2}{b^2} = 1$, in which 1) is transformed for $u^2 = 0$; this cylinder meets the surface 1) in another curve of the twelfth order, etc.

b. The intersection of the surface 1) with each of the planes ZOX, ZOY consists of four straight lines and a rectangular hyperbola counting twice. So $y = 0$ yields the four lines $x \pm z = \pm a$ and the hyperbola $x^2 - z^2 + a^2 = 0$, and likewise $x = 0$ yields the four lines $y \pm z = \pm b$ and the hyperbola $y^2 - z^2 + b^2 = 0$. So 1) contains besides the fourfold conic at infinity still two double conics; moreover it bears eight right lines, viz. the four pairs of lines into which the rectangular hyperbolae of the vertices of ε are degenerated.

c. The surface contains four separate double points, the four vertices of ε . By transporting the axes of coordinates parallel to themselves to one of these points as origin and equalling the terms of the second order to zero the equation of the corresponding osculating cone is found.

d. The curve limiting the projection of 1) on the plane of the circles is obtained by elimination of z between 1) and its differential quotient according to z . It consists of the intersection of 1) with the plane of the circles and of the projection of a curve in space. The first is the locus of the octuples of points common to the corresponding pairs of circles of the two circle involutions

$$\begin{cases} (x \pm a \sec \lambda)^2 + y^2 = a^2 \tan^2 \lambda \\ x^2 + (y \pm b \cosec \lambda)^2 = b^2 \cot^2 \lambda \end{cases}$$

and as such a quadricircular octavic; the isolated double points in the vertices of ε excepted, all its points are imaginary. The second is found by the elimination of u^2 between 1) and its differential quotient according to u^2 , which, v being substituted for u^2 , comes to the elimination of v between

$$\frac{4 a^2 x^2}{(v + a^2)^2} + \frac{4 b^2 y^2}{(v + b^2)^2} = 1,$$

$$\frac{4 a^2 x^2}{v + a^2} + \frac{4 b^2 y^2}{v + b^2} = 2v + a^2 + b^2.$$

By solution we find

$$(v + a^2)^3 = 4 a^2 c^2 x^2, \quad (v + b^2)^3 = -4 b^2 c^2 y^2.$$

and by elimination of v

$$(2a x)^{\frac{2}{3}} + (2b y)^{\frac{2}{3}} = \frac{4}{c^3},$$

i.e. the evolute of the ellipse $\frac{4x^2}{a^2} + \frac{4y^2}{b^2} = 1$. This result was to

be foreseen. For the normal at the plane of the circles in a point of this evolute meets two immediately succeeding rectangular hyperbolae and is therefore tangent to the surface on either side of the plane $X O Y$. The curve of contact itself, of which this evolute is the projection, is of the twelfth order. The cylinder of the sixth

order, of which this evolute is a right section, meets the surface 1) moreover in a curve in space of the order 24.

6. The found image 1) can be useful in different researches about the system S of the circles of JOACHIMSTHAL. By determining the number of points common to this surface and a rectangular hyperbola, a parabola and a straight line, we find that the system S has the characterizing numbers 4,8,16; in other words, it contains four circles passing through two given points, eight circles passing through a given point and touching a given line, sixteen circles touching two given lines. In the same manner is proved that it contains sixteen circles touching two given circles, etc.

7. If we are given a parabola instead of an ellipse, all the circles passing through three conormal points pass also through the vertex of the parabola. Here the found surface of the eighth order is reduced to the right cone $x^2 + y^2 = z^2$, of which the vertex of the parabola $y^2 = 2px$ is the vertex. And the case of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $z = 0$ leads to the surface

$$\frac{4 a^2 x^2}{(u^2 + a^2)^2} - \frac{4 b^2 y^2}{(u^2 - b^2)^2} = 1$$

and is quite analogous to that of the ellipse.

Physics. — „On maxima and minima of apparent brightness resulting from optical illusion.” By Dr. C. H. WIND. (Communicated by Prof. H. HAGA).

1. If we see on a surface two zones of different (real) brightness united by a transition-zone whose brightness decreases continuously from the brighter down to the darker zone, this transitionzone seems to be separated from the brighter zone by a still brighter line (maximum of brightness) and from the darker zone by a still darker line (minimum of brightness).

2. This phenomenon, which — as will be seen from what follows — presents itself under very different kinds of conditions was first observed by me in a drawing carefully and successfully executed by Mr. VAN GRIEKEN, of the firm VAN DE WEYER at Groningen, by means of lithography. This drawing of which fig. 1 is a photographic reproduction (reduced to $1/4$ of its size), which unsatisfactory as it is, yet enables us to observe the phenomenon, consists of a great number of parallel lines of equal thickness drawn at intervals of 1 m.M. in two outer zones, at intervals of 0.4 m.M. in a middlezone,

at continuously increasing distances in the transition zones between the lastnamed and the two firstnamed (The law of increase of the distances has been chosen so that the proximity of the stripes in the transition zones varies as a linear function).

3. This phenomenon may be produced even more obviously and in a simpler way by means of revolving discs, as are often used in physiological optics. If we paste a piece of white paper of suitable shape on a black disc and cause the disc to turn very fast in its plane we may get on the surface any distribution of light in which the brightness of the observed plane differs only for points situated at different distances from the centre of rotation and so we may compare it with the corresponding distribution of apparent brightness. We can also photograph the discs first in rest, afterwards in rotation, and then the first photograph gives an auxiliary figure, from which the real distribution of light on the figure given by the second photograph may be known.

Fig. 2, 3 and 4 show (reduced to $\pm \frac{1}{6}$ of their real size) some of the discs used by me; the outlines of the white parts of the discs are partly radii pointing towards the centre of rotation, and partly arcs of spirals of Archimedes, for which, as is wellknown, the variation of the length of the radius-vector is proportional to that of its angle of rotation; at a rapid rotating of the discs represented by these figures we get in accordance with these curved outlines transition zones in which the apparent brightness varies as a linear function.

The maxima and minima of brightness on the limits of the transition zones as mentioned in 1., are clearly seen especially by direct observation of the discs while rotating, but also by examining the photographs taken of them. Fig. 5 and 6 are reproductions of such photographs; fig. 5, corresponding to fig. 3, shows pretty clearly the maxima and minima, alluded to above (being circles in this case); fig. 6, corresponding to fig. 4, shows them hardly at all. The reproductions of several of the photographs made by me proved so inadequate that I preferred not to have them inserted here at all; all those published, without an exception, show the phenomena in question far less distinctly than the originals¹⁾.

4. Now the question arises whether this optical illusion cannot be classed among phenomena already familiar to us. Although I dare not give a definite answer to this question I may be allowed to offer the following considerations.

¹⁾ Before the assembly Prof. HAGA has demonstrated the phenomena described by me, as well by means of revolving discs as by photographs taken of them.

It is well known that two areas of different brightness, either adjoining each other, or at some distance of each other standing out against the same background, when viewed in one glance, influence each other's apparent brightness by contrast, so that the brighter area causes the brightness of the darker area apparently to decrease and conversely. Several psychologists LEHMANN¹⁾, EBBINGHAUS²⁾, HESS and PRETORI³⁾, KIRSCHMANN⁴⁾ and others have made this phenomenon a subject of investigation and have brought to light some regularities in its occurring; the laws that govern this phenomenon have however not yet become completely known in spite of the important investigations of these scientists.

Now we should be inclined to suppose that the maxima and minima of brightness described above are to be reduced to the said effects of contrast. But then it would seem strange that there should be no intensifying whatever of the lines, whenever the transitionzone disappears between two areas of equal brightness; for we might expect that especially in this case there would be a prominent maximum on the one side of the border and a prominent minimum on the other side of it. But as it is, each of the two zones shows almost uniform brightness (fig. 7a, which shows the disc of fig. 7b while rotating) although it is not to be denied, that in both areas there can be detected, towards the limits a slight variation of brightness in the sense to be expected. Similar changes appear a little more distinct in a zone of uniform brightness bounded on the one side by a brighter zone, on the other side by a darker zone as represented in fig. 8a (corresponding to fig. 8b); but although in this case the contrasting influence of the neighbouring zones is more visible, yet the phenomenon observed shows quite another character than in the case of the presence of a transitionzone.

5. Presently we shall say a few words more about a possible explanation of the optical illusion; first however I'll give some further information about the conditions under which it appears and about the laws which seem to govern it:

10. In the case of a transitionzone with constant gradient of decreasing or increasing intensity the maximum and minimum appear exactly on the borders of the transitionzone; at any rate the devia-

¹⁾ LEHMANN, Wundt's Philosophische Studien 3, S. 497, 1886.

²⁾ EBBINGHAUS, Berl. Sitzber. 1887, S. 995

³⁾ HESS a. PRETORI, Von Graefe's Archiv f. Ophthalmologie 40, 4.

⁴⁾ KIRSCHMANN, Wundt's Philosophische Studien 6, S. 417, 1891.

tions, which might still exist, seem to fall within the limits of probable error in the determination of the place of those lines.

2^o. If we examine successively several drawings in which a transitionzone occurs and which differ only in this regard that the transitionzone gets narrower and narrower, we see the bright and the black line growing finer and finer without there being any continuous increase in their distinctness; on the contrary the distinctness diminishes when the transitionzones are very narrow and it seems as if the narrowing transitionzone were a zone of almost constant brightness, with sharply defined limits towards the other zones. (Besides it seems to me that there is a general tendency to take the middle part of a transitionzone in the above sense, for a zone of uniform intermediate illumination). This gradual fading away of the lines at a narrowing of the transitionzone may easily be seen by slightly extending (for instance $\frac{1}{2}$ cM.) the opening in the middle of the rotating disc (fig. 7b), by means of which the disc is mounted on the axis, in the direction of one of the radial borders of the white paper and then causing the disc to rotate, it being mounted on the axis first as excentrically as possible afterwards with gradually decreasing excentricity. Indeed we get in this way a gradual narrowing transitionzone and at the same time the experiment shows very clearly the just mentioned fading away of the lines.

3^o. Not only do the two lines appear when in the transitionzone the brightness varies in a direction perpendicular to the border of the zone with constant gradient, but also under quite different laws of intensity-variation as may be gathered from the fact that also the *photographs* of the rotating discs show the lines clearly; they are very prominent for instance when this gradient is infinite on the brighter border and becomes finite and diminishes gradually towards the other border, where it attains a minimum differing from zero.

4^o. The lines can also be very conspicuous when the gradient continuously approaches zero on the borders of the transitionzone.

5^o. Lastly the lines appear in some cases even where the zones of equal brightness on either side the transitionzone are replaced by zones of which the brightness gradually decreases (resp. increases) towards the transitionzone. It may easily be stated by means of a rotating disc that the brightness between the two transitionzones still may appear to have two maxima as in fig. 5, two minima as in fig. 6 the real brightness having even a faint maximum, resp. a faint minimum, in the middle between the two transitionzones.

6. To return to the explanation of the optical illusion described above, it seems not impossible to me that we should have to look for it in the influences by contrast as mentioned in 4., starting for instance from the hypothesis that each element of the field in general influences the observed brightness of any definite element under consideration, an influence depending in a definite way on the distance between the two elements and on the brightness both of the „inducing” and the „reacting” element. But then we are not allowed, as we might be inclined to do at first sight, to assume that this influence increases as continually the distance between the inducing and the reacting element diminishes and as the difference in brightness between these two elements increases. For the peculiarity of the appearing of the lines mentioned in 5. sub 2^o. would be incompatible with an influence acting in this manner. Moreover LEHMANN's¹⁾ investigations have already brought to light that the influence by contrast reaches its maximum at a definite value of the proportion between the brightness of the inducing and the reacting field. If we may as seems quite natural apply this law to the contrast between any two *elements* of the field and if at the same time further investigations might prove this „critical” proportion between inducing and reacting brightness to decrease as the distance between these two elements diminishes, it may be conceived that the optical illusion we have described and the ordinary effects of contrast will be found to obey the same laws. Accurate investigations will however be required²⁾, especially concerning the influence of the distance between two contrasting fields on the amount of the influence, before we can arrive at exactly formulating these laws.

7. Among the methods which may be used to produce the optical illusion described I may still mention a very simple one: we send a beam of light through a not too narrow slit, so that it falls on a second slit parallel with the first, and receive the beam on a screen. If the second slit is *wider* than the first, a middle zone on the screen will be illuminated by all the elements of the first slit; on either side this middle zone transition-zones will be found illuminated by continuously decreasing parts of the first slit, and these zones will pass into other zones not at all illuminated by light through the slit.

¹⁾ LEHMAN l. c. S. 525.

²⁾ KIRSCHMANN announces in his treatise alluded to above the publication of his investigations already partly made on this influence of distance; but the publication seems not to have taken place yet.

The image on the screen will show very beautifully the bright and dark lines mentioned in 1. On photographs taken of this image, these lines are very clearly visible too; fig. 9, a reproduction of such a photograph also shows them, but much less well-defined.

If the second slit is sufficiently *narrower* than the first, the image on the screen will not show a part illuminated by the whole of the first slit, yet there will be again a middle band of maximum illumination across its whole extent; and for the rest the image on the screen will show as a whole the same characteristics as in the preceding case as to both real and apparent brightness. If the second slit is gradually narrowing towards the lower part we see an image projected on the screen in which two straight bright lines appear, these lines, parallel with the edges of the second slit, intersecting somewhere, but remaining clearly visible beyond the point of intersection.

If we cause the light emitted from the first slit to cast a shadow of a thin needle or thread, we get a silhouette with a middle zone having over the whole of its breadth uniform minimum brightness bounded by transition zones of uniformly increasing brightness which on their outsides again are bounded by areas of uniform maximum illumination. The maxima, and especially the minima alluded to in 1. are again very clearly visible here, even to such a degree that in some cases the appearing of the minima might lead one to speak of a doubling of the shadow cast by the thread.

In all these cases, if only the slits are wide enough, diffraction plays no perceptible part.

8. If we illuminate the first slit of 7. by a *X-ray* tube instead of by ordinary light, and if the rays are not caught on a screen but on a sensitive plate we get on developing, negatives of which the positives are exactly similar to the images described in 7. Fig. 10 shows a reproduction of such a positive, corresponding to a similar case as fig. 9.

Fig. 10 moreover shows a white rectangle, which covers part of the drawing. This effect was obtained by covering part of the negative, of which it is a reproduction, with a slip of paper during the copying. I did so in order to point out that the disappearing of the transition zone — which was effected on the spot in question, at least for the greater part, by this slip of paper — is sufficient to cause the line corresponding to it to vanish. Indeed the bright line which in the other parts of the image is still visible, however much it has lost of its clearness by repeated processes of reproduction is no longer to be traced where the transitionzone has been covered, which sufficiently proves the fictitious nature

Fig. 1.

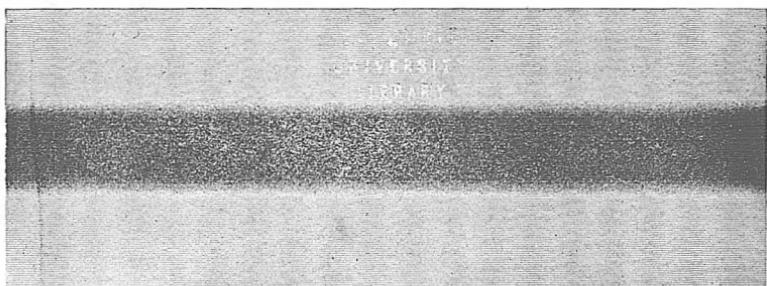


Fig. 2.

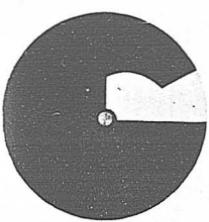


Fig. 3.

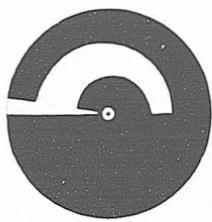


Fig. 4.

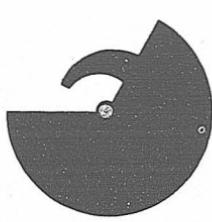


Fig. 5.

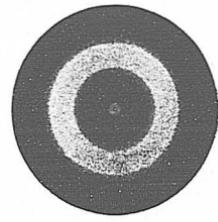


Fig. 6.

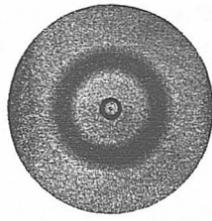


Fig. 7a.

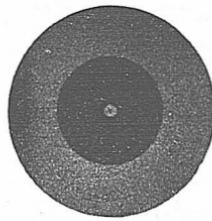


Fig. 7b.

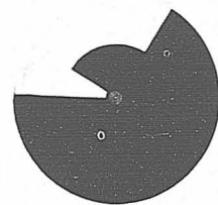


Fig. 8a.



Fig. 9.

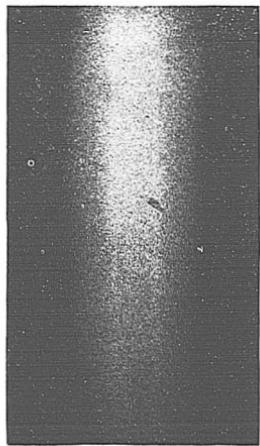


Fig. 10.

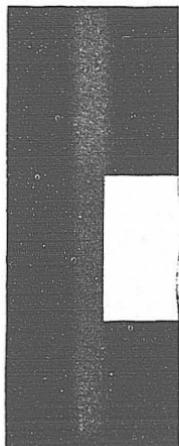
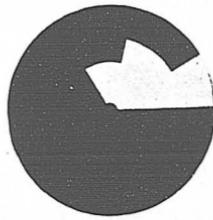


Fig. 8 b.



of the line. The same experiment may be made in the case of fig. 9 by covering somewhere the transitionzone (for the greater part) with a sharply outlined piece of paper. That the disappearing of the line is really due to the disappearing of the transition-zone and not to ordinary contrast in consequence of the covering piece of paper will appear if we take first a dark piece of paper and afterwards a light one, both producing the same effect.

9. Although the optical illusion described above may be produced especially by the methods given here, yet it may be often observed in a simpler way. In nearly all cases where we have an umbra passing into a penumbra we may observe a dark and a bright line as borders of the penumbra, and the optical illusion appears in all the various forms described above and even more.

10. Attention has been drawn to the appearing of bright and dark lines conjointly with a penumbra, as described in 7 and 8 by SAGNAC (*Journal de Physique VI* p. 169, 1897). Formerly I did not conclude from what he said on this subject that he took those lines for the result of optical illusion only, although now it seems to me, if not probable, yet possible that this has been the case. That I feel justified to communicate my observations on this subject somewhat extensively is owing to the great importance that, in my opinion, must be attached to this optical illusion.

In the first place it has deluded many a physicist, who thought that he observed diffraction lines or other important lines in cases where now it is certain that no real maxima or minima of brightness of any importance existed; so it has especially caused bright and dark lines to be observed in X-rays-shadows the true nature of which some have tried to explain in various ways, but no one had yet sufficiently accounted for. (Another time I hope to return to this subject).

In the second place it seems to me that this optical illusion may under special conditions lead to the observation of the doubling of bright or dark lines or bands where in reality there is nothing but a broadening of these lines or bands together with the borders growing less sharply defined (compare what has been said in 5. sub 5^o), a thing that may occur f. i. when the optical system we use in observing is not accurately adjusted (resp. accommodated). It may be that in some cases this optical illusion must be held responsible for doublings which have been observed and which have not yet been explained in the right manner or not yet been explained at all.

In the third place the optical illusion may lead to errors in the estimation of the place of the maxima and minima of brightness

in a system of bright and dark lines, as soon as the brightness in the neighbourhood of the lines is not perfectly symmetrical with respect to their centres.

At any rate we may conclude from what preceeds that we cannot be too critical while observing maxima and minima of brightness, and that in many cases we shall even have to convince ourselves of the existence or non-existence of real maxima and minima of brightness corresponding to the observed maxima and minima.

Bacteriology. — *On the relation of the obligatous anaerobics to free oxygen.* By Prof. M. W. BEIJERINCK.

The relation of the living cell to free oxygen is best to be judged from the influence of this gas on the *growth* and on the *mobility*. Of course, only the first method is of universal application.

As to the mobile microbes, some time ago I gave the name of „figures of respiration“¹⁾ to the peculiar groupings, which originate in preparations destined for the microscope, in consequence of the access of oxygen only along the edge of the examined drop under the cover-glass, the microbes being thereby enabled to seek that quantity of oxygen which is best adapted to their respiration. Three types may here be distinguished according as the microbes seek the highest tension of the oxygen along the edge, a middle tension at some distance of it, or the smallest tension in the centre of the preparation. These types I called the aërobic, the spirillous and the anaërobic type.

Further experience has shown that the anaërobic type, characterised by the accumulation of the moving microbes at that spot of the preparation where the oxygen tension is minimum, — commonly near the centre, — does not exist as a special type, but becomes visible only under particular circumstances, and further, that when the aëration of the preparation is sufficiently small, all anaërobics, examined till now, appear to belong to the spirillous type, that is to say, they not only don't fly those places in the preparation, where a small oxygen tension still exists, but they even seek them.

This tension, beneficent for the anaërobics, is however very slight, whence follows, that by using only a moderate number of microbes, consuming but very little oxygen, there may enter at the edge more oxygen than is wanted. In such a case the tension, most approaching

¹⁾ Centralblatt für Bacteriologie Bd. 14 pag. 837, 1893.

the required optimum, will be found in the centre. The accumulation of the microbes will then also be localised to the centre, causing the semblance of an anaërobic type as a special case. It is clear, that if this is the right explication, the true representatives of the second type, viz. the spirilli, must under certain conditions also accumulate at the centre, namely then, when all the spirilli together cannot absorb the total quantity of oxygen entering along the edge of the preparation, and this is indeed easily to be observed, by using a small number of spirilli and a large coverglass.

So, there is no sufficient reason to divide the mobile bacteria into three types according to their relation to free oxygen, as I formerly did, but only into two. It also seems to me that the names for the types, already mentioned, are not quite applicable, and that it is preferable to call *aërophilous* all organisms which seek the highest oxygen tension¹⁾, and *microaërophilous* those which require a lower tension. To this latter group then, belong the obligatous anaërobics as far as now observed, and the aërobic spirilli with regard to their mobility.

I am obliged here to speak of „aërobic spirilli”, as I have formerly shown that there also exists an obligatous anaërobic spiril, namely the organism of the reduction of sulphates, *Spirillum desulfuricans*. Though this kind is very mobile, yet the growth is so slow, that I have not succeeded in collecting a sufficient number of individuals to get distinct figures of respiration, — a difficulty which exists also more or less with other obligatous anaërobics.

The conviction that free oxygen is beneficial to all that lives, and in the long run probably even necessary, is based on the relation of the *growth* of the obligatous anaërobics to this gas, and here the mobile forms as well as those which are not mobile may enter into consideration. Before however describing the experiments which seem decisive, I must fix the attention on the following circumstance.

For alcohol yeast and the other facultative anaërobics, it must be admitted, that the possibility of their temporary anaërobiosis, is determined by the presence of a *provision of oxygen* in loose combination with the living matter of the cell itself, by which combination some cell divisions are rendered possible without the supply of new oxygen. Consequently there must be a difference between aërated and not aërated cells.

¹⁾ Here is only question of experiments in common air, not in pure oxygen.

If it is accepted that this relation holds also good for the obligatous anaërobics, then it is to be expected that their provision of oxygen will be much smaller than that in the yeastcell, so that it is necessary to take much more efficacious measures to render the influence of the oxygen visible in the former than in the latter case. It is therefore desirable, in some cases even necessary, to use for the experiments materials, taken from such cultures as have long been continued in absence of air, by which the provision of oxygen has been lessened. So far as I am now able to judge, strongly aërated anaërobics are, as to their growth, aërophobic, i. e. they grow best there where the oxygen tension is minimum or zero. As contact with air is in itself not sufficient to cause aération, — spores for instance seem less fit to be aërated than vegetative cells, — there now and then occur strange incidents which make the experiments troublesome.

The way in which I arranged my growth experiments is as follows.

Material of the species to be examined, is introduced, in a not aërated condition, and if possible, in the state of spores, into the culture mass still in fusion, in such a quantity, that the germs, developed into colonies, may render that culture mass, after solidification, rather opaque.

If such a culture mass, from which the free oxygen is completely withdrawn, is contained in a deep experiment-tube, where the air can only find access from above, then, if the growth is favoured by a certain oxygen tension, there must result at the very place where this tension becomes optimum, an opaque and distinctly visible *niveau* of colonies, which are greater than the colonies beneath and above this niveau.

The easiest way for completely removing the oxygen, is to sow simultaneously an aërophilous species, not acting injuriously on the development nor disturbing the observation of the anaërobic. Such an aërobic must have the following qualities: The oxygen must be completely absorbed, without exciting so much growth in the surface of the culture mass, that the colonies of the anaërobic become indistinct. Besides, an easy recognition in the microscopic preparation, and a simple separation of the aërobic and the anaërobic must be possible.

In trying various species of microbes, I found some kinds of yeast to be most efficient for the research of the anaërobics of putrefaction and of sulphate reduction, as for these processes no carbohydrates are essential, in which case yeast does not grow strongly, whilst it is distinctly recognisable under the microscope. Besides,

yeast can easily be separated from the anaërobics of proteine putrefaction, because it dies at 50° à 60° C., whilst the spores of the latter can be heated to 90° à 100° C. without dying. For the examination of those anaërobics which require sugar in their food, as for instance the butyric ferments, it is preferable, for oxygen absorption, to make use of certain blastomycetes (which grow and reproduce like yeast, whilst alkohol fermentation is absent) or aërobic bacteria, which don't produce acid, nor liquefy gelatine. Good results were obtained with a red blastomycete, isolated from garden-soil, and with *Bacillus fluorescens non liquefaciens*.

It is good (but not always necessary), to place the prepared experiment-tubes, in an exsiccator which is vacuated. For this vacuation a KÖRTING-waterjetpump with manometer will suffice, by which at the same time the pression of the gas used may be measured.

Another very suitable method to state the influence of oxygen on the growth, is to cultivate in the „humid room” on the object-bearer under the cover-glass, in a not too small quantity of the nutritious liquid, but in such a way as to keep the preparation thin enough for the microscope. In this way it is possible to observe, in the same preparation, first the figure of respiration and afterwards the growth.

The species of obligatous anaërobics which I have examined are the following.

Butyricferment (Granulobacter saccharobutyricum). This anaërobic is extremely common in garden-soil. Fit material for figures of respiration is to be obtained as follows. Water with some kalium phosphate and magnesium sulphate and 5 or 10 pCt. glucose is boiled in a little flask with so much fibrine that a thick paste is formed. During the boiling an infection with garden-soil is practised, in which only spores of bacteria remain alive. In the thermostat a vegetation of aërobics develops first, which, by the absorption of the oxygen, introduces butyric fermentation. Sometimes this fermentation will follow, even in absence of aërobics, i. e. notwithstanding the entrance of air into the mass of fibrine, thus showing that some aération is certainly no bar to this process. If perhaps an aërobic grows too strongly, reinfection in another flask with the same mixture, will suffice to make it disappear and at length still to obtain an almost pure butyric fermentation. If in the infection material there are too few spores, as well of the butyric ferment as of aërobics, some aërobic bactery, or a blastomycete must be purposely added for the absorption of the oxygen.

In this way a culture is obtained containing only the „oxygen-

form" of butyric ferment, i. e. only mobile staves and no clostridia. With them a figure of respiration may be produced consisting of a single fine line of quickly moving little staves, situated at some distance from the edge of the cover-glass and the meniscus of the preparation, which places the microaërophily of this ferment beyond all doubt.

If to the fermenting mass pure calcium carbonate is added, by which the acid is neutralised, the growth of the bacteria becomes much stronger, and the staves give place to clostridia rich in granulose, and which at length produce spores. Although the opaqueness caused by the chalk, spoils in some degree the purity of the figures of respiration, yet the experiment succeeds well enough, and leads to the same result, i. e. proves that the clostridia of the butyric ferment are microaërophilous in the same way as the oxygenform. With boiled milk,¹⁾ in which a spontaneous butyric fermentation had originated, the above observations could equally be made. That the same may be said with regard to the *butylic ferment* (*Granulobacter butylicum*) formerly described by me²⁾, I need hardly add here, as it was the continued study of this very ferment, which rendered the here described relations clearer to me.

Anaerobics of putrefaction of proteids. The most striking instances of obligatous anaerobics are met with in the putrefaction of pepton, or, generally speaking, of proteine substances. If one wishes to isolate the microbes concerned, efficacious measures must be taken for the exclusion of oxygen, as the quantity of air which these ferments admit, without their growth being impaired, is certainly much smaller than with the butyric and butylic ferments. Hence here in particular it was of importance to study their relation to oxygen.

Before entering upon my immediate subject, I think it necessary to say something about the different species concerned in the process of putrefaction, the literature on this subject being quite unsatisfactory.

Bacillus putrificus coli BIENSTOCK³⁾ is an anaerobic, found back by nobody, so that it cannot be typical for the putrefaction of proteids. Besides, an exact microscopic examination shows that more

¹⁾ Milk, boiled in a not too undepth flask, will sometimes get into butyric fermentation, even with free entrance of air, without the presence of aerobics.

²⁾ Archives Néerlandaises T. 29, pg. 1. As this ferment produces much more propylalkohol than butylicalkohol it would have been better to call it *Granulobacter propylicum*.

³⁾ Zeitschrift fur klinische Medicin, Bd. 8. pag. 1. 1884.

than one species must here be active. That however, the number of typical bacteria should be very great, I think doubtful, for the following reasons: The course of the process of putrefaction is quite the same when the material, after infection with soil, is for some moments heated to 90 à 100° C. as when this is not done. Hence it follows that only spore-forming microbes are typical for the process. The experiment shows further that exclusion of air acts favorably on its course, so that all aerobic microbes appear to be indifferent, except in so far as by absorption of oxygen, they favor the development of the properly so-called putrefaction bacteria.

By these two data the process was so much simplified from a bacteriological point of view, that there appeared some chance of further unravelling it. Though hitherto I have by no means entirely succeeded, I think, nevertheless, that what follows holds good.

Three species of obligatous anaerobics are in particular concerned with the putrefaction of proteids. In the first place *Bacillus septicus*, secondly a group of extremely variable forms, related to the tetanus-bacillus, and to which I will give the name of „skatol-bacteria”, and thirdly, an immobile, well-characterised species, called by me *B. pseudopulcher*. For separating these different species, I used a culture gelatine of the following composition: Destilled water, 10 pCt. gelatine, 3 pCt. pepton siccum, 0,05 pCt. dinatrium phosphate, 0,05 pCt. magnesium sulphate, using at the same time yeast or a blastomycete for withdrawing the oxygen. When put into a deep experiment-tube, the anaerobics develop even with free entrance of air.

B. septicus PASTEUR, is, according to my experience, one of the most spread species of bacteria, to be found wherever animal substances are tainting, and very common in dust and in the soil. It is an easily recognisable and well defined species. A virulent form of it goes in the German literature by the name of *B. oedematis maligni*¹⁾. Material of the latter, occurring in the laboratoria, I compared, also with a view to their relation to oxygen, with cultures of *B. septicus*, repeatedly isolated by me from putrefying albumen solutions, or fibrine, infected with garden-soil, but I could discover no difference.

The skatol-bacteria are to be known by the globular spores which are found in proteine putrefactions, in the swollen ends of thin, commonly long staves. One of the forms isolated retained

¹⁾ A bacteria, accepted by the medical men as a particular species, *B. Chauveani* (of the French) or *B. emphysematis* (of the Germans) is, in my opinion, only a variety of it.

at first, even in the pure cultures, globular spores, whilst in other isolations the spore-form proved not to be constant. The dimensions of spores and staves are most variable. Motion, if present, is slow, in pure cultures sometimes absent. Glucose, added to the above mixture, gives rise to the production of gas. The colonies cause the culture gelatine more to weaken than to liquefy; they are sometimes colorless, commonly, however, surrounded with a brownish aureole. The study of this species is difficult on account of the great variability in form and functions, which renders the experiments doubtful and often suggests infection with allied forms, to which their common occurrence gives particular cause.

While skatolbacteria never fail in putrefying substances, *B. septicus* may be absent and its place be taken by *B. pseudopulcher*. This name was chosen on account of its resemblance to a common earth-bactery, related to *B. megatherium*, and which I baptised *B. pulcher*¹⁾. Motion is never observed in *pseudopulcher*; the spores are oblong, larger than in *B. septicus*, frequently to be found in long rows within the threads, generally, however, they occur in short staves. The colonies, which liquefy strongly, have a smooth surface, by which they may be easily distinguished from *B. septicus*. They are characterised by a heavy sediment, consisting of staves and spores. This sediment is different, or wanting in *B. septicus*. The pure cultures develop gases but not many stinking products. There is often a distinct smell of cheese to be observed. The study of this bactery is still imperfect and I mention it only because it might be taken for *B. septicus*.

For the object I have in view, I studied in particular *B. septicus*, while I think, that there is not one indication in bacteriological literature which suggests any benificial effect of free oxygen on the functions of this bacillus. For the skatolbacteria on the other hand, such indications exist. It seems at least according to some authors, that certain varieties of the nearly allied and commonly obligatous anaërobic tetanus-bacillus, are suspect to change into aërobics, a transformation which I witnessed myself by other varieties isolated from putrefying albumen. Moreover *B. septicus* is a „bona species”, i. e. recognisable for everybody.

B. septicus is exceedingly mobile and generally consists of staves, covered all over with ciliae. Spores grow easily, especially when there is contact with air. They are more oblong than globular,

¹⁾ At present in trade by the name of „alinit.”

mostly enclosed in the somewhat swollen ends of the staves, and surrounded by a hollow space. Though this bacillus is evidently polarically constructed, it moves spore-end or tail-end forward, and may suddenly change the direction of the movement. When a little air accedes, the staves may grow out into long threads and the motion ceases.

With a total withdrawal of oxygen there is a disposition for the formation of clostridia, but without a marked difference between an „oxygenform” and a „clostridiumform” as found in *Granulobacter butylicum*.

If the nutrient matter is merely albumen or pepton, gases are produced, whose quantity increases more or less by the addition of glucose.

Fibrine and proteids produce volatile sulphides, sometimes in great profusion; production of merkaptan, too, is observed under unknown circumstances. The colonies liquefy the gelatine of the above composition; their surface is quite characteristically pointed, evidently because many small shoots pierce slightly into the gelatine, before the melting sets in. This may be compared with the behaviour of anthrax, where, however, there is no melting at all. Commonly whether spores or vegetative cells are sown out, only few germs develop, which proves, that the nutrient matter itself, — even the best I could procure, — acts in a high degree as a „bactericid” in relation to *B. septicus*. The growth is slow, even at brooding temperature, when compared to allied aërobics.

Concerning the necessity of oxygen for *B. septicus* and the skatol-bacteria, I could state what follows.

B. septicus I observed as well with regard to the figures of respiration, as to the growth. In both ways the microaërophily could with certainty be stated. As this bacillus is extremely mobile, and as the spores render the swarms of bacteria very opaque, the study of the figures of respiration offers no difficulty.

A small number of bacteria accumulating in the centre of the preparation, produce the impression of aërophobia. If on the contrary, the bacteria are very numerous, a circular accumulation is formed at some distance from the edge of the cover-glass and the meniskus, pointing out the place where the tension of oxygen is optimum. If we examine the inner field, i. e. the part surrounded by the accumulation and totally deprived of oxygen, there, too, all is in motion. This motion is however much slower, more staggering and uncertain, than in the accumulation itself. I think that this inner part is continually supplied with individuals from the accumulation, which individuals, after some time return to the latter, to

take in a new provision of oxygen. Outside the accumulation, near the edge of the cover-glass, where the pressure of oxygen increases, the number of bacteria diminishes quickly, together with the mobility of the remaining ones. At the edge itself all is in complete rest, and no motion sets in when the surrounding is freed from oxygen. Still I have no reason to consider the resting individuals as dead; I even think they function as an "oxygen filtrum", thus protecting the more inwardly swarming.

If some grains of fibrine are introduced into preparations of which the figures of respiration are being studied, and if placed at c. a. 25° C. in a "humid room", a considerable increase of bacteria may readily occur. Watching the process micros- and macroscopically, we find the growth almost exclusively limited to the accumulation parallel to the edge, which accumulation grows more and more dense by the increase of the spores, whilst the central part continues as clear as at first. Consequently, it may be taken for granted that *B. septicus* requires oxygen for its growth, as well as for its mobility.

On this occasion I wish to correct a mistake in my description of *Spirillum desulfuricans*. I there stated erroneously¹⁾ that *Spirillum tenue*, which is typical for microaërophily as to mobility, is also microaërophilous with regard to growth, so that, when sown in a fit culture mass, it shows its maximum growth, not at the surface, but at a certain distance below it.

This has proved to repose on "trophotropy", signifying that growth is more favored by the influence of the food than by the oxygen. It occurs only when a bad culture ground is taken for the experiment, and it is by the aërophily of the growing spirilli that it must be explained. For the intense growth will cause at the surface a rapid exhaustion, so that, if no abundance of food is present, and if the food can only come up slowly from the depth by the process of diffusion to the place of consumption, then, not the surface itself, but a deeper layer will, under the joined action of oxygen and food, be most favorably situated for growth and increase. Thus in fact, *Spirillum tenue* is aërophilous as to growth and microaërophilous as to mobility.

Beside to this peculiar form of "trophotropy" in the growth, one has to pay attention, when studying the figures of respiration, to a phenomenon of almost the same nature with respect to the mobility, and which may be called "trophotaxis". It consists in the accumu-

¹⁾ Archives Néerlandaises, T. 29, pag. 272.

lation of the mobile microbes, which are more attracted by the food than by the oxygen, not in the meniscus and at the edge of the preparation, but at some distance. I observed this in an aërobic species, which I have called *Bacillus perlibratus*, where trophotaxis may become so strong, that microaërophily is mimicked, and was erroneously described by me as such¹⁾.

With abundant food, however, nothing is to be seen of these phenomena, so that by attentive observation microaërophily may always distinctly be recognised.

I now return to the anaërobies of the putrefaction of proteids, and in particular to the second important form, the skatolbacteria. Of this polymorphous form I examined, as already said, material closely allied to the *tetanus-bacillus*, which material is strictly anaërobic, and perhaps ought to be considered as the most characteristic for the process of putrefaction in general. I isolated various varieties and by means of growth experiments I was enabled to state microaërophily. The mobility of my varieties was too insignificant to be of use for the production of figures of respiration. When using the above mentioned pepton gelatine as nutrient matter and *Saccharomyces apiculatus* for the absorption of oxygen, most convincing „niveaus”, of a light brown color and with much growth, originate in deep experiment-tubes at a certain distance from the surface, in the surface itself the transparent clear colonies of the *apiculatus*-yeast develop vigorously, the skatolbacteria not being able to develop there.

The spore-formation seems here also to be favored by a small quantity of oxygen. Certain it is that spores are the most profusely formed in the niveau, and as their production goes parallel, first to the weakening and then to the complete liquefying of the gelatine, it is clear that also the latter process must begin in the niveau, to become only much later perceptible in the depth, and without reaching the surface at all.

I wish to terminate this survey of the obligatous anaërobics, studied by me, with the statement that the existence of microaërophily could also be proved for *Spirillum desulfuricans* by means of growth experiments.

This species is, in opposition to *S. tenuis*, strictly anaërobic and belongs morphologically to quite another group than the butyric-ferments and the bacteria of putrefaction, which is clearly demon-

¹⁾ Centralblatt für Bacteriologie, Bd. 14, pag. 839, 1893.

strated as well by the vibrio- or spiril-form, as by the absence of spores ¹⁾.

If sown in pepton gelatine, with MOHR's salt and an aërobic bacterium (*B. termo*) for the absorption of oxygen, in deep experiment-tubes, the microaërophily becomes visible by a black „niveau” of ferrosulfid, first formed at some distance beneath the surface and thence, only slowly, growing towards the depth and upward. When microscopically examined this niveau proves to be richest in reducing spirilli, so that evidently not the function of the sulphate reduction as such, but the growth of the *Spirillum*, active in this process, is furthered by a low oxygen tension.

It is fit here to make a few remarks concerning the relation of facultative anaerobics to free oxygen. By far the greater part of facultatives is aërophilous. I mention for instance *Mucor racemosus*, all alcohol yeasts, *Bacterium coli commune*, *B. lactis aërogenes*, *Granulobacter polymyxa*, *Bacillus tuberculosis*, *B. prodigiosus*. If the production of figures of respiration is possible, then the width of the moving bacteria zone is very great, even in dense swarms, which indicates a slow consumption of oxygen. This is especially the case with the fermenting species, as *coli* and *aërogenes*, and sometimes, too, with not fermenting species, such as *Bacillus tuberculosis*²⁾.

Microaërophilous are among the facultatives, so far as I think I can assert now, only the true lactic ferments, which may be brought to two groups of which the most important representatives are: *Bacterium lactis* (of buttermilk), and *Bacillus longus* (of cheese and of the yeast industry).

As these forms have no motion and produce little living matter in growing, the total quantity of absorbed oxygen is very small, whence the experiments are difficult and subject to doubt. If sown, however, in a proper solid culture mass, to which calcium carbonate is added, in a deep experiment-tube, it may be observed that, under favorable circumstances, at a certain depth below the surface, the formation of acid is the most vigorous. This is caused by the existence of a niveau, very rich in bacteria if compared with deeper layers where less, as well as with those nearer to the surface, where more oxygen is present. After some time, however, so many colonies originate, as well at the surface as in the depth,

¹⁾ As I think the only well-described instance of a spore-free obligatous anaerobic.

²⁾ The mobility of *Bacillus tuberculosis* has first been seen by Mr. MAC GILLAVRY. The figures of respiration are troublesome to obtain and only with quite young cultures, as for instance of flesh bouillon agar, not older than 24 hours.

that the microaërophily grows indistinct, without changing into aërophily.

Recapitulating, and adding some instances not yet mentioned, I come to this conclusion :

Aërophilous are: All aërobic bacteria (except some spirilli), most facultative anaërobics, probably all cells of higher animals and plants, most infusoria.

Microaërophilous are: The few hitherto examined obligatous anaërobics, to which belong also the chromatia and other sulphur-bacteria, and *Spirillum desulfuricans*. Of the facultatives probably all lactic fermenters, besides some (perhaps many) species of monads, and some infusoria.

Aërophilous with regard to growth, microaërophilous with regard to motion are: Some true spirilli, perhaps also some monads.

Though nobody will be surprised that, in reason of the above observations, I believe that all living organisms known at present, require free oxygen for their existence, I am far from pretending to have furnished the entire proof for that belief. It may even be asked whether there is cause to speak of „want” of free oxygen, and if „use of it if accessible” were not more adequate.

With regard to the examined obligatous anaërobics I have only shown that an extremely small quantity of free oxygen is propitious to their growth and mobility, but not yet that in the long run they would perish without it¹⁾.

I must however insist on this being positively the case with the aërophilous facultative anaërobics, such as alcohol fermenters, *B. coli commune*, etc. If these are prevented from laying in a „provision” of oxygen, on which to live when this gas fails, the growth soon ceases and, even with the best food, life too²⁾. This fact is very singular, for the extremely small quantities here concerned, are nothing as to the development of energy.

Consequently it is not clear why the combined oxygen, which abounds in the food, cannot here fill the place of free oxygen. With

¹⁾ Experiments in this latter direction have not yet given any sure results and have only proved that, with apparently efficacious precautions, anaërobiosis without access of air can long go on. So I could, without supply of air, make seven butylic fermentations go on successively, but at the seventh there arose some doubt whether the bacteria had varied or that an infection from without with butyric ferment had occurred.

²⁾ For this reason I formerly proposed to call these organisms „temporary anaërobics”, but now that I am more and more convinced that also the „obligates” can exist only temporarily without free oxygen, I no more attach much value to that term.

the unknown signification of the latter it is, to be sure, quite uncertain whether there must exist a minimum limit beneath which the possibility of life is totally excluded; but as this limit does certainly exist for the facultatives, one is by analogy inclined to accept its existence everywhere, consequently for the obligatous anaerobics, too. That is, for them also, to recognise free oxygen as a necessity for existence.

This opinion has the more weight now that it has been proved how easily may be shown that they not only *use* free oxygen but if possible, *seek* it and that it may promote even such important functions as growth and mobility.

Without doubt, this points to something more than „use”, albeit the term „want” goes perhaps too far. As it is however a fact that the obligatous anaerobics can produce thousands of new generations without a renewed contact with free oxygen, the hypothesis demands the acceptance of a peculiar exciting action of the free oxygen, stored up as a provision in the body of the bacteria.

This action cannot be compared to that of kalium, or of magnesium, or of the other elements necessary for life in small quantities. In the first place, because the latter must be present in quantities of another order, quantities gigantic compared to that of the oxygen provision; secondly and especially, because these elements may be withdrawn from the most different chemical compounds. The very necessity of the oxygen being free, causes the difficulty of giving a definite representation of its function. Some light would go up if it could be proved, that in the food a loosely bound form of oxygen might occur, accessible to the anaerobics, and PASTEUR has indeed supposed that the oxygen, which is found in beer wort, and cannot be separated from it by pumping or boiling, makes the anaerobiosis of beer yeast possible.

Facts are however not in accordance with this hypothesis. Now, as in the case of beer yeast and the other facultative anaerobics, we are obliged to admit the existence of a store of free oxygen in the cell itself, which, in a way hitherto unexplained, makes a temporary anaerobiosis possible, analogy, supported by the observations here described, leads to the same conclusion for the obligatous anaerobics.

Physiology. — On the influence of solutions of salts on the volume of animalcells, being at the same time a contribution to our knowledge of their structure. By Dr. H. J. HAMBURGER.

(Will be published in the Proceedings of the next meeting).

Physics. — „*On an asymmetry in the change of the spectral lines of iron, radiating in a magnetic field*”. By Dr. P. ZEEMAN.
(Will be published in the Proceedings of the next meeting).

Physics. — *The HALL-effect in electrolytes*. By Dr. E. VAN EVERDINGEN JR. (Communication N°. 41 from the Physical Laboratory at Leiden, by Prof. H. KAMERLINGH ONNES).

1. The researches on the HALL-effect and the increase of resistance in the magnetic field for bismuth, communicated to the Academy in the Meetings of 30 May 1896, 21 April and 26 June 1897¹⁾, and afterwards treated more at large in my dissertation, induced me to put the question, whether these phenomena may justify a choice among various theories about the nature of the electric current and the resistance of metals. A first step towards answering this question was the deduction of a formula for the HALL-effect in electrolytes, with the aid of simplifying suppositions. Indeed it is generally assumed that in electrolytes the electric current consists in a convection of charges by the ions; the velocities of this motion are known in many cases, hence all the data for the calculation are present. This research, already begun in Chapter VIII of my dissertation, being concluded for the present, I wish here briefly to communicate the results.

2. Several physicists have tried to observe the HALL effect in liquids. They succeeded indeed in observing differences of potential, caused by a magnetic field in solutions of sulphate of zinc and copper which were traversed by currents, and changing their signs on the reversal of the magnetic field or of the current. Whereas however in most metals the HALL-effect is proportional as well to the strength of the current as to the field, and in all metals the difference of potential appears immediately on closing the magnetizing current, in the liquids this difference increases more than the current and less than the magnetic field, and after the applying of the field it grows slowly towards a maximum. Chiefly on account of the last named fact ROTTI²⁾, FLORIO³⁾ and CHIAVASSA⁴⁾ refused to acknowledge the differences of potential observed also by themselves as

¹⁾ Communications from the Phys. Lab at the Univ of Leiden. N°. 26, 37 and 40.

²⁾ Atti della R. Acc. dei Lincei 12 p 397, 1882; Journ. de Phys. 1883.

³⁾ Il nuovo Cimento, Ser. 4, T. 4, p. 106, 1896.

⁴⁾ Elettricista 6, 1897.

proofs of the existence of the HALL-effect in liquids, and attributed those differences rather to ponderomotive forces, exerted by the magnetic force on the particles of salt, to differences of concentration or to differences of temperature. The two physicists first mentioned confined themselves to experiments, in which disturbances of this kind were avoided, and the HALL-effect had really disappeared; CHIAVASSA however also demonstrated the existence of differences of temperature and of concentration, determined the influence of these differences on the readings of the electrometer, and proved that in a non homogeneous magnetic field vortical motions occur in the liquid, which depend on the strength of the current, the magnetic field etc. in the same manner as the observed apparent HALL-effect.

BAGARD¹⁾ on the contrary believed to have avoided all disturbances in his experiments and was therefore convinced of the reality of the HALL-effect in liquids.

The observations of CHIAVASSA were made in the same manner as those of BAGARD; hence the greater part of the difference of potential observed by BAGARD was very likely due to disturbances. In the controlling experiments however, which were to prove that without disturbances no difference of potential appears at all, strongly concentrated solutions were always used; so it was still possible that in the diluted solutions used by BAGARD a part of the observed effect would remain even in the absence of disturbances. Hence the question has not yet been solved by experiment.

3. We proceed to the theory of the phenomenon and for the present confine our attention to the state of matters in the inner part of the liquid, i. e. far from the lateral borders. If we put for the E. M. F. in the direction of the axis of X E_x , and for the velocities of the ions caused by a slope of potential of 1 C.G.S. unit per cM. U and V , then the velocities are

$$E_x U \text{ and } E_x V$$

respectively in the directions of the positive and the negative axis of X . We assume with LORENTZ²⁾ that an ion, moving with the velocity v in a magnetic field of intensity H is acted upon by a force represented by the vector product $[v. H.]$ for each unit of elec-

¹⁾ C. R. T. 123, p. 77 and 1270, 1896. Journ. de Phys. Sér. 3, T. 5, p. 499, 1896.

²⁾ See f. i. Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpern. Leiden, 1895.

tricity. In real cases the numerical value of this product is small as compared with unity; the variation in the direction of motion is only slight; hence we are allowed to assume this force to be always perpendicular to the axis of X . If the magnetic force is along the axis of Y , the ions will obtain an additional velocity in the direction of the axis of Z ¹⁾.

These velocities will cause differences of potential and of concentration. When the state of equilibrium has been reached it is not necessary that the velocities should be zero, but it will be sufficient if

1^o. The velocities of positive and negative ions are equal.

2^o. As much of the dissolved material wanders back by diffusion in the molecular state as is transported in the form of ions by the said velocities.

For completing the image formed in this way we ought to imagine at the borders of the liquid, or at all events outside the space now considered, on one side molecules dividing into ions, on the other side ions combining again into molecules. For the moment we suppose the reaction-velocities for these processes to be infinitely great.

We will now work out the two conditions.

If we call the E. M. F. in the direction of the axis of Z E_z , the ions obtain velocities in the direction of this axis equal to

$$(U H E_x + E_z) U \text{ and } (V H E_x - E_z) V.$$

Writing c_1 for the concentration of both ions and c_2 for the concentration of the salt, we may represent the velocities of diffusion of the ions by

$$\frac{A}{c_1} \frac{dc_1}{dz} \cdot U \text{ and } \frac{A}{c_1} \frac{dc_1}{dz} \cdot V$$

and that of the salt by $\frac{B}{c_2} \frac{dc_2}{dz} \cdot U$.

Here A is a constant closely related to the diffusion constant of completely dissociated electrolytes; B has a similar signification for the diffusion of a non dissociated salt.

¹⁾ A calculation which takes into account the irregular thermal motion of the ions leads to the same results.

(30)

The first condition reads

$$\left(U H E_x + E_z - \frac{A}{c_1} \frac{dc_1}{dz} \right) U = \left(V H E_x - E_z - \frac{A}{c_1} \frac{dc_1}{dz} \right) V . \quad (1)$$

the second

$$\frac{B}{c_2} \frac{dc_2}{dz} \cdot U c_2 = \left(U H E_x + E_z - \frac{A}{c_1} \frac{dc_1}{dz} \right) U c_1 . \quad (2)$$

Taking into account that the difference of concentration will be very small, and that we supposed the reaction-velocities to be infinitely great, we shall put $c_1 = k c_2$, k being a constant.

(2) may now be written

$$\frac{B}{c_1} \frac{dc_1}{dz} = \left(U H E_x + E_z - \frac{A}{c_1} \frac{dc_1}{dz} \right) k .$$

so

$$\frac{1}{c_1} \frac{dc_1}{dz} = \frac{(U H E_x + E_z) k}{B + A k}$$

Substituting this in (1) and writing a for $\frac{Ak}{B+Ak}$ we obtain

$$(U H E_x + E_z) (1 - a) U = \{(V H E_x - E_z) - a (U H E_x + E_z)\} V .$$

$$E_z \{U + V - a (U - V)\} = - E_x H \{U^2 - V^2 - a (U^2 - U V)\}$$

$$\frac{E_z}{E_x} = - H (U - V) \cdot \frac{U (1 - a) + V}{U (1 - a) + V (1 + a)}$$

and

$$\frac{1}{c_1} \frac{dc_1}{dz} = E_x H \cdot \left\{ U - (U - V) \cdot \frac{U (1 - a) + V}{U (1 - a) + V (1 + a)} \right\} \frac{a}{A} .$$

The limiting cases are $k = 0$ and $k = \infty$.

First case (very slightly dissociated) $a = 0$.

$$\frac{E_z}{E_x} = - H (U - V) \quad \frac{dc_2}{dz} = 0 \quad (3)$$

Second case (completely dissociated) $a = 1$.

$$\frac{E_z}{E_x} = - \frac{H}{2} (U - V) \quad \frac{1}{c_1} \frac{dc_1}{dz} = E_x H \cdot \frac{U + V}{2 A} \quad (4)$$

Both these results may be deduced directly. If c_1 tends to zero, no differences of concentration of the free ions are possible and the velocities must be equalised by the action of electromagnetic and electric forces alone.

The equation

$$(E_x U H + E_z) U = (E_x V H - E_z) V$$

immediately gives the relation (3).

In a completely dissociated solution on the contrary no molecules of salt can go back by diffusion, so the velocities of the ions in the direction of the axis of Z should both become equal to zero; hence

$$E_x U H + E_z - \frac{A}{c_1} \frac{dc_1}{dz} = 0 \quad \text{and} \quad E_x V H - E_z - \frac{A}{c_1} \frac{dc_1}{dz} = 0$$

from which the relations (4) follow immediately.

As a matter of fact, the reaction-velocities are not infinitely great. By supposing these velocities to be equal to zero we may however perceive that the result is not very much altered by this. In this case no decomposition or combination occur; the concentration of the molecules of salt is not altered, these have therefore no influence at all upon the phenomenon. So we then find, whatever the real degree of dissociation may be, the same result as in the case of complete dissociation.

Hence it is very likely that the HALL-effect in liquids may generally be represented by

$$\frac{E_z}{E_x} = -\mu H(U - V)$$

μ lying between 1 and $\frac{1}{2}$.

This result gives the rotation of the equipotential lines in the inner part of the liquid. The question rises, whether this is what has been observed in the experiments. In these namely the difference of potential between two electrodes of metal is read; if these electrodes are immersed in liquids of different concentration, another difference of potential will appear, the value of which may be

estimated by means of the formula of HELMHOLTZ¹⁾

$$V_2 - V_1 = \frac{2 p_0}{\epsilon} \cdot \frac{V}{U + V} \cdot l \frac{c_2}{c_1}$$

Here p_0 means the osmotic pressure for a concentration of 1 gram-ion per cM³, ϵ the charge of a gram-ion in electromagnetic units.

We take a definite case for this estimation, namely a solution of sulphate of copper with the concentration $c_1 = 3.10^{-5}$, the lowest value we find among the numbers of KOHLRAUSCH²⁾ for this salt. We shall suppose that, in this case, applies the formula for completely dissociated solutions.

$$U = 218.10^{-14} \quad V = 373.10^{-14}$$

so — $(U - V) = 155.10^{-14}$. Taking $E_x = 1$ volt or 10^8 C. G. S., $H = 10^4$, we find $E_z = 0.78$.

This is the difference in C. G. S. units between two points on the axis of Z , 1 cM. apart.

The difference of concentration is determined by

$$\frac{1}{c_1} \cdot dc_1 = E_x H \cdot \frac{U + V}{2A}.$$

Comparing our formula for the velocity of diffusion with that of NERNST³⁾, it appears that $A = \frac{p_0}{J} \frac{u}{U}$ where $\frac{J}{u}$ = force required to move a gram-ion, with migration-velocity u , with the velocity of 1 cM. per second through the solution.

$$l \frac{c_2}{c_1} = l \left(1 + \frac{dc_1}{c_1} \right) = \frac{dc_1}{c_1} = \dots$$

We retain only the first term, so

$$l \frac{c_2}{c_1} = E_x H \cdot \frac{U + V}{2A}$$

¹⁾ Wied. Ann. 3. p. 201, 1878. In this form the formula is given by NERNST, Zeitschr. f. ph. Chem. 4, p. 129, 1889.

²⁾ Wied. Ann. 50, p. 404. 1893.

³⁾ Zeitschr. f. ph. Chem. 2, p. 619, 1888.

$$V_2 - V_1 = \frac{2 p_0}{\epsilon} \cdot \frac{V}{U + V} \cdot E_x H \cdot \frac{U + V}{2 p_0 u} \cdot J \cdot U = \frac{E_x H}{\epsilon} \cdot J \frac{U V}{u}$$

$$= \frac{10^8 \cdot 10^4}{10^4} \cdot 0,89 \cdot 10^{10} \cdot \frac{110}{10^8} \cdot 373 \cdot 10^{-14} = 3,6$$

These calculations are sufficient to show that this difference of potential is of the same order of magnitude as the HALL-effect.

If the electrodes are placed in little vessels communicating through long tubes with the points on the axis of Z , as frequently occurred in the experiments, then no difference of concentration exists at the electrodes, and the last mentioned difference of potential disappears.

4. Comparison of theory and experiments.

For this comparison we choose the experiments of BAGARD with a half normal solution of sulphate of copper. We find for this solution $V - U = 11 \cdot 10^{-13}$ ¹⁾.

The experiments of BAGARD however in a field of 385 C.G.S. give for $\frac{E_z}{E_x} : H$ a value of $13 \cdot 10^{-7}$.

Taking the sixteenth normal solution in the strongest field, 962, the theory gives $14 \cdot 10^{-13}$, the experiment $35 \cdot 10^{-7}$.

If we moreover consider, that even the sign of the apparent effect, which agrees here with the theoretical sign, in the experiments of CHIAVASSA with sulphate of zinc proved to be variable, it will appear that we may safely conclude that the hitherto observed galvano-magnetic differences of potential in liquids are *not* caused by the phenomenon of HALL.

Researches much more accurate than have been attempted till now²⁾ should decide whether the weak theoretical effect can be observed. It may be doubted however whether the disturbances, which can assume such large proportions, will ever be completely avoided.

5. So the result of this research is: for the present we can not

¹⁾ In the dissertation the factor 10^{-9} (ratio of volt and C.G.S. unit) has been omitted. As moreover the accurate researches of CHIAVASSA had not yet appeared then, the experiments of BAGARD were, at that time, trusted more than they now appear to deserve. Also mr. F. G. DONNAN at Holywood (Ireland) has noticed the omission and has been so kind as to tell me so.

²⁾ The smallest observable difference of potential was in the experiments of CHIAVASSA $5 \cdot 10^{-6}$ volts. The largest effect calculated above would give for a slope of potential of 1 volt per cm in a field of 10000 C.G.S. units $1,4 \cdot 10^{-8}$ volts.

yet use the phenomena in electrolytes in order to get a better insight into the nature of the electric current in metals. In this respect more may perhaps be expected from researches as those published lately by RIECKE¹⁾ under the title „Zur Theorie des Galvanismus und der Wärme,” where relations are sought between the different thermic, galvanic, thermo-magnetic and galvano-magnetic phenomena in bismuth. I. a. RIECKE calculates with the aid of certain suppositions U and V for bismuth and finds

$$U = 0,05 \cdot 10^{-5} \quad V = 7,21 \cdot 10^{-5}.$$

I intend shortly to publish the results of measurements of the various phenomena in the *same* plate of bismuth, and to compare those results with RIECKE's theory.

¹⁾ Gött. Nachr. 1898.

(June 24 1898).

Royal Academy of Sciences. Amsterdam.

PROCEEDINGS OF THE MEETING of Saturday June 25th 1898.

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The following papers were read:

Chemistry. — „On chemical and microscopical examination of antimonial alloys for axle boxes.” By Prof. H. BEHRENS also in the name of Mr. H. BAUCKE.

By the direction of the „Holl. IJz. Spoorweg-Mij” several cushions of Babbitts-metal (82 % Sn, 9 % Sb, 9 % Cu) were put aside for chemical and microscopical examination. Alloys of this kind, when slowly cooling from a melting heat, will split up into a nearly amorphous mother liquor, rich in tin, into rectangular crystals (probably cuboidal rhombohedrons) of an alloy of tin and antimony and into a whitish bronze, forming radial clusters of brittle rods, composed of hexagonal plates.

The chemical examination of these components has been carried out by Mr. BAUCKE, analytical chemist at Amsterdam, partly by means of analytical, partly by means of synthetical methods, while I have undertaken the microscopical and mechanical investigation of the properties of Babbits metal.

For separating the products of liquation I proposed pressing of the alloys in a semi-liquid state. This suggestion has been adopted by Mr. BAUCKE and the method, worked out and perfected by him, has given very good results. When Babbits metal in a pasty condition is pressed between hot slabs of iron, *tin* will flow out, containing 3 % of antimony and copper and a hard, brittle cake will be left, formed principally by the crystals described above. An alloy composed of 90 % tin and 10 % copper gave a hard residue, from which a remnant of mother liquor was removed by treating it with hydrochloric acid and subsequently with caustic soda ley. Powdery copper was washed away in a stream of water, and then analysis gave 35.1 % Cu, 64.8 % Sn for the composition of the crystalline alloy, while the formula Cu Sn would require 34.9 % Cu and 65.1 Sn. Repeated heating and pressing will drive out more tin, so that this alloy may be said to comport itself in a similar manner as some hydrated salts, e. g. crystallized sulfate of sodium.

In alloys with *antimony* the tin was found to be more strongly combined. From an alloy, containing 10 % of antimony, after heating and pressing thrice, a residue was obtained, resembling closely an alloy of 70 % tin and 30 % antimony. Purified by extraction with hydrochloric acid and washing in a stream of water it was found to be composed of 33.7 % Sb and 66.3 % Sn. Calculation from the formula Sb Sn₂ gave 33.8 % Sb, 66.2 % Sn. With 42 % Sb an alloy was obtained, showing prismatic crystals of higher melting point between the cuboïd ones of the compound Sb Sn₂. By hot pressing and treatment with hydrochloric acid an alloy was isolated, answering to the formula Sb Sn. Found: Sb 50.55, Sn 49.65 %; calculated: Sb 50.37, Sn 49.63 %. In alloys, containing 80 % of antimony, this element can be made to crystallize in a nearly pure state. Its crystals are enveloped by a compound of prismatic form, probably answering to the formula Sb₂ Sn. This point has hitherto not been settled in a satisfactory manner.

Microscopical examination of cushions, that had done duty under railway cars led speedily to the conviction, that their behaviour depends on the frequency and size of rectangular crystals. In cushions, marked as having been unduly heated by running, the rectangular crystals of the compound Sb Sn₂ were found poorly developed or

absent. By mechanical engineers I was told, that the alloy for bearings must not be heated further than necessary for giving it sufficient fluidity, then stirred and cast without delay in moulds heated to the boiling point of water.

In Babbitt's metal, heated till its surface becomes smooth and bright the majority of the crystals is not liquified. Castings, made of such metal, without previous stirring, consist of tin, with about 4 % of copper and antimony, while at the bottom of the melting pot a hard porous mass is found, melting at the same temperature with zinc. Made quite liquid, and then chilled, Babbitt's metal becomes nearly amorphous, sonorous and very smooth when filed or turned. Nevertheless it will stick to an axle, even when liberally lubricated, and when heated to softening it is liable to recrystallization, crystals being formed in a groundmass of liquid tin. Tinning of the axle, sticking, and as an inevitable consequence heating will occur, whenever a heavily weighted axle is run in a box filled with such metal. Finally recrystallization sets in and liquid tin is squeezed out, the newly formed crystals accumulating around the axle. In one case the crystals had formed a compact cylindrical layer, at first sight puzzling, but now easily explained.

With a view to test this theory, experiments were made with model cushion blocks, cast under varied conditions. The blocks were fitted on a mandril of polished steel, running with a speed of 1600 revolutions per minute in a wooden casing. The apparatus was contrived in such a manner, that the pressure on the mandril could be varied at pleasure and the temperature observed on a thermometer fitted into the blocks. The following table gives for some of the experiments the pressures reduced to kilogrammes on the square centim. of the longitudinal section of the mandril and the mean increase of the temperature after a minute of running. Block I was cast in a mould, cooled by running water, block II in a mould heated to 100°, block III in a mould heated in molten zinc.

	0.3 kg.	0.6 kg.	1.2 kg.	3.0 kg.
I	0.50	1.12	1.50	3.80
II	0.64	0.74	0.75	1.64
III	0.65	0.72	2.62	4.64

After the series of experiments with a charge of 3 kgr. block I presented a running surface scarred by irregular grooves and scratches. On block II the rectangular crystals stood out in a relief, similar to that, produced by etching with hydrochloric acid. They were smooth and bright, the interstices deadened by small scars or dimples. On block III the particulars shown by I and II were found combined.

An unexpected result was obtained by examining the metallic sediment from the oil that had been used as lubricant. Mixed with fine dust (essentially tin) were found: in sediment from block I shavings, threads and angular fragments; in sediment from block II spherical and egglike bodies (0.08—0.1 mm.), looking like small drops of mercury; in sediment from block III spheroids and angular fragments. A few of the spheroids from block II were subjected to microchemical tests and found to consist of tin with a considerable admixture of antimony.

These observations suffice to explain the slow increase of temperature in block II: the oil has been more evenly spread, owing to the peculiar relief, developed on the running-surface of this block, and at the same time a *ball cushion* has been formed, whereby rolling has been brought in the place of sliding friction. Spheroids, similar to those of block II were formed in experiments with model bearings of magnolia metal (77.8 % Pb, 16.3 % Sb, 5.9 % Sn) and of aluminium brass, but not with bearings of common brass and of grey cast iron. In this way I have been able to trace their origin to cubical or polyhedral crystals, scattered in a softer metal and accompanied by smaller crystals of a hard and brittle alloy. In Babbitt's metal fragments of the brittle rods of bronze act as a grinding powder, undermining and rounding the rectangular crystals of the compound $SbSn_2$, in the same way, as pebbles are formed in the bed of a river. The tin comes in as a soft cement, possibly its powder has also a favourable influence, augmenting the viscosity of the lubricant. Metal of coarse structure (cast in overheated moulds) is not evenly eroded; the majority of the rectangular crystals, weakened by cracks, are scarred and crushed, instead of being rounded and loosened.

Further reasoning would be misplaced, as it may be expected, that continued experiments and observations in the directions, pointed out above, will speedily throw more light on the subject.

Chemistry. — „*The condition of substances insoluble in water formed in gelatine.*” — By prof. C. A. LOBRY DE BRUYN.

The condition of matter called the colloid, is in many respects a subject of interest. The physiologist, the physicist, the chemist take, each in his department, an interest in the peculiar qualities of the colloids. For to a great extent the processes of life take place in a colloidal medium or between colloids; the part taken by colloid bodies in osmose is generally known; every chemist knows a certain number of examples in which some substances appear in a colloid form, between liquid and solid-amorphous or as so-called colloidal solutions.

An important study on the qualities of colloidal mixtures or solutions, especially in a physic-chemical direction, we already owe for many years to Mr. VAN BEMMELEN. In these important experiments Mr. VAN BEMMELEN has studied, besides other qualities, quantitatively the changes taking place in a colloidal mixture prepared with water (a hydrogel) in varying the temperature or the relative humidity of the surrounding atmosphere.

The observations of which I beg leave to give a concise survey, do not refer in the first place to changes of a colloid mixture itself, but to the influence exercised by a hydrogel on the physical condition of amorphous, insoluble substances, created in a hydrogel as a medium. These observations are partly the result of former experience gained in the daily practice of the laboratory, as the non-appearance of precipitates if in a qualitative analysis, say on metals, there is an admixture of substances like gum; the non-precipitating of chromate of silver in a gelatine solution a. s. o. Such observations are in my opinion not indicated with sufficient accuracy by stating that the substances formed remain in suspension.

Experiences like those just mentioned were recalled to my memory by a communication of Dr. ERNST COHEN in a meeting of the „Amsterdamsch Genootschap.” Mr. GAEDICKE had asserted in a photographic journal that if equivalent qualities of AgNO_3 and KBr (somewhat more of the latter) are mixed in a gelatine solution of 5%, these two salts are only partially transformed, while this reaction is complete in aqueous solution in consequence of the excessively small solubility of AgBr . This assertion of GAEDICKE will appear strange to no one who performs this reaction; he will namely not see a precipitate of AgBr forming itself, but only a mild opalescence; while in aqueous solutions he will notice a strong troubling arising. GAEDICKE rested his assertion on the observation (proved to

be incorrect by Dr. COHEN) that when placing the mixed, solidified Ag Br-gelatine in water, there appeared in the latter Ag Br-flakes; he ascribed their formation to the diffusion of non-transformed K Br and Ag NO₃ from the gelatine into the water, where the reaction then takes place and the ordinary flaky Ag Br appears.

This opinion of GAEDICKE's seemed improbable to Dr. COHEN; and he has proved it to be incorrect by determining the conductivity of the gelatine solutions before and after mixing. The reasoning was simple; if the transformation is complete, the conductivity of the gelatine solution (tested in a liquid state at 30°) must be equal to that of K NO₃; if no reaction has taken place, the conductivity should be greater and reach the total of the conductivities of Ag NO₃ and K Br in ratio to the imperfectness of the transformation effected. The experiment decided in the former sense; the conductivity was equal to that of K NO₃ alone and the hardly visible transformation had yet completely taken place; the Ag Br is not dissociated into ions, does therefore not conduct.

So we have here before us some examples of phenomena that must strike the chemist; the apparent non-appearance of transformations and of precipitation when gelatine is present in cases in which a strong precipitation is observed in water alone. I have further extended these observations over a long series of reactions in which precipitation takes place. It has appeared then that what has been observed in different cases under peculiar circumstances for purely aqueous solutions viz. the interesting phenomenon that some bodies (silicic acid, titanic acid, chrome- and ferrichydrate, arsenious-, antimony- and coppersulphide, silver, gold, selenium) can remain in colloidal solution, is a rule for solutions in aqueous gelatine. The colloidal medium keeps amorphous bodies which are created in it, colloidally dissolved. This holds good for instance for the halogenous combinations of silver, the sulphides, and many hydrates of the heavy metals, for metallic silver, gold and mercury, for chromate of silver, Prussian blue, copper-ferrocyanide, iodide of lead, peroxyde of manganese a. s. o. They all give transparent gelatines which solidify on cooling, preserve their transparency, but often show an internal reflection or fluorescence. Especially for the coloured bodies, which I have cited here in the first place, the phenomenon is still remarkable in so far as, by the very appearance of the color one need not be in doubt about the question of the chemical change (as in the case of Ag Cl or Ag Br) and need therefore not convince oneself of its having taken place by determining the conductivity.

The modus operandi is nearly the same in all cases; deviations

in the concentration of some of the salt-solutions, required by the precipitation of the gelatine itself, betray themselves. I started with a 10 pCt. gelatine solution, which was filtered through a sievefunnel¹⁾ when warm. By adding to this the same quantity of the salt-solutions to be mixed (which were generally $\frac{1}{10}$ à $\frac{1}{20}$ normal) I got solutions containing 5 pCt. gelatine with the salt as $\frac{1}{20}$ or $\frac{1}{40}$ normal. These still liquid solutions (temp. $\pm 30^\circ$) were mixed under strong stirring and cooled quickly to the point of solidification. I then get a perfectly transparent gelatine, more or less opalescent according to the concentration and the thickness of the layer, which reflect a light internally and as it seems to me also show fluorescence. The formation of colloidal silver, which as we know, was first made by CAREY LEA in an aqueous solution, already takes place in gelatine with Ag NO_3 alone, especially under the influence of light; it appears in a very concentrated form after the addition of some formaldehyde; it is then black-brown, still perfectly transparent in thin layers and homogenous with the strongest magnifying; only on the glass wall it deposes very slowly as a metallic layer.

Without investigating the different cases more particularly, I only wish to observe that I have not succeeded in keeping substances, that precipitate crystalline or at all events crystallize very quickly, colloidally soluted in a medium of gelatine. Calcium oxalate, barium sulphate, ammonium- magnesium-phosphate and some other substances form in gelatine microscopically correct crystals or crystalline particles. In general it seems to me that from the many cases I have investigated, this conclusion may be drawn that the visible precipitation of amorphous bodies is prevented by gelatine, that of crystalline bodies not or not so easily.

Now the question is which is the degree of division of the substance, the size of the particles which are prevented by their origin in a colloidal surrounding to join to visible matter? It is here, in my opinion, not a phenomenon of suspension or emulsion; these terms are applied to microscopically visible particles; and therefore I have spoken of colloidal solutions in a colloid. The size of the particles is undoubtedly far beyond the limit of the microscopically visible and therefore embraces the vast field extending between the molecules themselves and the particles that are visible under the strongest magnifier. According to the nature of the substances and

¹⁾ The gelatine contains a little chlorine ($\pm 0.1\%$, H Cl) and some sulphuric acid. This must be taken into account in some reactions f. i. when forming Ag_2CrO_4 .

the modus operandi (concentration of the gelatine and of the salts) greatly different dimensions may be expected. PICTON and LINDNER who among others have proved the extremely small conductivity of the aqueous colloidal solution of arsenious sulphide, admit four different "grades" of division of this substance¹⁾. It will be found impossible however to make clearly defined distinctions in this respect, a gradual change in the size of the particles being much more probable.

The study of the phenomenon of the internal light-reflexion and fluorescence, that shows itself more or less distinctly in several substances formed in gelatine, may perhaps lead to further insight; these optical phenomena are also found in bodies as milky colored glass, that also contains invisible, extremely small particles. Others however of the colloid solutions in gelatine are not only perfectly transparent, but do not show any reflection of the light or fluorescence; it may therefore be admitted that the division of matter in gelatine approaches that of bodies soluted in water. In view of phenomena as those stated here, the idea of being soluted seems to lose its sharpness of outline.

Equally interesting are the chemical reactions, which are the effect of diffusion, on the substances formed in gelatine and in which one may be passed into the other. If a Na Cl-gelatine is surrounded by a Ag₂CrO₄ gelatine the Na Cl diffuses from the former gelatine into the latter, the colloidal Ag₂C₁O₄ being not (or very little) diffusible, while it is known (f i. by experiments of GRAHAM and HUGO DE VRIES) that the salts dissociated into ions diffuse as quickly in gelatine as in water. Now when the Na Cl ions penetrate in the bright red Ag₂CrO₄ the latter is converted into Ag Cl which, equally colloidal in solution, remains transparent and spreads in the form of a ring.

Astronomy. — *On the motion of the Pole of the Earth according to the observations of the years 1890—1896.* By Dr. E. F. VAN DE SANDE BAKHUYZEN. (Communicated by Prof. H. G. VAN DE SANDE BAKHUYZEN.)

1. In treating the meridian-observations made at Leiden in the last 20 years, it was necessary to arrive at a knowledge as exact as possible of the change in latitude during that period and, before investigating the Leiden observations in this respect, it seemed de-

¹⁾ Jour. Chem. Soc. **61**, 187, **67**, 63, **71**, 568.

sirable first to make endeavours to derive information from other sources.

Since in 1891 the 14-monthly period in the motion of the Pole of the earth had been found by CHANDLER, many computations have been made on that subject by him. On the other hand the chief series of observations of the last 50 years were submitted by H. G. v. d. SANDE BAKHUYZEN ¹⁾ in 1894 to a searching and complete examination, in which the result of the Leiden observations of the years 1864—1874 obtained by Mr. WILTERDINK's ²⁾ elaborate researches, as compared to those of the other observatories, proved to be entitled to considerable weight.

Meanwhile a beginning had been made with special series of observations having expressly in view the investigation of the problem of the latitude and soon important results were furnished by the voluntary co-operation of a number of astronomers. More than once Prof. ALBRECHT has summarised those results in the reports of the "International Geodetic Survey" and of late an ample report has been given by him about the state of the problem in Dec. 1897 ³⁾, in which he adjusted the whole series of the obtained results by a continuous curve for the motion of the pole.

But a small part of these results could yet be used by H. G. v. d. S. BAKHUYZEN and though it be true that CHANDLER has already made many computations on this subject, yet often there is a lack of judgment in the treatment of the observations, which in my opinion justifies a new investigation covering now the whole of the period of 1890—1897. The results I found being perhaps of some importance apart from the particular aim for which I undertook my investigation, I take the liberty of making them the subject of a concise communication.

2. ALBRECHT could use the observations of 19 observatories, viz : of *Tokio*, the *Cape of Good Hope*, 10 observatories in *Europe*, 6 in *America* and finally *Honolulu*, a temporary observatory having been erected there in 1891—1892. With a few exceptions the observations have been made according to the Horrebow method. Not in a single place however do they cover the whole period, so ALBRECHT had to undertake extensive computations to derive in successive approxima-

¹⁾ Report of the Meeting of the Roy. Acad. Amsterdam, Febr. 1894 and also Astr. Nachr. vol. 136 and 137.

²⁾ Report of the Meeting of the Roy. Acad. Amsterdam, Dec. 1892.

³⁾ TH. ALBRECHT, Bericht über den Stand der Erforschung der Breitenvariation in December 1897.

tions the mean latitude for each place and to join all the results to a connected system of co-ordinates, by which the instantaneous positions of the pole in respect to a mean one are represented. ALBRECHT chose as co-ordinate axes the meridian of Greenwich and one 90° westward, passing centrally over America, and finally he gives, besides a curve for the motion of the pole, the said co-ordinates x and y for every tenth of a year from 1890.0 till 1897.5.

The curve of ALBRECHT is rather intricate and so we may take for granted that his adjustment, performed undoubtedly with great care, has strained the observations but slightly. Therefore I thought myself justified in avoiding for my further computations the prolix work of falling back upon the original observations, and in making use of ALBRECHT's x and y co-ordinates. Finally however I have also compared my results with the original observations.

It being tolerably certain that the motion of the pole consists at least in the main of a 14-monthly and a yearly motion, I have examined.

1^o. the 14-monthly motion;

2^o. the yearly motion;

3^o. in how far the observations may be represented by the combination of a *constant* 14-monthly and yearly motion.

3. *The 14-monthly motion.*

Leaving the results 1897.0—1897.5 out of consideration for my computation, I had 70 x - and 70 y - co-ordinates at my disposal. Assuming for the present 432 days for the length of the period we find 6 periods = 7 years and 35 days. So the data allow of a good separation of the two motions, but the mutual commensurability is not so approximate that an easy determination of the elements of both at the same time can be founded thereon. So I began by deriving in 1st approximation the yearly motion and proceeded to use corrected x - and y - co-ordinates to determine the 14-monthly motion.

The computation of the x and y was made quite independent of each other and, assuming 432 days for the length of the period, I united in both cases the 70 values to 8 means, which then formed the basis for the computation of periodic formulae.

So I obtained:

$$x = + 0''.151 \cos 2 \pi \frac{t - 2412439}{432}$$

$$y = -0''.143 \sin 2 \pi \frac{t-2412438}{432}$$

in which the epochs are expressed in Julian dates.

By these my 8 means are represented as follows:

x obs.	$O-C$	y obs.	$O-C$
+ 0''.127	- 0''.017	- 0''.046	- 0''.003
+ 076	+ 7	- 115	+ 13
- 040	+ 7	- 118	+ 18
- 145	- 10	- 085	- 22
- 164	- 21	+ 036	- 12
- 056	+ 10	+ 157	+ 28
+ 059	+ 7	+ 141	+ 7
+ 120	- 15	+ 042	- 21

and

$$\sqrt{\frac{\sum \Delta x^2}{n}} = \pm 0''.013 \quad \sqrt{\frac{\sum \Delta y^2}{n}} = \pm 0''.017$$

ALBRECHT assumes 0''.04 as mean error of his co-ordinates, according to which that of the mean of 9 values, with which we operated here, would be 0''.013.

We thus see that the amplitudes found for x and y are very nearly equal and that the difference of phase amounts to 90° with the difference of only a single day. The motion thus proves to be, with considerable approximation, *circular* and to be *direct*, that is with the rotation of the earth from west to east.

To determine the length of the period these results must be united with the results of anterior observations. For that purpose I did not retain the whole of the data used by H. G. v. d. SANDE BAKHUYZEN, but chose only the most certain results, namely those from Leiden 1864—1874 and those from Pulkowa 1882—1892, for which the mean errors are by far the smallest. So I obtained as epochs of maximum of x , = epochs of maximum of the latitude for Greenwich:

		O-C	O-Ch
Leiden 1864—68 Fundam. Stars	2403394	+ 2	— 19
> 1864—74 Polaris	2403386 ¹⁾	— 6	— 27
Mean	2403390	— 2	— 23
Pulkowa 1882—92 Vert. Circle	2410298	+ 8	— 7
Observ. 1890—96 Summ. of Albrecht	2412439	— 6	+ 13

Giving equal weight to the three results, we get:

$$\begin{aligned} \text{Epoch of maximum} &= 2408565 \\ \text{Period} &= 431.11 \text{ days} \end{aligned}$$

and the differences Obs.—Comp. contained in the last column but one.

This epoch of maximum falls in 4 days before the one according to the results of H. G. v. d. S. BAKHUYZEN, whilst the length of the period found by him was first 431.22, later 431.55. Had I combined his mean epoch of maximum with my result then 430.36 would have been found for the length of the period.

This result again agrees well with the supposition that the length of the period has remained unchanged for the last 35 years. In that entire period it cannot have differed much from 431 days and such a great variability as CHANDLER assumes is now already contradicted by the observations. This appears, first in the differences: Obs.—Ch. which I inserted in the last column of the above table. In the second place however the consideration that just in the very last years, according to both suppositions, the epochs must begin to differ widely led me to investigate separately the three last years of ALBRECHT's summary. I operated only with the α co-ordinates and determined from these the epoch in the same way as before. This result, though necessarily less certain, an error in the assumed yearly motion being now of influence, agrees however almost perfectly with that of the 7 years together.

I found :

$$\begin{array}{lll} \text{Ep. of Max.} & \text{Obs.—C.} & \text{Obs.—Ch.} \\ 2413299 & - 8 & + 27 \end{array}$$

¹⁾ These numbers deviate 3 and 4 days from those in the summary of H. G. v. d. S. BAKHUYZEN, as I interpreted the results of WILTERDINK somewhat differently.

The error of the hypothesis of CHANDLER is thus already rather considerable in 1895.

Let us now consider the amplitude. Uniting according to the weights my results obtained by the x and by the y and adding to them some values, formerly found, we obtain

	Amplitude
Leiden 1864—68 Fundam. stars.	0".156
» 1864—74 Polaris	0 .158
Pulkowa 1882—92	0 .189
Final result of H. G. B. 1860—92	0 .168
Result 1890—96	0 .148

We have to remark here, that the result of Pulkowa is probably too small on account of the unequal distribution of the observations of the individual stars, for which WILTERDINK applied a correction in dealing with the Leiden observations.

So a change in the amplitude since 1860 is no more probable now than it appeared before to H. G. v. d. S. BAKHUYZEN. On the other hand we may not, as he already remarked, combine the observations before 1860 with the later ones, certainly not so far as the amplitude and perhaps not so far as the phase is concerned. This might be reconciled with the conception of the 431 day-period as a time of oscillation proper to the earth, when we assume¹⁾ that from time to time sudden causes may change the mutual position of the axis of rotation and the axis of inertia.

Finally I should like to lay down as most probable elements of the motion of 431 days since 1860

Time of transit through the posit. axis of x	2408565
Period	431.1 days
Amplitude	0".155

consequently

$$x = + 0".155 \cos 2\pi \frac{t - 2408565}{431.1}$$

$$y = - 0".155 \sin 2\pi \frac{t - 2408565}{431.1}$$

¹⁾ See also GYLDEN Astr. Nachr. Vol. 132. №. 3157.

4. *Yearly motion.*

To the original x 's and y 's of ALBRECHT the values according to the 431 day-motion were applied, in order to derive from the residuals the yearly motion in 2nd approximation. I employed for that purpose however values deviating slightly from those found above as the most probable. For the amplitude I adopted 0".151, for the period 431.0 and for the epoch 2412439 (or 2408560), which is that found by me from ALBRECHT's summary only.

Taking the means of the corresponding values for the different years, I obtained for x and y 10 mean values, from which periodic formulae, depending only on the *sine* and *cosine* of the single angle, were derived, just as had been done in the 1st approximation.

The results found for the 7 years together deviated but slightly from those which the 1st approximation had given, but I had now also the opportunity of dividing the period into two and of deriving the yearly motion from each half separately.

This seemed desirable, it being *a priori* very well possible that this motion, probably caused by meteorological influences, might differ considerably in the different years.

So I found

	from x		from y
1890—96	$x = + 0''.116 \cos 2\pi \frac{t-261}{365}$	$y = + 0''.067 \cos 2\pi \frac{t-143}{365}$	
1890—92	$x = + 0''.123 \cos 2\pi \frac{t-252}{365}$	$y = + 0''.087 \cos 2\pi \frac{t-126}{365}$	
1893—96	$x = + 0''.113 \cos 2\pi \frac{t-269}{365}$	$y = + 0''.058 \cos 2\pi \frac{t-163}{365}$	

The epochs of maximum are expressed here in days from the beginning of the year; if they are expressed in dates they are :

$t_0 = \text{Sept. } 18$	$t_0 = \text{May } 23$
Sept. 9	May 6
Sept. 26	June 12

By the formula found for the whole of the period the means employed are represented as follows:

<i>x</i> obs.	<i>O-C</i>		<i>y</i> obs.	<i>O-C</i>	
— 0''.034	—	0''.009	— 0''.060	—	0''.008
— 085	+	2	— 011	+	6
— 117	—	2	— 032	+	8
— 107	—	7	— 060	+	4
— 061	—	15	— 054	—	13
+	025	—	0	+	1
+	089	+	2	+	7
+	109	—	6	— 017	+
+	089	—	11	— 058	— 2
+	045	—	1	— 067	0

$$\sqrt{\frac{\sum \Delta x^2}{n}} = \pm 0''.007 \quad \sqrt{\frac{\sum \Delta y^2}{n}} = \pm 0''.007$$

The mean residuals found by testing the mean values for the half-periods have about the same magnitude.

For the entire period, as well as for both halves, the amplitudes in *x* and in *y* prove to be considerably different. At the same time the difference in phase clearly deviates from $90^\circ = 91$ days; it amounts in the three cases to 117° , 124° and 105° .

So the orbit of the yearly motion of the pole is a rather eccentric ellipse, whose principal axes are inclined to the meridian of Greenwich.

The motion of the pole in this orbit is very approximately a simply harmonic one. The mean deviations of the observations are here even considerably slighter than in the case of the 431-day motion.

In order to investigate this elliptic motion more closely, I have in the 3 cases turned the co-ordinate axes through an angle such that they coincided with the principal axes of the ellipses. Doing this it appeared in the first place that the major axes of the ellipses fall east of the meridian of Greenwich and form with it the following angles :

ellipse	1890—96	19°	East.
»	1890—92	29°	»
»	1893—96	10°	»

Further we find for the components of the motion in the direction of the principal axes:

$$1890-96 \quad x = +0''.121 \cos 2\pi \frac{t-\text{Sept. 28}}{365} \quad y = -0''.057 \sin 2\pi \frac{t-\text{Sept. 28}}{365}$$

$$1890-92 \quad x = +0''.136 \cos 2\pi \frac{t-\text{Sept. 23}}{365} \quad y = -0''.065 \sin 2\pi \frac{t-\text{Sept. 23}}{365}$$

$$1893-96 \quad x = +0''.114 \cos 2\pi \frac{t-\text{Oct. 1}}{365} \quad y = -0''.055 \sin 2\pi \frac{t-\text{Oct. 1}}{365}$$

So the motion is *direct*, like that of 431 days, and the times t_0 express the times of transit through the positive halves of the major axes.

The motions found for the two half-periods do not differ so much that there is any guarantee for a real difference existing. It is my opinion therefore, that for the present we must assume as the most probable orbit for the yearly motion of the pole an ellipse, whose major axis lies 19° east of the meridian of Greenwich and whose semi-axes amount to $0'.12$ and $0''.06$.

As early as 1894 CHANDLER had found¹⁾ an excentric orbit for the yearly component; so on the whole I can fully confirm his result. His ellipse however has a greater inclination (45° east of the meridian of *Greenwich*) and also a somewhat greater excentricity (semi-axes $0''.16$ and $0''.05$) than mine.

The results of absolute determinations of zenith distances are so liable to systematic perturbations of a yearly period, that to my idea they cannot contribute to a more accurate knowledge of the yearly term of the motion of the pole.

The results obtained by the Horrebow-method too are certainly not quite free from these perturbations and particularly there would be reason to fear for them the influence of the inclination of the strata of air either in or outside the observing room. (see a. o. ALBRECHT

¹⁾ Astron. Journal, No. 323, 329 and 402.

„Report etc.” page 11—12). Yet I believe, especially on account of the good mutual agreement of the results 1890—92 and 1893—96, obtained by the cooperation of observatories partly different for the two periods, that the results obtained are on the whole trustworthy.

To derive the yearly term H G. v. d. S. BAKHUYZEN also exclusively made use of the results then known of the Horrebow-observations, namely the series of observations of Berlin, Potsdam, Prague and Strasburg made in 1889—1892. Of course only the x co-ordinate can be derived from these and, computing the corrections for the small differences in longitude by considering the motion as circular, the result is :

$$x = + 0''.112 \cos 2 \pi \frac{t - \text{Sept. } 12}{365}$$

nearly coinciding with that which has now been derived from so much fuller data.

Let us finally consider what may be said about the meaning of the results obtained. In 1890 RADAU¹⁾ for the first time drew attention to the fact that by cooperation of periodic displacements of mass, depending on the season, with the period proper of the axis of the earth, then evaluated at about 304 days, motions of the axis of inertia, caused by the above mentioned displacements, may be transmitted greatly magnified to the axis of rotation. By the relatively small difference between the period of the perturbing action and the period proper, we have to do with phenomena which can be regarded as resonance and consequently the possibility arises that the relatively small displacements of mass which may be brought about by meteorological influences, may cause a rather considerable motion of the axis of rotation. Shortly afterwards the matter was more fully investigated by HELMERT²⁾ who showed that the general elliptic motion of the pole of inertia causes motions of the pole of rotation in orbits, which in the various cases may have every shape from the circle with direct motion through more and more excentric ellipses and the straight line, to ellipses with retrograde motion and finally the retrograde circle. In the most favourable case (a direct circular motion of the pole of inertia) the motion is transmitted magnified about 6 times, in the most unfavourable case (a retrograde

¹⁾ Bulletin astron. T. VII, page 352.

²⁾ Astron. Nachr. Vol. 126, N°. 9014.

circle) reduced to about one half. If the motion of the pole of inertia is not simply elliptic, but, more generally, expressible by a series of periodic terms, these are transmitted more and more reduced to the pole of rotation the higher the terms we reach. So in general to irregularities in the motion of the former will correspond much smaller ones in that of the latter.

All this remains not only true in principle, when we take the period proper of the axis of the earth to amount to 431 days, but also the numerical values remain about the same¹⁾. So it is possible and important to consider which motion of the pole of inertia must be assumed to explain the yearly motion found for the pole of rotation. Substituting my final results for 1890—96 in the general formulae, we find as co-ordinates for the *pole of inertia* with respect to the axes finally adopted :

$$x = + 0''.055 \cos 2\pi \frac{t - \text{Sept. 28}}{365}$$

$$y = + 0''.084 \sin 2\pi \frac{t - \text{Sept. 28}}{365}$$

that is, the motion must be *retrograde*, the principal axes have the same direction as those of the pole of rotation, but the major and minor axes have changed places, and the motion of the pole of inertia is but slightly smaller than that of the pole of rotation. We would have found a more considerable proportion had the ellipse of the latter been assumed less excentric.

So after all little has been gained for the explanation of the phenomenon. Only it is perhaps more intelligible that the yearly motion of the pole of rotation may be pretty regular, though *a priori* the reverse be probable for the pole of inertia.

5. Comparison of the observed motion with the sum of the two adopted terms.

In the first place the x and y of ALBRECHT have been compared with the computed values. I adopted for the motion of 431 days the same elements which have served for the derivation of the yearly motion, and for the yearly motion my final results for the whole period. So I found the following differences between observation and computation expressed in hundredths of seconds.

¹⁾ See also NEWCOMB Monthl. Not. 1892, March.

x Albrecht — x computed.

	. 0	. 1	. 2	. 3	. 4	. 5	. 6	. 7	. 8	. 9
1890	- 10	- 6	- 2	+ 1	+ 2	+ 3	+ 1	- 1	- 6	- 7
91	- 8	- 4	- 1	0	+ 3	+ 3	+ 1	0	+ 2	+ 5
92	+ 6	+ 6	+ 1	- 3	- 5	0	+ 3	+ 2	- 1	- 2
93	- 1	+ 1	+ 2	+ 2	0	+ 4	+ 1	- 1	- 2	0
94	+ 2	+ 4	+ 6	+ 7	+ 4	+ 2	0	- 2	0	+ 3
95	+ 4	0	- 7	- 10	- 10	- 8	- 3	- 2	- 2	0
96	+ 1	+ 1	- 1	- 3	- 4	- 3	- 2	+ 1	+ 2	+ 3
97	+ 6	+ 5	+ 6	+ 3	- 2	- 6				

y Albrecht — y computed.

	. 0	. 1	. 2	. 3	. 4	. 5	. 6	. 7	. 8	. 9
1890	- 4	+ 5	+ 9	+ 8	+ 4	+ 2	- 1	- 2	- 3	- 6
91	- 7	- 3	+ 1	+ 7	+ 4	- 3	- 9	- 11	- 6	+ 1
92	+ 7	+ 8	+ 2	- 5	- 8	- 1	+ 4	+ 6	+ 1	0
93	0	+ 1	0	- 2	- 4	- 1	- 2	- 1	0	- 1
94	0	0	+ 3	+ 5	+ 1	- 1	- 2	- 2	- 1	0
95	- 1	- 5	- 6	- 7	- 8	- 6	+ 6	+ 6	+ 5	+ 1
96	- 1	- 2	- 3	- 2	+ 2	+ 8	+ 9	+ 9	+ 2	+ 4
97	+ 1	- 4	- 1	- 3	- 4	- 1				

$$\sqrt{\frac{\sum \Delta x^2}{n}} = \pm 0''.040 \quad \sqrt{\frac{\sum \Delta y^2}{n}} = \pm 0''.045 ,$$

whilst ALBRECHT assumes as mean error of his co-ordinates the value $0''.04$, differing but slightly from the former. The residuals however show more than once a systematic character.

In the second place the original results of observations — usually monthly means — as they are communicated by ALBRECHT, were compared with my formula, and for each observatory¹⁾ the mean devia-

1) If there exist for one observatory various separate series of observations, I reckon them each separately as ALBRECHT does; besides as he I use only the observations till 1896.5. So, if we leave out 4 very short ones, we arrive at 27 series. Neither did I take into consideration the observations which ALBRECHT laid aside on account of clearly appearing systematic error.

tion of such a result was computed; by ALBRECHT himself the comparison of the observations with his curve had already been made. Moreover for every observatory ALBRECHT's residuals as well as mine were united to half yearly means, that is to say means of these residuals were taken for each summer- and each winter-half year, and finally from these, mean deviations for those half-yearly periods were computed.¹⁾

It would take too much space to communicate these comparisons *in extenso*, so I shall restrict myself to a few results derived from them.

If we call Δ_A and Δ_B the mean residuals of the single results of observation as found by ALBRECHT and by myself, computed for each observatory; Δ'_A and Δ'_B the corresponding quantities for the half-yearly periods, I find, taking the means of the 27 values of each of these quantities:

$$\begin{array}{ll} \Delta_A \text{ (mean)} = \pm 0''.061 & \Delta'_A \text{ (mean)} = \pm 0''.033 \\ \Delta_B \text{ (mean)} = \pm 0.071 & \Delta'_B \text{ (mean)} = \pm 0.048 \end{array}$$

Among the 27 series Δ_B is smaller than Δ_A in 5 cases. (Pulkowa 2nd series, Berlin 1st s., Cape of Good Hope, Potsdam 2nd s. and Lyons) and larger in 22 cases; for the Δ' the corresponding numbers of cases are also 5 and 22.

If I use only the 18 series with the smallest mean deviations from ALBRECHT. I find

$$\begin{array}{ll} \Delta_A \text{ (mean)} = \pm 0''.051 & \Delta'_A \text{ (mean)} = \pm 0''.029 \\ \Delta_B \text{ (mean)} = \pm 0.061 & \Delta'_B \text{ (mean)} = \pm 0.041 \end{array}$$

The fact that the Δ_B are in general larger than the Δ_A may be attributed to real deviations from my formula and to systematic errors of the observations, which for a part will have been transmitted to the curve of ALBRECHT. As mean values for the sum of those influences (deviation from my formula and partial influence of the systematic errors), we find for all observatories together:

$$\sqrt{\frac{\sum(\Delta_B^2 - \Delta_A^2)}{27}} = \pm 0''.037 \quad \sqrt{\frac{\sum(\Delta'_B^2 - \Delta'_A^2)}{27}} = \pm 0''.036$$

and for the eighteen most accurate series only:

$$\sqrt{\frac{\sum(\Delta_B^2 - \Delta_A^2)}{18}} = \pm 0''.033 \quad \sqrt{\frac{\sum(\Delta'_B^2 - \Delta'_A^2)}{18}} = \pm 0''.033.$$

So it appears: 1^o that the combined influence of both circumstances is very appreciable; 2^o that influences are at work which

1) Means depending only on a single monthly result were neglected.

operate in a constant direction for rather long periods, 3°. that in the most accurate series of observations they appear in a less degree. So it appears certain that systematic errors have been at work, as indeed had already been found before (see above).

How great however is the share of those systematic errors? In this respect only the deviations of the various observatories compared mutually for the same period can give us some indications and we can use for this end the half-yearly means. If we look into these more closely we often find considerable differences between observatories situated close to each other, which must thus be due to systematic error, but side by side with these we sometimes also find criteria that real deviations from my formula may exist.

To show the first I have derived for the 4 observatories which have furnished observations during 6 successive half-years (Potsdam, Strasburg, Carlsruhe and Lyons), by comparing the result of each one with the mean of the 4, the mean value of the total error of observation in the half-yearly mean of an observatory. The result found is $\pm 0''.049$, that is a considerable amount and which, as it agrees exactly with the value of Δ'_B obtained above, would as far as it goes tend to show that in this case¹⁾ systematic errors of the observations alone may account for the deviations from my formula.

On the other hand a case pointing to a real deviation from my formula is offered by the summer of 1895. Here follow for the European observatories the deviations from ALBRECHT as well as from me:

	<i>O-A</i>	<i>O-B</i>
Kasan	+ 0''.025	- 0''.022
Prague	+ 10	- 48
Potsdam	+ 33	- 30
Carlsruhe	- 104	- 170
Strasburg	- 26	- 90
Lyons	+ 17	- 87

If a real deviation from my formula is indicated by this, it would prove that the combination of a constant motion of 431 days and a *constant* yearly motion does still somewhat fall short of the reality. That the latter may be different in the various years is *a priori* not improbable. It must be left to further observations, which will have to be kept still more carefully free from systematic error, to furnish more certainty upon this point.

¹⁾ The same criterion is not so easily applicable to other observatories and in other years.

Physics. — „*On a 5-cellular quadrant-electrometer and on the measurement of the intensity of electric currents made with it.*” By Prof. H. HAGA.

During the last years I was repeatedly obliged to measure with accuracy the intensity of a constant current of about ten ampere.

From the many methods which might be used, I chose that by which the difference of potential is measured between the ends of a known resistance, inserted in the circuit. As this method is exceedingly simple, capable of measuring currents of greatly different intensities with an accuracy of $\frac{1}{10}$ percent, it seems desirable to me to draw the attention to this method.

A good quadrant-electrometer is required. In 1893 HIMSTEDT¹⁾ described a 4-cellular quadrant-electrometer; the needles were suspended by a silvered fibre of quartz; the damping was obtained, by hanging two vertical magnets at the lower end of the small rod which bears the needles, the poles to opposite sides, so that they could move within an annular space in a piece of copper. The magnets not forming a *perfect* astatic system, a small directing force was left, because of which HIMSTEDT was obliged to make the whole apparatus moveable round a vertical axis.

It seemed to me that this difficulty might be avoided, by reversing the HIMSTEDT method of damping, that is to say by hanging a hollow copper cylinder movable in a magnetic field²⁾. A 5-cellular quadrant-electrometer with this damping was constructed in the physical laboratory at Groningen. The principal part of this apparatus is represented by fig. 1; the brass cylinder round the quadrants and the cylindric case round the mirror are removed.

The base of the instrument is a brass plate five mm. thick on three levelling screws; the four quadrants are placed on the plate, insulated by glass columns; one of them may be moved micrometrically. The terminals are attached under the plate, in order to protect them from dust as much as possible. They are perfectly insulated from the plate by ebonite, glass and shellac.

The whole apparatus for damping also is attached to the underside of the plate; it consists of a circular magnet three cm. high (fig. 2), provided with the armaments *a* and *b*; in the latter notches have been filed, in which the poles of the magnet fit closely;

¹⁾ Wied. Ann. Bd. 50, p. 752, 1893.

²⁾ Beiblätter, Bd. 19, p. 896, 1895.

H. H A G A. A 5-cellular quadrant-electrometer.

Fig. 1.

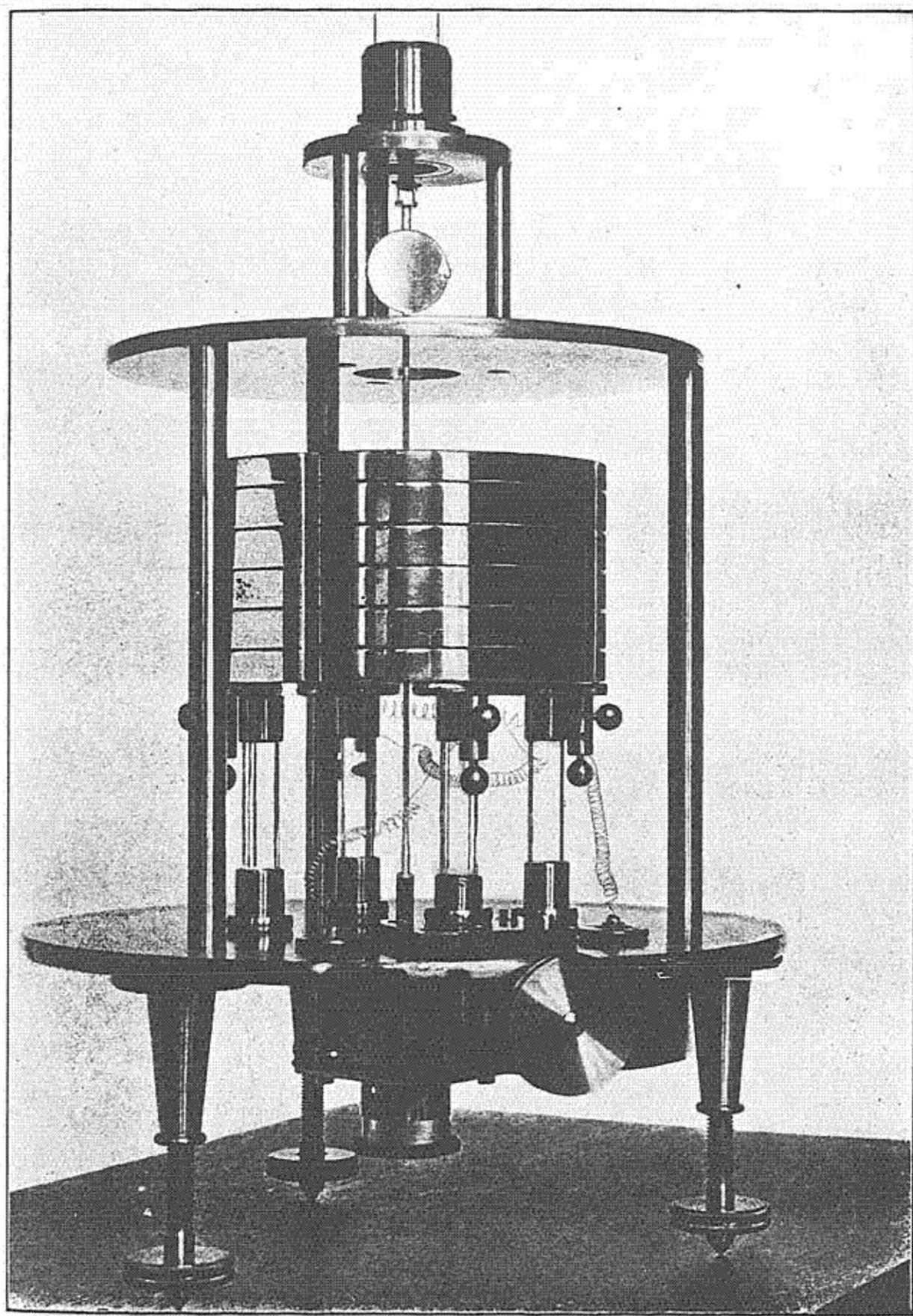
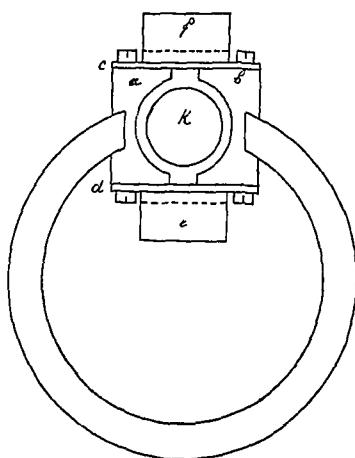


Fig. 2, $\frac{1}{2}$ of the real size.

two brass plates c and d unite the armaments and the magnet, so that they form one whole. Another pair of brass plates e and f are bent under a right angle, the vertical parts are soldered to c and d ; the horizontal parts serve to attach the apparatus to the under side of the plate. Also the under sides of the armaments are covered by a brass plate, in which a copper tube is soldered, in which the iron core k fits closely (fig. 1 and 2). The hollow copper cylinder can move in the annular space between the outer part of the iron core and the inner part of the armaments a and b ; in

the middle of the plate is a hole as large as the space between the armaments, so that a small aluminium tube may be slid over the lower end of the aluminium rod, to which the needles are attached. The tube bears an ebonite rod, which is screwed on the lid of the hollow cylinder. In this way the cylinder is suspended exactly in the centre of the magnetic field, the currents of air being excluded by the brass plates round the armaments.

Moreover the apparatus bears a torsionhead and an arrangement to move the needles in a vertical direction without their being discharged. The length of the suspending fibre is 17 cm.; the distance between the lower and the upper plate is 13.5 cm.

As to the sensitiveness of the electrometer, it depends on the suspending fibre to a great extent. With a silvered quartz fibre of 55μ diameter a CLARK-cell caused a deviation of 760 mm. by reversal, the needles being charged at 180 volt; distance of the scale 2 M. In about half a minute the position of equilibrium was reached after three oscillations.

As the total weight which the fibre had to bear, was 20 grams (the hollow copper cylinder weighed 10 grams), a quartzfibre of 24μ may also be used, through which the deviation would be increased more than 16 times. The period, however, would be increased to four times its former value.

If this very great sensibility is not necessary, a platinum wire¹⁾ annealed in a candle-flame is to be preferred because of its

¹⁾ HALLWACHS. Wied. Ann. Bd. 55, p. 170, 1895.

greater convenience in use. The elastic hysteresis¹⁾ moreover is not to be perceived, as is proved by the observations mentioned below. The WOLLASTON wire of 25μ , supplied by HERÄUS can bear but 4 grams, a platinum wire of 50μ more than 30 grams; most of the later measurements have been made with this platinum wire. The time of oscillation ($\frac{1}{2}$ period) is about 12 sec., and as the damping was so regulated that the needles were in rest after three oscillations, the observation could already be made after 36 sec.

In order to be able to regulate the damping, the hollow copper cylinder is so long thinned and shortened, till (the whole iron core being in its copper tube) the movement is aperiodic. By moving the core downward, the required damping may be obtained. Every wire has of course its own hollow copper cylinder which fits it best.

In using the quadrant-electrometer the charging-battery is of great importance. If measurements are to be made only for a short time, a ZAMBONI pile or a battery of water-cells, consisting of small test tubes filled with water, in which copper and zinc rods have been put, is sufficient. If the charging-battery, however, must be kept ready for use for some months, these arrangements do not suffice any more; nor have I succeeded in reaching this by replacing the water by paste of plaster or by gelatine with carbol. I was very much satisfied with 300 LECLANCHÉ-cells, supplied by P. J. KIPP AND SONS, J. W. GILTAY successor. In the following measurements the positive pole of this battery was connected with the needles, the negative one with the earth.

Table I contains the comparison of six CLARK-cells, constructed according to the instructions given by KAHLE in Wied. Ann. Bd. 51, p. 203, 1894, with a normal CLARK-cell furnished by R. FUESS, at Steglitz near Berlin and tested by the Physikalisch-Technische Reichsanstalt. This cell is called *N*, while the cells, here constructed are marked *A*, *B*, *C*, *D*, *E* and *F*.

The negative pole of the cells remained connected with the earth, the positive one with one of the pairs of the quadrants, while the other pair of quadrants was kept on zero potential, the position of equilibrium was determined. By reversal these two connections were interchanged and this was continued till five observations were made.

¹⁾ GERMAN: Elastische Nachwirkung.

T A B L E I.

February 11th 1897.

Needles charged by 300 Leclanché cells; distance of the scale two metres.

Clark cell.	Observed position of equilibrium.	Deviation.
N. 18°.0	787.8	
	188.1	
	787.8	599.7
A. 18°.3	188.1	
	788.0	
	188.2	599.8⁵
B. 18°.2	788.0	
	187.2	
	787.0	599.9
N.	187.2	
	787.1	
	187.3	599.8
B. 18°.2	787.1	
	187.3	
	787.1	599.7⁵
A.	188.2	
	188.3	
	788.1	599.8
N.	188.3	
	788.2	
	188.3 ⁶	599.8
C. 18°.0	188.2 ⁶	
	788.1	
	188.9 ⁶	600.0
D. 18°.0	186.0	
	785.9	
	185.9	600.1
	784.9 ⁵	
	184.8	
	784.9 ⁶	
	185.0	

Clark coll.	Observed position of equilibrium.	Deviation.
N.	785.2 ^b	185.1
	785.2	185.1
	785.2	185.1
		600.1
D. 18°.0	785.1	185.1
	785.2	185.1
	785.3	185.1
		600.1
C. 18°.0	785.6	185.2
	785.5	185.1
	785.4	185.1
		600.3 ^b
N.	790.9	191.1
	790.7	191.1
	790.8	191.1
		599.7
E. 18°.1	791.1	191.6
	791.0	191.6
	791.0	191.6
		599.4
F.	792.5	193.2
	792.5	193.1
	792.5	193.1
		599.4
N.	792.7	193.0
	792.5	193.1
	792.5	193.2
		599.5
F. 18°.2	792.4	192.9
	792.6	193.1
	792.6	193.2
		599.4
E. 18°.1	791.1	191.9 ^b
	791.3	192.0 ^b
	791.2	
		599.3
N. 18°.0	791.9	192.0
	791.7	192.0
	791.7	192.0
		599.8

From these observations we deduce:

<i>N.</i> 599,8	<i>N.</i> 599,9	<i>N.</i> 599,7
<i>A</i> 599,8	<i>C</i> 600,2	<i>E</i> 599,3
<i>B</i> 599,8	<i>D</i> 600,1	<i>F</i> 599,4

The difference of deviation, caused by *N*, must in my opinion, be ascribed to the influence of temperature, in consequence of which the potential of the charging battery was changed. If greater accuracy is desired, it is necessary to keep the temperature of the charging battery and of the CLARK cell constant; in these experiments the CLARK cells were placed in glass vessels filled with paraffine oil, and care was taken that the temperature could not differ much during the night preceding the experiments.

From this series of observations is seen: 1^o. that with this platinum wire the elastic hysteresis may be quite neglected and 2^o. that with one observation a difference of potential of 1,4 volt may be measured to at least $\frac{1}{10}$ percent.

Another great advantage of this method is that the CLARK cell does not yield a current.

If, without examining in how far the deviation of the electrometer and the difference of potential are proportional, we wish to measure the intensity of an electric current, the resistance must be chosen in such a way, that at its ends the difference of potential is nearly equal to that of a CLARK cell, and this must be compared with a CLARK cell. So a resistance of 0.14 ohm is required for currents of about ten ampere; this resistance must be constructed in such a way that the current cannot cause great differences of temperature; therefore ten German Silver wires of $\frac{1}{2}$ mm. diameter and about $\frac{1}{2}$ M. long were arranged in parallel and placed in a large basin filled with paraffine oil; because of the small temperature coefficient it is sufficient to know the temperature to within three degrees without making a mistake greater than $\frac{1}{10}$ percent. The resistance itself was determined in the usual way with the WHEATSTONE bridge. Many measurements of the intensity of currents have been made and ampere-meters have been tested with this resistance 1).

1) That it is necessary to test ampere-meters repeatedly is proved by the fact, that on an ampere-meter of CARPENTIER a current of 10 ampere was marked in

March '95 with 12 amp.
 January '96 " 13,5 "
 March '97 " 14 "
 October '97 " 16 "

Evidently the magnet has been losing its strength continually.

As it is, however, of importance to be able to use the same resistance for currents of different intensity, it was investigated, in how far the deviation of the electrometer and the difference of potential between the pairs of quadrants are proportional.

To this purpose the current of two accumulators was conducted through a resistance of 1050 ohm, viz. $400 + 300 + 200 + 100 + 100 + 30 + 20$ ohm of a tested rheostat. By means of the side plugs the difference of potential could be measured between 500, 400, 300, 200 and 100 ohm, in doing which the plug on one end was always connected with the earth.

In order to eliminate the difference of the indication of the instrument between the positive and the negative potential of the same absolute value, the current was led in both directions through the resistance; the two CLARK-cells *A* and *B* were placed together in one cylindric glass vessel, filled with paraffine oil; the negative pole of *A* and the positive pole of *B* were continually in connection with the earth and the others by turns with the electrometer.

The following table contains the observed positions of equilibrium and the deviations, the smaller values of the deviation caused by 1 CLARK compared with those of table I are to be ascribed to the gradual weakening of the charging battery; measured with an electrometer of BRAUN the 300 LECLANCHÉ cells had no more than 320 volt. Distance of the scale was 2 meters.

TABLE II

May 5th 1898.

$t = 15^\circ, 1$

Observed position of equilibrium.		Deviation.
Clark B —	{ 283 0 283.0	781 1
15°.1		
Clark A +	{ 781.6 781 6	284.8
500 Ohm	{ 203.8 203.8	866 0
	—————	
	866 3	206.4
	866 3	

	Observed position of equilibrium.	Deviation.
400 Ohm	{ 798.2 798.0 ————— 267.6 267.5	528.4 529.9⁵
300 Ohm	{ 331.1 331.3 ————— 728.8 728.8	397.0 396.1
500 Ohm	{ 866.2 866.2	659.9
200 Ohm	{ 660.2 659.8 ————— 395.1 395.1	264.0 264.0
100 Ohm	{ 458.9 459.1 ————— 591.1 591.1	131.9 131.6
500 Ohm	{ 866.1 866.1	659.8
Clark B —	{ 282.0 282.2	498.5
Clark A +	{ 781.2 781.2	497.0

The observation of the difference of potential at the ends of the resistance of 500 ohm has been repeated a few times for one of the two directions to find out, whether the intensity of the current has changed during the experiment; it appears that it has remained perfectly constant.

The results of this experiment have been collected in table III; the two numbers between brackets represent the deviations caused by the current in its two different directions.

T A B L E III.

Deviation.	Difference of potential in volt.		Intensity of the current in ampere.		
	Observed.	Calculated.			
Clark B — 498.1					
Clark A + 496.8	497.5				
500 Ω	662.2 659.9	661.1	1.900	1.905	0.003801
400 Ω	528.4 529.9 ⁵	529.2	1.524	1.524	0.003810
300 Ω	397.0 396.1	396.6	1.143	1.143	0.003811
200 Ω	264.0 264.0	264.0	0.7617	0.7615	0.003811
100 Ω	131.9 131.6	131.8	0.3806	0.3810	0.003805
Clark B — 498.5					
Clark A + 497.0	497.7				

Because of the slight difference between the deviation of 1 CLARK and the difference in potential at the ends of a resistance of 400 ohm, the proportionality between the deviation and the difference of potential was assumed, from which 1.524 V. was derived; accordingly the intensity of the current was 0.003810 ampere; if we multiply this value with the value of the resistances — taking the very slight corrections into consideration — we find the „calculated” differences of potential. Under „observed” differences of potential the values have been given which are found in the superposition of proportionality between deviation and difference of potential. It will appear that this proportionality ranges within pretty wide limits. The same fact is also shown in the last column where the intensity of the current is calculated by dividing the observed difference of potential by the resistance. It is a matter of course that the deviations are reduced to angles.

If care is taken that the angle through which the needles are moving, does not exceed $7^{\circ}\frac{1}{2}$, (agreeing with the deviation 529 by reversal) and that the deviations do not become too small, so that

the errors of the readings are less than $\frac{1}{1000}$ of the deviation, very different intensities may be determined with the same resistance to at least $\frac{1}{10}$ percent. In this case the current was weak, and it is clear, how with the quadrant-electrometer we may determine among others the constant of mirror galvanometers without any difficulty. If on the other hand we have a „Normal Widerstand” of 0.001 ohm, as it is constructed by SIEMENS and HALSKE for instance, a current of 1000 ampere may be measured with the same accuracy.

To conclude I shall point out a circumstance which may occur in some cases with such measurements, and which may cause very great mistakes, and has rendered a great many of my own experiments worthless. In this laboratory 30 accumulators are coupled in series, and different groups may be used in different rooms; in my experiments the current, which passed also through the resistance, mentioned on page 6; was taken from a group of 5 accumulators; one end of this resistance was connected with the earth, the other with the electrometer; when however in another room another group of accumulators is used at the same time and part of the circuit is in connection with the earth in that room too, or is not quite insulated, the end, connected with the electrometer may be between two points which are in connection with the earth, and the true difference of potential is not measured. A mistake of this kind is at once found when we change the direction of the current; moreover this mistake may be easily avoided by insulating the cells used.

Physical Laboratory. Groningen.

Physics. — „*On the influence of the dimensions of the source of light in diffraction phenomena of FRESNEL and on the diffraction of X-rays.*” (Third communication.) By Dr. C. H. WIND (Communicated by Prof. H. HAGA).

17. In my former communications on this subject¹⁾, I have pointed out (cf. Arts. 10 and 16) that the theory concerning the influence of the widening of the illuminated slit, in the simple form at least in which it was given there, cannot explain the fact that, when the diffraction slit gradually narrows towards one end, the two principal maxima continue to appear as two distinct bright or (on photographic negatives) dark lines, even after the point of inter-

¹⁾ Versl. K. A. v. W. 5, p. 448 and 6, p. 79, 1897.

scetting. According to theory there should be soon after the point of intersection an area of nearly uniform maximal illumination in the middle of the diffraction image, covering the whole region between the two places where in fact the maxima are seen. This difficulty is solved by the optical illusion, which I have described in the Proceedings of the May-Meeting. In cases like that mentioned — viz: those, where a zone of uniform maximal illumination gradually passes on both sides into zones of continuously decreasing illumination — there are indeed two separate maxima to be observed on the borders of the bright zone (cf. fig. 9 and 10 in the former communication).

18. This proves already that the optical illusion mentioned may largely influence our observations of diffraction phenomena. As also appears from some experiments, which I have lately made for this purpose, this influence may even be so great, that the real distribution of light in the diffraction images can hardly be deduced from immediate observation in many cases. So, for instance, it has struck me repeatedly that the diffraction image of a slit still shows the principal maxima with great distinctness, even when the luminous slit has a considerable width, whereas theory indicates the excess of intensity in the maxima, compared with the field round them, to become smaller and smaller, when the slit is widened more and more. As we know now, that under the influence of the optical illusion distinct maxima of brightness are observed even on the borders of a zone of uniform illumination, provided that it passes into zones of decreasing intensity, we may by no means wonder, if we see the principal maxima apparently continue to be clearly visible, though their excess of intensity become very small. We may ask however, whether, because of the optical illusion, the place where we observe our maxima of *brightness* be still the same as the place where those maxima of *illumination* really occur. And without any doubt the answer ought to be: no. Yet it may be presumed and experiments have confirmed, that, as long as the principal maxima themselves are of a considerable distinctness (excess of *intensity*), their apparent place is only slightly influenced by optical illusion, that i. o. w. the place, where our eye observes a maximum of *brightness*, is chiefly determined by the principal maxima themselves. This explains why some measurements of secondary diffraction images, mentioned in my second communication on the subject, may have led to a taxation of the wave-length of light which was not quite wrong.

19. Another remark relating to my former experiments is this. The maxima observed in the secondary diffraction images are of a pure white, though we should expect them to be slightly coloured,

as their place, as deduced from the theory of those diffraction images, is not quite independent of wave-length. We now easily understand this particular to be also due to the optical illusion. The differences of colour, which should really exist to some extent, have intensities, which correspond to the real excesses of *illumination* in the principal maxima and by no means to the much greater excesses of *brightness*, which our eye perceives there.

20. Whereas our optical illusion leads in this way to the explanation of some difficulties, it compels us on the other hand to give up our former conclusion (cf. Art. 15), that the analogy between the shadow images of X-rays and of ordinary light justifies the supposition that the X-rays consist of undulations. The facts mentioned in my former communication sufficiently show, that even without a trace of diffraction the X-rays could produce shadow images, which present to our eye the well-known bright and dark fringes. The photo a reproduction of which is given in that communication as fig. 6 shows in originali a character which agrees perfectly with the (negative) X-shadow images, obtained formerly by FOMM and others. Its dark fringes are by no means less distinct than those of the X-shadow images mentioned; and on the other hand the latter show very distinct bright fringes on their outer sides — though there may have been paid little attention to these fringes hitherto —, these bright fringes being exactly those corresponding to the bright circular lines in fig. 6. Moreover it is by no means difficult — as I pointed out in Art. 8 of my former communication — to obtain the X-shadow images with the characteristic bright and dark fringes by using slits of a considerable width (cf. fig. 10 of that communication). This however would contradict with expectations founded on the supposition of those fringes being caused by diffraction.

21. The optical illusion alluded to leaves, of course, beyond all doubt the correctness of the theoretical considerations, communicated before, on the influence of the widening of the luminous slit on diffraction phenomena. Especially the simple method, developed in order to take that influence into account, holds quite good as far as concerns an exact interpretation of the diffraction images observed and a calculation of the distribution of light, which may be expected in diffraction images under given conditions¹⁾.

¹⁾ It is easy to see that my method for calculating the influence of the widening of the slit on the distribution of light in diffraction images also applies, in principle at least, to other diffraction phenomena than those of FRESNEL, e. g. to those of FRAUNHOFER, and it is easy to conceive how this method is to be modified for being applicable to such cases.

The following question seemed rather important. What are the different stadia through which the primary diffraction image, (projected by the edge of a screen f.i.), the luminous slit being gradually widened, passes into the shadow image, belonging to a slit so wide that diffraction has finished to be of any consequence; and how does the diffraction image present itself to our eye in the different stadia under the influence of the optical illusion mentioned. In order to be able to answer this question I have photographed a series of fifteen diffraction images of a rather wide slit, the luminous slit having throughout the whole series an ever increasing width, but the conditions of the experiment being left quite unaltered in any other respect. The width of the latter slit ranged between such limits, that the space through which the primary diffraction curve must be displaced, in the construction of Art. 5, in order to obtain a correct geometrical representation of the secondary diffraction image we should expect, corresponded in the consecutive experiments to displacements of the starting-point of the effective arch on the spiral of CORNU, as by which the well-known quantity v of FRESNEL varied from 0,2 (when the slit was narrowest) to 8,4 (when the slit was widest). Applying the construction alluded to, I have on the other hand drawn common-type graphical representations of the distribution of light in the diffraction images belonging to the consecutive widths of the slit. It is not the place here for a description in details of the distributions of *light* expected and of the distribution of *brightness* observed on the negatives; it may be sufficient to remark that most of the plates of the series showed a considerable difference between the distribution of light and that of brightness, a difference even as concerns the general aspect of the image. There is, however, not a total want of regularity in these differences. I generally found that the eye perceives *maxima* resp. *minima* of *brightness* in those places of the zones of continually increasing intensity (transition zones), where — in consequence of undulations of the primary diffraction curve — the rate of increasing of the intensity changes considerably. For the rest the diffraction appeared to come forth pretty clearly in the first plates of the series (the luminous slit being rather narrow), so far as concerns the general aspect of the images, whereas its influence further on in the series decreases more and more and is at last scarcely recognised. The latter remark is of some importance especially with a view to the following considerations.

22. The leading idea of the whole investigation has always been the desire to get an estimation of the wave-length of X-rays, or

at least a determination of a possibly low upper limit for such a wave-length. With a view to this purpose Prof. HAGA and myself have continued the experiments begun by Mr. TIDDENS; in the beginning, however, always with a negative result.

Finally two experiments have been made these last days under the following conditions. In both of them the width of the luminous slit was $\sigma = 49 \mu$ and this slit received the radiation of an X-ray tube (with self-regulating vacuum according to the newest system of MULLER at Hamburg), placed in such a way that the anticathode made a very small angle with the axis of the arrangement (cf. Art. 1) in order to procure the greatest possible concentration of the pencil of X-rays, terminated by the slit.

One of the ends of the diffraction slit was narrower than the other, the latter being of a width of $\pm 400 \mu$, the other of a few microns only. In the experiment A the distances were: $a = 293$, $b = 298$ cM., in the experiment B: $a = 605$, $b = 615$ cM.

Both the slits as well as the sensitive plate were attached to firm stands, fastened by means of plaster to columns of compact lime-stone, which were attached with plaster to the pillars or the stone floor of the building. The time of exposition was a little more than 8 hours for the experiment A, 40 hours for the experiment B.

After development each of the two plates clearly showed a thin black line, sharpening towards one end and being the image of the second slit.

By first observation there is no influence of diffraction to be perceived on either of the plates, the borders of the black line mentioned appearing almost absolutely sharp defined and the line itself ending in a very sharp point. Closer observation, however, shows on both sides a transition zone limited by a dark line on the inner side, by a bright one on the outer side. Finally, by looking carefully at the lines under a microscope, magnifying about 14 times, I have been led to the following observations.

1^o. On plate A the dark and the bright lines are pretty sharply defined, much like those appearing in transition zones, where there is no diffraction. The distance between the dark and the bright fringe has for each of the transition zones an average value of about 67μ over nearly the whole plate, while on the other hand the width of the transition zones as calculated from the values of σ , a and b would amount to 50μ , in the supposition of rectilinear propagation.

Comparing the plate with the series of images mentioned in Art. 21, one may feel inclined to assume that the difference between

those 67 and 50 μ be possibly caused by diffraction; but at all events it is certain that the image of the slit on plate A does agree pretty well in its aspect with the images on those plates of the series for which $v > 2, 3$, but does not agree with the other images — a peculiarity which is quite compatible with the rather small value of the difference just mentioned. Hence follows, however, as is easy to calculate, that — if we may speak of a wave-length of X-rays — we may assume:

$$\lambda_x < 0,2 \mu\mu .$$

2^o. The dark and the bright lines on plate B are less sharply defined; the dark lines seem even to have passed into rather broad bands with a gradual decay of darkness towards the outer side, the image beginning in this way to resemble, as far as concerns its general character, the former plates of the series of experiments mentioned in Art. 21. It might even be considered, from its general aspect, as being exactly similar to those of the images of the series, for which v is about 1,5. If we might take this similarity for granted, it would follow that

$$\lambda_x = \pm 0,18 \mu\mu .$$

Influence of diffraction in this experiment is also made probable to some extent by the fact that there have been found values for the distance between the dark and the bright fringes in the transition zones, which range between 76 and 91 μ over nearly the whole plate and amount to 78 on an average, so as to point to a distance actually bigger than on plate A; in this respect we should, however, remember that the measurements could not be effected with a high degree of accuracy. On the other hand the point of intersection of the dark fringes on this plate correspond with a larger width of the diffraction slit than on plate A, which also seems to point out an influence of diffraction. Nevertheless, though all these particulars may be considered as possibly being caused by diffraction, we ought to be aware it being by no means impossible that they might have been caused by continual, though very slight, vibrations of one or more of the stands, if only the manner in which these stands have been mounted may have not quite prevented them from any such continual motion.

3^o. In the neighbourhood of the point-end of the image of the slit on plate A, my attention was repeatedly drawn to something, that in my opinion may possibly be considered as a slight indication

of a fan-like broadening of the image of the slit, chiefly manifesting itself in a diverging of the two bright lines of the outer side. If we might assume, that this fan-like broadening really exists and is a consequence of diffraction, and further that such a broadening of the image does not occur before the diffraction slit is so narrow as to correspond at the utmost to a value 2 for the quantity v , we should find by calculation, as the diffraction slit has proved by measurement to have a width of $2,3 \mu$ on the spot where the phenomenon is seen:

$$\lambda_x > 0,12 \mu\mu .$$

Though on one hand I am aware that what I have described as seeming to be a fan-like broadening of the image of the slit is in its appearance so utterly mean, that every one might feel justified in not accepting my interpretation, yet it is on the other hand of some importance, that this particular presented itself to my eye quite spontaneously, I having had not the slightest expectation to observe anything of the kind.

4º. After having observed the phenomenon mentioned under 3º on plate A, I tried to find something of the same kind on plate B, and indeed I succeeded in finding a place which gave an indication of a phenomenon somewhat like it. The phenomenon being even less distinct here than on plate A is in itself not astonishing at all, as plate B on the whole seems to be much less affected by the processus of insolation and development than plate A. The spot where I believed to observe the fan-like broadening lies higher here than on plate A, viz: on such a height as to correspond with a width of the diffraction slit of about 30μ .

If the reality of this phenomenon might be accepted, we might deduce from it by calculation:

$$\lambda_x > 0,15 \mu\mu ,$$

suppositions of the same kind being made as before.

It is in agreement with the above remarks that the image of the slit extends considerably farther, at the point-end, on plate A than on plate B and less far on both the plates than would correspond to the real length of the diffraction slit. We may, however, not attach too much importance to this fact, 1º. because plate B shows on the whole a less intensive darkness than plate A, as has already been said, and 2º. because the diffraction slit becomes so

exceedingly narrow on one end, that we could not expect much effect of the insolation on the corresponding parts of the negative, even if diffraction were to be excluded from before hand.

I do by no means intend to say that by what precedes the conclusion might be justified, that X-rays are diffracted and that *they have a wave-length of about $0.1 \text{ \AA} 0.2 \mu\mu$* . Yet, as we have arrived through two or three independent ways at results which agree pretty well, those results — though each of them, the first excepted, may separately be of nearly no value at all — if taken together make me believe, that the conclusion mentioned *may possibly be not very far from truth*. Moreover they make a further investigation in the same direction desirable. As to the result mentioned under 1^o. I think it may be accepted without any restriction ¹⁾.

Physics. — *The galvanomagnetic and thermomagnetic phenomena in bismuth*, by Dr. E. van EVERDINGEN JR. (*Communication No. 42 from the Physical Laboratory at Leiden*, by Prof. H. KAMERLINGH ONNES).

In order to explain the galvanomagnetic and thermomagnetic phenomena RIECKE ²⁾) assumes that a galvanic current is always accompanied by a current of heat, and a current of heat by a galvanic current. The basis of these hypotheses lies in WEBER's theory of the conduction of electricity and heat in metals; in this theory also the conduction of heat is ascribed to the motion of charged particles *alone*.

The velocities of the positive and negative particles are :
for a slope of potential of 1 C. G. S. unit per cM. u and $-v$
for a slope of temperature of 1° per cM. g_p and g_n .

In the magnetic field H a positive ion with the velocity U is acted upon by a force $H U$ for each unit of electricity.

RIECKE assumes further that the velocities in the direction of this force of the particles are zero in the state of equilibrium in the case of the phenomenon of HALL; the equilibrium is reached by the combined action of a difference of potential and of temperature between the sides of the plate.

In this way he finds for the coefficient of the galvanomagnetic

¹⁾ With great pleasure I acknowledge the assistance given by Mr. C. SCHOUTE in the measurements, necessary for the experiments.

²⁾ Gott. Nachr. 1898.

difference of temperature (i. e. the difference of temperature per linear cM. for a slope of potential 1 and a magnetic field 1)

$$\frac{uv(u+v)}{ug_n+vg_p} = a \dots \dots \dots \dots \quad (1)$$

and for the rotation of the equipotential lines occurring in the HALL-effect

$$\frac{u^2g_n - v^2g_p}{ug_n + vg_p} = b \dots \dots \dots \dots \quad (2)$$

In a similar manner he deduces:

Coefficient of the thermomagnetic difference of potential

$$\frac{g_pg_n(u+v)}{ug_n+vg_p} = c \dots \dots \dots \dots \quad (3)$$

Coefficient of the thermomagnetic difference of temperature (i. e. the rotation of the isothermal lines)

$$\frac{uv(g_p-g_n)}{ug_n+vg_p} = d \dots \dots \dots \dots \quad (4)$$

When these four phenomena have been observed, u , v , g_p and g_n may be calculated. Proceeding to this calculation, RIECKE says: „Eine thermomagnetische Temperaturdifferenz scheint hier (bei Wismuth) nicht vorhanden zu sein.” Therefore he takes $g_p = g_n$, which considerably simplifies the formulae.

This remark however is not justified. As early as 1887 LEDUC¹⁾ observed the rotation of the isothermal lines in bismuth; this observation was corroborated by the experiments of VON ETTINGSHAUSEN²⁾. The rotation however is not very large and hence one might think, that RIECKE's assumption does not remain far behind the truth. In order to decide this question I have repeated the calculation with the data used by RIECKE, augmented by VON ETTINGSHAUSEN's result for the rotation of the isothermal lines. The latter found with a plate of bismuth, 2,2 cM. broad and probably³⁾ 4,8 cM.

¹⁾ C. R. 104, p. 1784, 1887.

²⁾ Wied. Ann. 33, p. 135, 1888.

³⁾ See VON ETTINGSHAUSEN and NERNST, Wied. Ann. 33., p. 477, 1888.

long, when two opposite borders of the plate were maintained respectively at 100° and at the temperature of the room, in a magnetic field of 9500 C G. S. a difference of temperature of $\frac{1}{8}^\circ$ between the two other borders. The slope of temperature in the middle of this plate was probably no more than 10° per linear cm. Hence a was about $-6 \cdot 10^{-7}$, and the four quantities must be calculated from

$$a = 2,98 \cdot 10^{-10} \quad b = -7,16 \cdot 10^{-5} \quad c = 0,132 \quad d = -6 \cdot 10^{-7}$$

Eliminating g_p and g_n from the equations 1 to 4, we obtain

$$v^2 + (b + d)v + (bd - ac) = 0 \dots (5) \quad \text{and} \quad u = v + (b + d) \dots (6)$$

or

$$uv = ac - bd \quad \text{and} \quad u - v = b + d.$$

Only one of the two sets of values for u and v , furnished by these equations, satisfies all conditions

$$u = -0,005 \cdot 10^{-5} \quad v = 7,215 \cdot 10^{-5},$$

whence we calculate by means of the relations 3 and 4

$$g_p = -0,012 \quad g_n = 0,133.$$

It is evident that g_p and g_n are not even approximately equal.

The quantities $\frac{g_p}{u}$ and $\frac{g_n}{v}$, which according to RIECKE's theory are proportional to the kinetic energy of positive and negative particles, appear now to be

$$\frac{g_p}{u} = 2,4 \cdot 10^5 \quad \frac{g_n}{v} = 0,0185 \cdot 10^5$$

2. An objection which may be raised both against this calculation and against that of RIECKE is, that the various coefficients were not determined all with the same sample of bismuth, and not all in the same magnetic field. Indeed we know how different the results of experiments upon bismuth from different sources may prove to be; and though theoretically all the phenomena are proportional to the first power of the strength of the field, practically this proportionality is sometimes far from complete; VON ETTINGSHAUSEN and NERNST f. i. obtained as constants of the HALL-effect in mag-

netic fields from 1650 to 11100 values from — 10,27 to — 4,95. Moreover in the formulae the value of the resistance of bismuth must be used, which depends largely on the strength of the magnetic field. These considerations induced me to repeat the measurements of the various phenomena with the same plate of bismuth and in the same magnetic field¹⁾.

In order to avoid as much as possible the disturbances which might be caused in cast plates by irregularities in the crystalline structure²⁾, these experiments were made with a plate of bismuth deposited by electrolysis in a solution of nitrate of bismuth.

During the experiments the plate was fastened on a little wooden board under two copper strips, soldered to the copper tubes serving to supply or absorb heat and at the same time as electrodes for the galvanic current through the plate. The thermo-electric needles could be introduced between the plate and the wood at three different, fixed distances from the copper plates, for which purpose three grooves had been sawn in the wood. In measuring differences of potential the copper wires of the thermo-electric couples (German silver-copper) were used. The space between the plate and one of the poles of the magnet was filled up with cotton.

A similar arrangement of the experiments was used by VON ETTINGSHAUSEN and NERNST in their research on the phenomena in pure bismuth and in alloys of bismuth and tin³⁾, with this difference, that as well the copper tubes as the thermo-electric needles were soldered by them to the plate.

3. *Measurements.* Dimensions of the plate: length 3,3 cM., breadth 1,15 cM., thickness 0,089 cM.

The experiments in which the thermo-electric needles were on the middle of the plate are indicated by the letter *A*; those in which they were placed on the side of the heated tube by *B* and the third series by *C*. The distance between the two copper strips was 2,96 cM.; the distances between the heated strip and the 1st, 2^d and 3^d groove were respectively 0,65, 1,53 and 2,37 cM.

In judging the results, entered under *B* and *C* we should keep

¹⁾ See my communication in the Meeting of May 28, 1898, p. 58. Communications Phys. Lab. Leiden, №. 41, p. 13.

²⁾ See my communication in the Meeting of April 21, 1897, p. 501 and June 26, 1897, p. 71, 72. Comm. №. 37, p. 17, №. 40, p. 8, 9.

³⁾ Wied. Ann. 38, p. 474. If in the course of this research also the rotation of the isothermal lines had been measured, it would have furnished good data for a calculation.

in mind that the proximity of the well conducting copper strips must cause a decrease of the occurring differences of temperature and of potential.

Some observations were made in magnetic fields differing a little from those mentioned below. The communicated numbers were then obtained by graphical interpolation, just as the numbers of von ETTINGSHAUSEN and NERNST, entered under v. E. and N., which are given for the sake of comparison.

a. *Galvanomagnetic difference of temperature.* The method of observation was the following: when the current through the plate and the magnetizing current had been closed during about 15 minutes and the temperature had become approximately constant, the deflections with both thermo-electric couples were alternately read at constant intervals, f. i. 4 times that with the first and 3 times that with the second. Then the magnetic field was reversed, after some minutes the series of experiments repeated, thereupon the magnetic field reversed again and a third series of experiments made. Afterwards this whole series was repeated with the direction of the current through the plate reversed.

Each series of 7 observations furnishes a mean difference of temperature between the upper and the lower border of the plate. The difference between the mean of series 1 and 3 and series 2 gives double the galvanomagnetic difference of temperature. Finally the mean was taken of the results with both directions of the current.

During these experiments water at the temperature of the room was flowing through both tubes. But for this precaution a difference of temperature of about 30° might arise between the two borders where the current entered and left the plate, caused by PELTIER-effect; in consequence of the rotation of the isothermal lines a sensible error might be caused by this difference of temperature.

Current through the plate 3,0 ampères.

Magnetic field	Difference of temperature in °C.			
	A	B	C	v. E. and N (current 5,6)
1350	—	0,00	—	
2730	+ 0,03	+ 0,02	—	+ 0,95
4800	0,16	0,10	+ 0,09	1,66
6100	0,29	0,20	—	2,12

b. HALL-effect.

The observations were made by means of the method of compensation¹⁾.

Magnetic field.	HALL-constant			
	A	B	C	v. E. and N
1350	— 14,2	— 11,7	— 11,5	— 10,5
2730	13,1	11,1	11,1	9,3
4800	12,4	10,2	10,2	8,0
6100	11,8	9,9	9,7	7,1

c. Transverse thermomagnetic difference of potential.

For measuring this phenomenon the upper and lower borders of the plate must be connected through the galvanometer; as moreover the current of heat cannot be reversed momentaneously, we are obliged to observe the change in the deflection on the reversal of the magnetic field. These reversals occurred every minute; the readings were taken just before reversing.

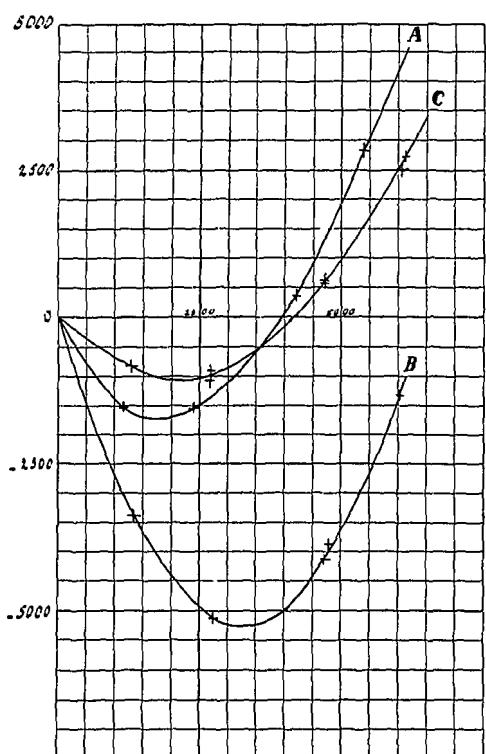
In this measurement the rotation of the isothermal lines causes a disturbance. Though the rotation is not considerable, yet the disturbances are not to be neglected; for on the reversal of the field the sign of the difference of temperature between the borders is changed; moreover we have to do with a thermo-electric couple bismuth-copper; in fact the disturbances assumed sometimes a value of 60 scale-divisions, which would cause an error in the difference of potential of 1800 C. G. S. units.

The current of heat was obtained by conducting steam of 100° through one of the tubes, and water at the temperature of the room through the other tube. The slope of temperature, which we must know in order to calculate the coefficient of this effect, is not the same in different sections, in consequence of the loss of the heat absorbed by the surroundings. Therefore I have measured the temperature in the three places and found for A ± 34°, B 50° and C 28°. Thence the slope of temperature given below was deduced by graphical interpolation.

¹⁾ See communication of May 30, 1896, p. 47. Comm. N°. 26, p. 1.

Magnetic field.	A	B	C	v. E. and N
	Slope of temp. 14	28	8	< 4
Transverse thermomagn. diff. of temp in C. G. S. units				
1350	- 1650	- 3390	- 900	-
2730	- 1300	- 5130	- 825	+ 4900 ¹⁾
4800	+ 1350	- 4020	+ 700	+ 8600
6100	+ 4480	- 1200	+ 2600	+ 10700

In the graphical representation all observed differences of potential have been inserted; the abscissae are strengths of the field, the ordinates differences of potential, both in C. G. S. units.



d. Transverse thermomagnetic difference of temperature (rotation of isothermal lines).

This difference of temperature might have been observed after the same method as the galvanomagnetic difference of temperature; just as the latter it arises gradually. In consequence of the heating of the surroundings however the temperature in the plate was not so constant as in the experiments without a current of heat; moreover it was desirable, with a view to the correction of the thermomagnetic difference of potential, to measure the rotation under quite the same circumstances as in

those experiments. Therefore the variation of temperature on the reversal of the field was here determined for each of the thermo-electric couples separately.

¹⁾ The breadth of the plate was here 2,2 cM.

Magnetic field.	A	B	C
	Slope of temp 14	28	8
Transverse thermomagn. diff. of temp. in °C			
1350	— 0,04	— 0,06	—
2730	0,09	0,12	— 0,04
4800	0,14	0,18	0,06
6100	0,16	0,23	0,07

4. Generally these results differ rather considerably from those of VON ETTINGSHAUSEN and NERNST. The galvanomagnetic difference of temperature is smaller and increases nearly in proportion to the second power of the strength of the field instead of the first. The HALL-effect is larger — larger even than has ever been observed in bismuth — and increases more with the magnetic force. For the thermomagnetic difference of potential we find in the weaker fields even another sign than was observed before; only in the stronger fields the sign agrees again with the observations of VON ETTINGSHAUSEN and NERNST; accordingly the effect, when positive, increases more than proportional to the first power of the magnetic force.

Probably the most important cause of this difference is the different purity of the bismuth; also the circumstance, that my bismuth was deposited by electrolysis may have some influence.

The behaviour of the alloys of bismuth and tin may now serve in order to indicate which bismuth is purer. VON ETTINGSHAUSEN and NERNST experimented upon bismuth, considered as chemically pure, and four alloys with increasing quantities of tin. They found:

a. The galvanomagnetic difference of temperature has a maximum in the alloy with 1% of tin. In the alloys the increase is less than proportional to the first power of the magnetic force, in bismuth it is almost proportional to the magnetic force.

b. The HALL-constant is in the alloys much smaller than in bismuth; the relative decrease with increasing magnetic forces grows with increase of the quantity of tin.

c. The transverse thermomagnetic effect is largest in bismuth; the longitudinal thermomagnetic effect on the contrary has a maximum in the alloy with 1% of tin. The increase in the alloys is less than proportional to the magnetic field; in bismuth it is nearly proportional to that field.

Hence in respect to a and b my bismuth and the alloys differ

in opposite senses from the bismuth of von ETTINGSHAUSEN and NERNST; also with respect to c , if we regard only the relation with the magnetic force, whilst the discrepancy of sign is not found in the alloys.

That indeed the bismuth of the physicists mentioned was not completely pure is moreover proved by what follows: the temperature coefficient of the resistance was found to be negative, whereas with the electrolytic bismuth from HARTMANN and BRAUN according to FLEMING and DEWAR¹⁾ it is positive and just as in other pure metals; the conductivity was $4,8 \cdot 10^{-6}$ instead of $8,6 \cdot 10^{-6}$ with electrolytic bismuth, and the increase of resistance in a magnetic field 8400 was 30% against 40% with the electrolytic bismuth according to HENDERSON²⁾.

Hence very likely the bismuth used by me was very pure. By this result the confidence in the results of the experiments is augmented; yet I will indicate a possible, though improbable disturbance, which might account for the change of sign in the transverse effect. It is namely possible that the thermo-electric needles did not follow completely the variations of temperature of the plate, and that therefore the disturbances caused by rotation of isothermal lines was greater than was calculated. If we consider however, that I found for that rotation greater values than von ETTINGSHAUSEN found with soldered thermo-electric needles, that after 1 minute the deflections varied rarely more than one scale-division, and that my values for the rotation would have to become at least 6 times greater in order to render the effect positive in all cases, then it will be agreed that this explanation is very improbable. I intend however to repeat one of the experiments with soldered electrodes.

5. Before proceeding to the calculation of u , v etc. from the new data I wish to point out a remarkable discrepancy between RIECKE's theory and my observations.

According to formulae 1 and 3 the ratio of the coefficients of thermomagnetic difference of potential and galvanomagnetic difference of temperature is equal to $\frac{g_p}{u} \cdot \frac{g_n}{v}$

$\frac{g_p}{u}$ and $\frac{g_n}{v}$ however are equal to

¹⁾ Proc. Roy. Soc. 60, p. 73 and 425, 1896. A similar result I obtained with bismuth from *Oberschlema*. (See my dissertation p. 99).

²⁾ Wied. Ann. 53, p. 912, 1894.

$$\frac{1}{2e} \mu_p c_p^2 \quad \text{and} \quad \frac{1}{2e} \mu_n c_n^2$$

(see § 12 of RIECKE's paper) where e denotes the charge in electro-magnetic units, μ_p and μ_n denote masses and c_p and c_n velocities of the charged particles. Of course the product of these quantities can never be otherwise than positive. Now for the galvanomagnetic difference of temperature always a positive value, for the thermomagnetic difference of potential in weak fields however a negative value was found. The regularity with which this phenomenon showed itself, appearing f. i. from the graphical representation, proves that there can be no question of errors of observation; none of the known thermomagnetic phenomena can cause a systematic error; hence the negative sign of the thermomagnetic difference of potential in weak fields is not to be reconciled with RIECKE's theory in its present form.

But even if we will not insist upon this negative sign in the weak fields, the positive value in stronger fields leads in connection with the other quantities to conclusions, which give rise to the question whether in the theory everything has been taken into consideration.

For the calculation I chose the magnetic field 6100 and the series of observations *A*. We find then

$$a = 1,05 \cdot 10^{-10} \quad b = -8,83 \cdot 10^{-5} \quad c = 4,56 \cdot 10^{-2} \quad d = -1,63 \cdot 10^{-6}$$

In the calculation of a and b for the resistance of the bismuth, which has not yet been determined by me, was taken the value which FLEMING and DEWAR found in a field of 6100, namely $134 \cdot 10^3$ in C. G. S. units.

The formulae 5 and 6 give

$$u = -0,157 \cdot 10^{-5} \quad v = 8,84 \cdot 10^{-5}$$

whence we deduce further

$$g_p = -1,304 \quad g_n = 0,046$$

so

$$\frac{g_p}{u} = 8,3 \cdot 10^5 \quad \frac{g_n}{v} = 0,0052 \cdot 10^5$$

This result does not appear to me very probable.

6. I wish to defer a comparison between theory and experiments entering into more particulars until also the longitudinal phenomena: the conductivity for electricity and for heat, the change of these in the magnetic field and the longitudinal thermomagnetic effect have been investigated in this plate.

I think it probable that RIECKE's theory will have to be modified. Firstly also the variation in the magnetic field of the conductivity for electricity and heat might be included in it. The observations of these phenomena prove that the variation is much smaller for the thermal conductivity than for the electrical conductivity. This indicates, I think, that the theory wrongly ascribes the *whole conduction of heat to the charged particles*¹⁾. It would be desirable to try whether it is possible, taking this into account, to explain the phenomena with the assumption that the kinetic energy is the same for positive and negative charged particles, which seems at first sight more probable. At the same time it might perhaps be recommended to omit, in the deduction of the formulae for the four transverse phenomena, the condition that the velocities of the particles in the direction of the electromagnetic force should be zero, and to replace it by a similar condition as was assumed by me in the theory of the HALL-effect in electrolytes²⁾. Finally it will appear necessary to make some hypothesis in reply to the question, what happens with the charged particles when they reach the borders of the metal RIECKE himself indeed has observed³⁾, that it is not possible to formulate a complete theory of thermo-electricity with the aid of his hypotheses without accounting for the conditions at the junction of the two metals and forming an idea about the reciprocal influence of ponderable molecules and electric particles.

Physics. — *On the deviation of DE HEEN's experiments from VAN DER WAALS' law of continuity.* By Dr. J. VERSCHAFFELT
(Communicated by Prof. H. KAMERLINGH ONNES).

(Will be published in the Proceedings of the next meeting).

¹⁾ See my dissertation p. 114

²⁾ See communication of May 28, 1898, p. 48. Comm. N°. 41, p. 6.

³⁾ § 8 of his paper.

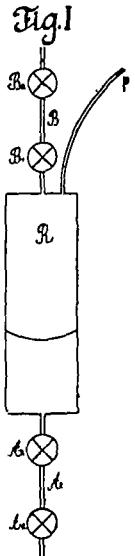
Physics. — *The composition and the volume of the coexisting vapour- and liquid-phases of mixtures of Methylchloride and Carbonic acid.* By CH. M. A. HARTMAN (*Communication N°. 43 from the Physical Laboratory at Leiden*, by Prof. H. KAMERLINGH ONNES).

§ 1. VAN DER WAALS' theory of mixtures has already given rise to numerous observations. KUENEN f. i. has determined the isothermal lines for three different mixtures of methylchloride and carbonic acid in the gaseous phase in order to construct the ψ -surface up to the border-curve for different temperatures and to obtain data about the values of $a_{1,2}$ and $b_{1,2}$. The phenomena in the neighbourhood of the plaitpoint have also been investigated by him for different substances. Through this the shape of the connodal lines in the neighbourhood of the plaitpoints has become fairly well known.

It was very desirable that now also a first plait crossing the ψ -surface from one side to the other should be traced experimentally with special indication of the coexisting phases. If at a given temperature the composition and the density of these phases are known we may draw the projection of the connodal line in the x - v plane and in this figure we may draw the projections of the tangents which unite the coexisting phases on the surface.

To obtain the data required for this has been the purpose of this investigation. I have chosen methylchloride and carbonic acid for substances because many data about the mixtures of these two have already been gathered by KUENEN.

§ 2. Fig. 1 may serve to explain the principle of the method of investigation that has been followed.



The coexisting phases are contained in a wide reservoir R . At the upper end of the reservoir we may isolate a definite volume of the vapour-phase by shutting the cock B_1 and draw it off by means of the cock B_2 and receive this above mercury in a common gas tube. The quantity of this is found by measuring the volume, pressure and temperature of the gas and the composition is found by having the carbonic acid absorbed by potash.

In the same way we may isolate a definite volume of the liquid-phase at the lower end of the reservoir.

The volumes of the liquid-space A and of the vapour-space B have been determined by measuring the volume of oxygen which they can contain at high pressure.

The equilibrium between the two phases is obtained by stirring with the aid of an electro-magnetic stirring-apparatus, as has lately been described by VAN ELDIK¹⁾ (see § 6). In this manner a perfect equilibrium must be obtained within the wide reservoir. The cocks A_1 and B_1 are wide open during the stirring. Care should be taken that the composition of the substance in the space B should be the same as that of the rest of the vapour-phase in R , and therefore, after sufficient stirring (or while carefully continuing the stirring) as much of the vapour as is contained in the space B must be slowly blown off by slightly opening the cock B_2 , after which the cocks B_2 and B_1 are successively shut. In tapping from the liquid-phase the same is done with respect to the cocks A_1 and A_2 .

The spaces A and B are 0,0778 and 3,25 c.c.m., while the volume of R is about 150 c.c.m. So we may assume that the change of the composition of the liquid and of the vapour during the tapping may be left out of consideration, if the tapping be done with the necessary care in the way described above, and that the compositions of liquid- and vapour-phases, found by analysis, indicate the compositions of coexisting phases at the temperature and under the pressure at which the tapping is done. I intend to connect with the present apparatus the compound hydraulic pump for moving coexisting phases (described in the Proceedings of the Meeting of May 27th 1897 p. 21) by means of which the permanent equilibrium between the phases may be still better secured.

In order to keep the temperature constant, the apparatus is placed in a water bath (see § 5).

The pressure of the coexisting phases is measured before and after the tapping by means of the manometer connected with p (see § 4). The average of both measurements is taken as the pressure to which the coexisting phases belong.

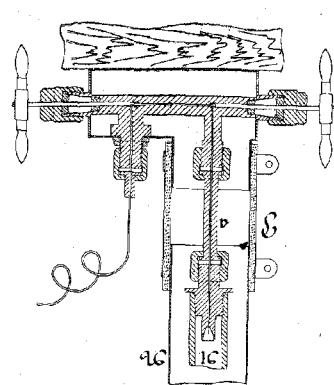
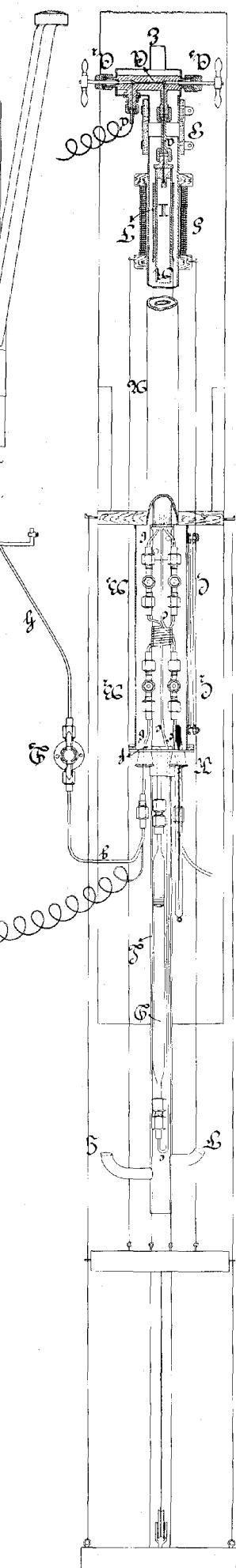
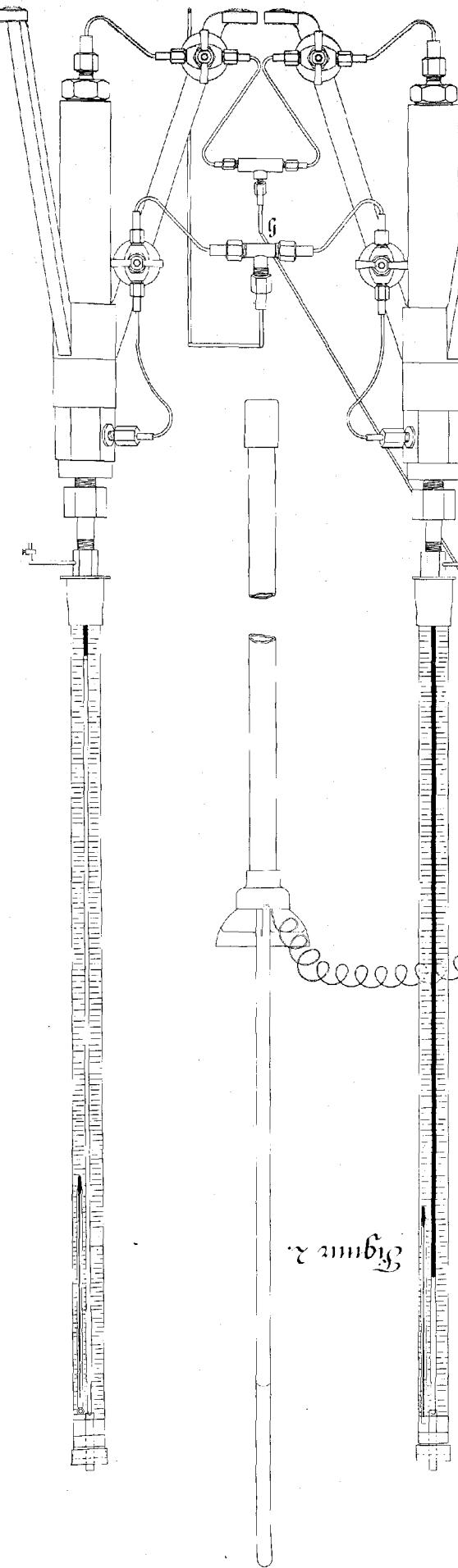
When one measurement has been performed we may make a second determination by tapping once more, and so on, as long as the quantity of the liquid phase in the reservoir is sufficient. At each successive determination we find a new value for the pressure to which the new pair of coexisting phases belongs.

In this way a series of a certain number of successive determinations was generally made at the same temperature.

§ 3. *Description of the apparatus* (fig. 2—4).

The reservoir R consists of a drawn brass tube (long 55 c.M. internally) closed at both ends by a plug screwed and soldered in

¹⁾ v. ELDIK, Proceedings of the Meeting of May 29 '97 p. 20.



it. The liquid-space A is contained between the needle points of a double cock; the chamber of cock A_1 is connected with the reservoir by a brass overpipe a , which is provided with two nuts, and in which a narrow steel capillary tube has been soldered which is so long that the lower end of it reaches as far as the chamber of A_1 and the upper end not only goes through the plug but emerges nearly 5 m.m. beyond it.

Thus has been obtained that the communication of R with A has as little volume as possible; while at the same time at the lower end of the reservoir an annular space has been formed, in which may settle particles from the walls or from the packings which during the distilling of the gas might get into the reservoir.

A rim on the plug protects the capillary tube from being damaged by the stirring-rod.

In the plug at the upper end of the reservoir three copper capillary tubes have been soldered.

One of these, b , leads to the cock B_1 ; the vapour-space B consists in a spiral made of a copper capillary tube (long 80 cm.) of the same diameter. Care has been taken that the channel through which the vapour streams out has the same diameter over its whole length in order to prevent as much as possible the condensing of the vapour by adiabatic expansion.

A second tube c leads to the cock C_2 and serves to connect the purifying apparatus with the reservoir. The third tube d connects this reservoir with the apparatus for the measuring of the pressure.

§ 4. The measuring of the pressure.

Special care is required for this. By filling the tube which leads to the closed air-manometer with mercury, we may prevent condensation or evaporation in that tube caused by differences of temperature between that tube and the water-bath of the apparatus. But then the mercury-meniscus on which the vapour presses must be within the water-bath and moreover it must be visible in order to enable us to read the difference in height with the meniscus of the closed manometer.

Therefore the tube d has been connected through the cock C_1 with the copper capillary tube e , which ends in the upper part of the glass reservoir D , which is again connected with the mercury in the air-manometer through the steel capillary tubes f , g and the steel cock D_1 . With a view to greater accuracy two air-manometers are used, the one for pressures from 4 to 20, the other for pressures from 18 to 90 atmospheres, so that their indications may be compared together.

The pressure-cylinders of the two manometers are connected at their upper ends with an hydraulic pump; from the latter and from the tube h they are separated by cocks, so that each of them may be put out of use separately. By means of the hydraulic pump the mercury may be forced from one of the pressure-cylinders into the capillary tubes h , g and f , and so at the required height into D .

The difference in height of the mercury in D and in the manometers is taken into account as a correction for the pressure.

The volume of D is so large that after a change of pressure the mercury-meniscus in it always remains visible, without the hydraulic pump being required continually.

§ 5. *The water-bath.*

A constant temperature is obtained by causing water to flow through the water-bath in which the apparatus is placed. Whenever water from the main was used the temperature was regulated by means of the same apparatus which was used by Dr. VAN ELDIK in his investigations on capillary ascension¹⁾.

If the temperature of the water from the main was higher than was required for the observations it was cooled by means of ice and a circulation was brought about, on the whole agreeing with that used by Dr. DE HAAS in his researches on the viscosity of liquids²⁾.

The water-bath is made so that the whole apparatus may be put into it from above. For this the four cocks B_1 , B_2 , C_1 , C_2 are mounted on a base-plate H , to which the cover K of the water-bath is fastened. In the centre of this cover is an opening to let through the tubes e and f ; a rim surrounding that opening serves to fasten a glass tube L in it. Besides in the four corners of the cover are smaller openings in which the tubes b' and c' , the other end of and a thermometer are tightly fixed by means of corks. At the lower part of the base-plate is screwed an anglepiece with sliding tube supporting the reservoir.

The water-bath itself consists of a rectangular box M , containing the four cocks and a cylinder N , surrounding the reservoir. In the front of the box are four openings with necks corresponding to the steel pins of the cocks; when these have been placed into the water-bath we apply brass lengthening-pieces to those pins and the openings are shut water-tight by means of corks. The double cock A has a water-bath of its own soldered to it; an india-rubber ring P (high 10 c.m.), which can slide along the cylinder, and when it is

¹⁾ VAN ELDIK, l. c. p. 22 Comm. etc. N°. 39.

²⁾ DE HAAS. Dis. Leiden 1894, p. 8.

tightly fixed completes the waterjacket. One of the glass side-walls of the box *M* enables us in the case of a leak in the apparatus, to find out where the gas escapes. As in the water-bath alone there are twelve connections with overpipes and nuts, this measure is not superfluous.

During my observations at 9 to 10° the temperature in the room exceeded that of the water-bath. In this case the water flows into the apparatus through *E* and can leave it through the pipe *F*; we may then expect the temperature of the water when flowing through to rise a little, but by no means to fall. Thus we prevent the liquid from distilling from *R* into *D*. Yet from time to time a small quantity of liquid appeared on the mercury in *D*; however no perceptible disturbance of the equilibrium between the two phases ensued, at least the manometer showed no permanent change.

In case the temperature of observation should exceed that of the room the same result may be obtained by letting the water in through *F* and by causing it to flow off through *E*. *G* may serve here to prevent the level from rising too much if *F* for the time being does not lead off a sufficient quantity.

§ 6. The Stirring.

The soft iron stirring-apparatus consists in a little rod (long 15 m.m.) provided at both ends with a disc (broad 12 m.m.) and moved electro-magnetically by means of a coil *S*, outside the water-bath. A cylinder *T* of sheet iron fitting round the reservoir moves together with the coil and serves to concentrate the lines of magnetic force in its axis. The current for this coil is supplied by four portable accumulators.

§ 7. The analyses.

The composition of the gas-mixture, received above the mercury, is determined by having the carbonic acid absorbed by pieces of caustic potash (cylindres of 10 to 15 m.m. length). They must not be too dry, else we are not sure that all the carbonic acid will be absorbed. But on the other hand care must be taken that those pieces are not too moist, as a separate experiment showed that the methylchloride is dissolved by a saturated solution of caustic potash. After some practice I soon succeeded in introducing potash in the desired state of humidity into the gasometric tubes.

For some days the gas is left in contact with the potash. In order to hasten the absorption the tubes are plunged as far as possible into deep reservoirs filled with mercury, each consisting in an iron gaspipe, closed at the bottom and carrying at the top a wide glass basin. The absorption of the carbonic acid may also be hastened by moving the tube in the mercury up and down.

In determining the quantity of gas in the tube the latter is sunk into the mercury so deep that the measuring of the volume and of the pressure of the gas may be done with the same accuracy. Fitting round the tube are elastic cylinders of thin sheet iron, provided with sharp points, which are bent downwards. The needle is pointed at the mercury surface, and the rim allows to read the divisions on the glass tube. In doing this an india rubber stopper bearing a glass water-jacket is fixed round the glass tube. By means of this simple contrivance we know also the temperature of the gas to be measured and so the volume may be reduced to 0° C and 1 atmosphere.

In determining the quantity of the liquid-mixture and of the vapour-mixture drawn off, we also take into consideration the portion of the mixture which remains in the spaces *A* and *B* after the flowing off under the barometric pressure. Especially for the vapour at low pressure this should not be disregarded.

As the measuring tubes were filled with mercury in the ordinary way, there remained a small quantity of air in the theoretical vacuum; this quantity was determined and taken into account.

§ 8. Results.

The plait on the ψ -surface of carbonic acid and methylchloride

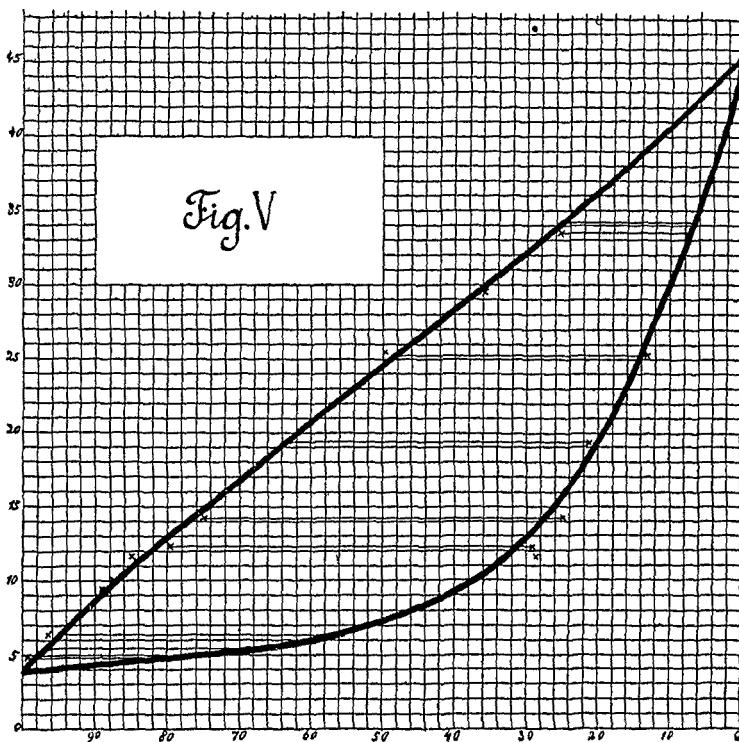
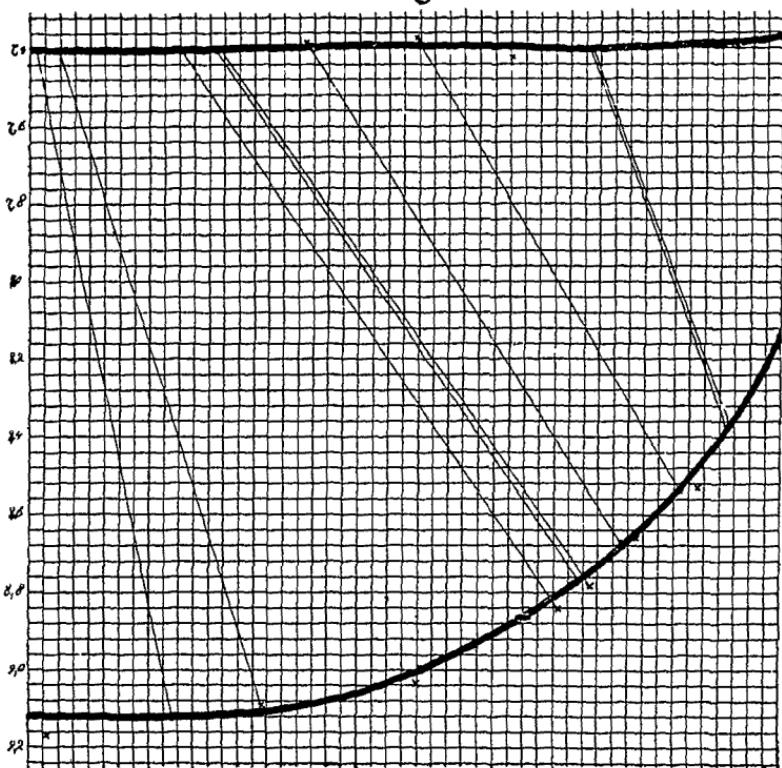


Fig.VI



at $9^{\circ}5$ has been investigated. As it was especially important to learn as well as possible the character of the plait at one definite temperature, corrections were applied for slight differences in order to reduce all the determinations to the same temperature. The results obtained as derived from the determination of ten coexisting phases are only provisional; but yet they are sufficient to show us the character of the plait as resulting from the observations. A graphical representation of the results will be sufficient.

Fig. 5 gives the quantity of methylchloride contained in the mixture (x of VAN DER WAALS) in function of the pressure in atmospheres (as ordinate) for the liquid- and for the vapour-phase. Fig. 6 shows further the value of the logarithm of the volume in function of x . As unit of volume of a mixture we have chosen that at 0° C. and 1 atmosphere. For the sake of a convenient representation the logarithm of the volume instead of the volume itself has been chosen as ordinate (turned towards the bottom of the page). From this figure and some lines drawn in it, which connect the coexisting phases, we may immediately deduce the projection on the $x-v$ plane of the connodal curve with the conjoint points of contact.

It is remarkable that the liquid-curve in fig. 5 should differ so little from the straight line. So it seems that for these mixtures at the chosen temperature the vapour-pressure of the liquid-phase may be represented with near approximation by:

$$P = P_1 (1-x) + P_2 x$$

in which P_1 and P_2 represent the vapour-pressures of the components. If we compare this with the formula given by VAN DER WAALS¹⁾, it appears that the exponents in the latter differ little from zero.

Physics. — „*Considerations concerning the Influence of a Magnetic Field on the Radiation of Light.*” By Prof. H. A. LORENTZ.

§ 1. The assumption that every molecule of a source of light contains a single movable ion, which can be displaced in all directions from its position of equilibrium and is always driven back to that position by the same force, proportional to the displacement, leads to the elementary theory of the phenomenon discovered by Dr. ZEEMAN. Viewed across the lines of force, a single spectral line must, by the action of the field, be tripled, and viewed along the lines of force, be doubled; besides, the components of the triplets and doublets must be polarised in a well known manner.

Whilst the first observations of ZEEMAN were consistent with this theory, and he soon could confirm the theoretical predictions by the observation of distinct triplets and doublets, yet it has become apparent that the case is often less simple. CORNU proved, that f. i. in the case of one of the sodium lines, viewed across the lines of force, the central component of the triplet is doubled, so that in reality a quadruplet is seen. MICHELSON and Th. PRESTON observed in many cases not only a far more complicated structure of the central component, but a similar structure of the outer components of the triplet. According to these observations, the word „triplet” is hardly applicable, though there is always an important difference between the central part of the appearance in the spectrum (the two central lines, f. i. of CORNU’s quadruplet) and the outer parts; the first is plane polarised, the plane of polarisation being perpendicular to the lines of force, whereas in the right and left part the plane of polarization is parallel to the lines of force.

§ 2. The facts mentioned evidently make it necessary to replace

¹⁾ V. D. WAALS, Verslag Kon Akad May 28 '91 p. 416.

the elementary theory by a more complete one. Some time ago, I examined¹⁾ therefore what phenomena are to be expected, if a molecule, having an arbitrary number of degrees of freedom and arbitrarily distributed electric charges, is oscillating about a position of equilibrium.

Before returning to this subject, I will consider the conclusions which may be obtained by arguments from symmetry, without entering into the details of the mechanism of radiation.

There can be no doubt, that we may consider the source of light as a system of extremely small particles, oscillating partly with the frequency of the light vibrations; in virtue of their electric charges, these particles must excite in the surrounding aether periodically oscillating dielectric displacements. These constitute the luminous motion radiated by the source.

For brevity's sake this entire system will be indicated by S .

We may now conceive a second system S' , which is the image of S , relatively to a fixed plane P . The meaning of this is as follows.

If A is a particle in S , there is in S' a particle A' , which is the image of A and of the same physical nature as this particle. Especially, the mass and the electric charge are the same; or, to speak more accurately, in corresponding points of A and A' the same material density and the same density of electric charge will be found. Moreover the particles A' will be *at every moment* the image of the particles A , or, as we may say, the motion of the ions in S' will be the image of the motion in S . If this is the case, the luminous motion in the aether in S' will likewise be the image of the motion in S , in this sense that the vector representing the dielectric displacement in S' is always the image of a corresponding vector in S .

Of course all this will only be possible, if the forces operating on the particles A' are the images of those to which the particles A are subjected. So far as the *mutual* action of the particles is concerned, we may regard this as a consequence of the supposed equality in physical nature. In order that the forces, originated by the external magnetic field, may satisfy the same condition, we will suppose that the vectors, representing the magnetic force in S' , may be derived from the corresponding ones in S by first taking their images, and then reversing the directions of these images²⁾.

¹⁾ Wied. Ann. Bd. 68, p. 278, 1897.

²⁾ If the magnetic field is generated by electric currents, we may imagine the required field in S' to be produced by currents, which are the images of the currents in S .

It will also be assumed that the properties of the image of the source of light, as far at least as we are concerned with them in really observable phenomena, are the same as those of the source itself, so that the latter may be substituted for the image. Finally, we suppose that the entire luminous motion in the aether is developed by means of FOURIER's theorem into simple harmonic motions; when the total luminous motions in S and S' are each other's images, the same will evidently be true of those parts of the luminous motions, which have a determinate period T — or rather periods between two definite limits T and $T + dT$.

§ 3. Let Q be a straight line, drawn from any point in the source of light parallel to the lines of force, and let L denote the luminous motion *with a definite period T* , existing at a distant point of Q . By taking the image of the whole system, relatively to a plane parallel to the line Q , it is easily seen that the image L' is exactly the luminous motion, that would exist in the point considered, if, the source of light remaining unchanged, the direction of the field were reversed. Hence L' may very well differ from L , but, in all observable properties, L' must remain unchanged, if the reflecting plane be turned around the line Q as axis.

Whence it follows, that, if all vibrations of L are resolved parallel to a line R , perpendicular to Q , the intensity produced by the components must be independent of the direction of R . Indeed, R_1 and R_2 being two lines perpendicular to Q , and I_{r1} and I_{r2} the intensities corresponding to them in the manner indicated, we may give to the reflecting plane two positions P_1 and P_2 in such a way that the image of R_1 , relatively to P_1 , coincides with that of R_2 , relatively to P_2 . Indicating by R' the direction of these coinciding images and by $I'_{r'}$ the intensity corresponding to this direction of vibration in L' — this quantity remaining the same, as was remarked above, for every position of the reflecting plane — we may write $I_{r1} = I'_{r'}$ and $I_{r2} = I'_{r'}$; hence $I_{r1} = I_{r2}$.

In this way we come to the conclusion that the light propagated along the lines of force, and having a definite period T , or, in other words, occupying a definite place in the spectrum, cannot be polarised plane or elliptically, neither completely, nor partially. It can only be unpolarised, or circularly polarised; in the latter case the polarization can be partial as well as complete.

The light would be unpolarised, if an influence of the magnetic field did not exist at all. As far as we know, the components of the doublets seen along the lines of force are completely circularly pola-

rised. From the above considerations it however appears that the radiation might also be partially circularly polarised. We see at the same time that, if in a given place of the spectrum the polarization is righthanded, it must become lefthanded at the same place by reversing the magnetization.

§ 4. Arguments of the same kind may be used when the observations take place across the lines of force. Now, we place the reflecting plane perpendicular to these lines. The magnetic field remains unchanged; consequently, the luminous motion must have the same properties as its image. Hence the light, observed in a given place of the spectrum, cannot be circularly nor elliptically polarised — neither completely nor partially. It must be either unpolarised light, or plane polarised — wholly or in part — the plane of polarization being parallel or perpendicular to the lines of force.

It needs scarcely be mentioned, that all observations are in agreement with this conclusion.

§ 5. A closer examination of the mechanism of radiation gives us a relation between the light radiated along and the one radiated across the lines of force. At least one conclusion concerning this point lies at hand.

Let M be a single molecule of the source of light, and let three rectangular axes OX , OY , OZ be drawn, the first along the line of force. Let e be the electric charge in a point of the molecule, having the coordinates x , y , z ; then we may call Σex , Σey , Σez — calculated for the entire molecule — the components of the electric moment of the particle.

These quantities are continually changing, and will perhaps be extremely complicated functions of the time. By means of FOURIER's theorem, we may however separate the parts that have a determinate period T . We will confine ourselves to these parts and denote them by M_x , M_y , M_z .

If now the dimensions of the molecule are very small compared with the wave-length, then, the observer being supposed at a distance of a great many wave-lengths, it may be deduced from theory, that in all points of OY , light is produced merely by the variations of M_x and M_z , M_x producing vibrations along, and M_z across the lines of force. Similarly for points of OX and OZ .

Suppose, that, when viewing across the lines of force, f. i. from a point of OY , in a given place of the spectrum light is seen

which is entirely plane polarised, the plane of polarization being perpendicular to the lines of force.

Then, at the place in question, there will not be any luminous motion produced by Mz , and, because the molecules are vibrating independently of each other and hence light emitted by one can never be totally destroyed by that originating from another, Mz must vanish in all molecules. Of course the same argument applies to My ; hence it follows, that no light can be observed from any point of OX , that is to say in the direction of the lines of force.

BECQUEREL and DESLANDRES have found¹⁾ that one of the iron lines, when viewed across the lines of force, becomes a triplet, the central and side components of which, as compared with those of the ordinary triplets, have interchanged their states of polarization²⁾. The foregoing reasoning entitles us to predict, that only the middle component of this triplet will be visible, when the phenomenon is observed in the direction of the lines of force.

§ 6. In the paper cited above, I have established the equations of motion for infinitely small vibrations of a molecule, having n degrees of freedom, and placed in a magnetic field. I called p_1, p_2, \dots, p_n the general coordinates, chosen in such a manner, that they are 0 in the position of equilibrium, and that they are principal coordinates as long as there is no external magnetic force. I obtained for the equations of motion

$$\begin{aligned} a_1 \ddot{p}_1 + b_1 p_1 - (c_{1,2} \dot{p}_2 + c_{1,3} \dot{p}_3 + \dots + c_{1,n} \dot{p}_n) &= 0, \\ a_2 \ddot{p}_2 + b_2 p_2 - (c_{2,1} \dot{p}_1 + c_{2,3} \dot{p}_3 + \dots + c_{2,n} \dot{p}_n) &= 0, \end{aligned} \quad \left. \right\} \dots (1)$$

etc.,

where a and b are constants, independent of the magnetic force. The influence of the field is expressed by means of the terms containing the quantities c , which are all proportional to the intensity of the field.

They further satisfy the relations

$$c_{r,s} = -c_{s,r} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (2)$$

To determine the possible periods of vibration, we put, according to a known method, in (1):

¹⁾ Comptes rendus, 4 avril 1898.

²⁾ The same phenomenon has been observed in the case of some iron-lines by AMES, EARHART and REESE, Johns Hopkins Univ. Circular. Vol. 17, No. 135.

(95)

$$p_1 = \mu_1 e^{lt}, \quad p_2 = \mu_2 e^{lt}, \quad \dots \quad p_n = \mu_n e^{lt},$$

and eliminate $\mu_1, \mu_2, \dots, \mu_n$. If

$$\frac{b_1}{a_1} = k_1^2, \quad \frac{b_2}{a_2} = k_2^2, \quad \dots \quad \frac{b_n}{a_n} = k_n^2.$$

and

$$\frac{e_{r,s}}{a_r} = -e_{n,s},$$

the result may be put into the form

$$\left| \begin{array}{cccccc} l^2 + k_1^2, & e_{1,2} l, & e_{1,3} l, & \dots & e_{1,n} l \\ e_{2,1} l, & l^2 + k_2^2, & e_{2,3} l, & \dots & e_{2,n} l \\ \vdots & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ e_{n,1} l, & e_{n,2} l, & e_{n,3} l, & \dots & l^2 + k_n^2 \end{array} \right| = 0 \dots (3)$$

In consequence of the relation (2), the development of the determinant will contain only even powers of l^2 . Hence an equation is obtained, of the n^{th} degree in l^2 . From the circumstances of the case it follows, that the roots of this equation are all real and negative; hence n pairs of imaginary values of l are obtained. If $+i k'_r$ and $-i k'_r$ are two of those values, there will be a mode of vibration with the frequency (number of vibrations in the time 2π) k'_r .

Evidently, without the field, the frequencies would become

$$k_1, \quad k_2, \quad \dots \quad k_n,$$

and it is clear that, if there is a magnetic field, each of these frequencies is modified into a value k'_r , differing very slightly from k_r .

In the cited paper I had restricted the development of (3) to terms containing the products of two factors e . Denoting by Π the product

$$(l^2 + k_1^2) \quad (l^2 + k_2^2) \quad \dots \quad (l^2 + k_n^2)$$

and by $\Pi_{r,s}$ the value, got from this by omitting the factors $l^2 + k_r^2$ and $l^2 + k_s^2$, we obtain

$$\Pi - \sum l^2 e_{r,s} e_{s,r} \Pi_{r,s} = 0, \quad \dots \dots \dots \dots \dots \dots \quad (4)$$

where the sum is to be extended to all combinations of unequal indices r and s .

I inferred from this equation, that a triplet can only be observed, if three of the values k are equal, or, in other words, if the system has three equivalent degrees of freedom. This will also be clear when it is considered, that by a continuous decrease of the magnetic field, the three components of the triplet may be made to coincide, so that the simple spectral line, as seen out of the field, may be considered as consisting of three coinciding lines. Applying the same argument to CORNU's quadruplet, it seems natural to suppose that the lines, which are apt to undergo this modification, consist already, under ordinary circumstances, of four coinciding lines, or otherwise, that now we have four equivalent degrees of freedom, or four equal values k .

Yet, the origin of a quadruplet cannot be explained by equation (4). Indeed, if k_1, k_2, k_3, k_4 are the frequencies having the same value k , there are in each term of (4) at least two factors $l^2 + k^2$. Hence the equation must still have two equal roots $-k^2$, and besides only two roots, differing very little from $-k^2$.

§ 7. It was however brought to the notice of the author by Mr. A. PANNEKOEK that in this case equation (4) is incomplete, because some of the terms neglected are of the same order of magnitude as those retained, and that, by returning to equation (3), an explanation of the quadruplet may be arrived at.

If $k_1^2 = k_2^2 = k_3^2 = k_4^2 = k^2$, certainly four roots of equation (3), if not $-k^2$, will differ only very little from this value.

If l^2 is one of these values (we need not occupy ourselves with the other values of l^2), then the four quantities $l^2 + k_1^2, l^2 + k_2^2, l^2 + k_3^2, l^2 + k_4^2$ will be small. On the other hand, the quantities

$$l^2 + k_5^2, \quad l^2 + k_6^2, \quad \dots \quad l^2 + k_n^2 \quad \dots \quad (5)$$

will have values, which by no means become small. Since all the quantities el are likewise small, the elements (5) of the determinant will exceed by far all other elements, and we shall obtain a sufficient approximation, when we take in the development of the determinant only those terms, which contain all the quantities (5). Evidently, the equation serving for the determination of the values of l^2 , which differ only slightly from $-k^2$, will therefore be

$$\begin{vmatrix} l^2 + k^2, & e_{1,2} l, & e_{1,3} l, & e_{1,4} l \\ e_{2,1} l, & l^2 + k^2, & e_{2,3} l, & e_{2,4} l \\ e_{3,1} l, & e_{3,2} l, & l^2 + k^2 & e_{3,4} l \\ e_{4,1} l, & e_{4,2} l, & e_{4,3} l, & l^2 + k^2 \end{vmatrix} = 0.$$

If we develop this determinant, all terms, which have an odd number of factors $e_{r,s} l$, are excluded by (2). Hence the equation may be written

$$(l^2 + k^2)^4 + A(l^2 + k^2)^2 + B = 0, \dots \dots \dots \quad (6)$$

where A contains terms with 2 factors $e_{r,s} l$, and B terms with 4 factors of this kind. In all these factors l^2 may be replaced by $-k^2$. Consequently, A and B can be found, and A is now proportional to the square and B proportional to the fourth power of the intensity of the field.

From (6) we get two values of $(l^2 + k^2)^2$, which are both real and positive, because, as was already remarked, real values must be found for l^2 . Hence the solution of (6) may be represented by

$$(l^2 + k^2)^2 = \alpha^2, \dots \dots \dots \dots \quad (7)$$

and

$$(l^2 + k^2)^2 = \beta^2, \dots \dots \dots \dots \quad (8)$$

where α and β are known, say positive, quantities. By reason of what has been remarked about A and B , the values of α and β will be proportional to the intensity of the field.

Finally from (7) and (8) the following *four* values of l^2 are obtained:

$$l^2 = -k^2 + \alpha, \quad -k^2 - \alpha, \quad -k^2 + \beta, \quad -k^2 - \beta,$$

so that in fact there must be seen a quadruplet in the spectrum. In order that the four lines of this quadruplet may be perfectly sharp, it is however necessary, that in a given magnetic field the quantities α and β are independent of the direction into which the molecule is turned, or, what comes to the same thing, that, for a given position of the molecule, α and β are independent of the direction of the magnetic force.

Mr. PANNEKOEK has also remarked, that a similar reasoning applies when an arbitrary number, e. g. p , frequencies k are equal. In this case we come to the conclusion that, for a given position of the

molecule in the field, the simple spectral line must be separated into a p -fold line, in such a way, that the position of the different components is symmetrical to the right and left of the original line. From this it follows, that if p is odd, one component remains at the place of the original line.

It seems however very difficult to conceive a system, having really, as is necessary for quadruplets, four equivalent degrees of freedom, especially if in addition to this, it is required that the values of α and β must be independent of the direction of the magnetic force, relatively to the molecule. I have not been able to find out a system, really fulfilling these conditions. It is true, it might be argued that the very existence of a quadruplet proves the equality of four frequencies, when there is no magnetic field, and hence that the above theory of the quadruplet must be true, even though the mechanism has not yet been found out. However I have some scruples to be satisfied with this view of the case, for I think, it is not yet quite certain, that the vibrations which produce light are really to be described by equations of the form (1).

Physics. — On an Asymmetry in the Change of the spectral Lines of Iron, radiating in a Magnetic Field. By Dr. P. ZEEMAN.

1. It is known that in the elementary treatment of the influence of magnetic forces on spectral lines according to LORENTZ's theory it is sufficient, if only one spectral line is considered, to suppose that in every luminous atom is contained one single moveable ion moving under an attraction proportional to the distance from its position of equilibrium. All motions of such an ion can be resolved into linear vibrations parallel to the lines of force and two circular vibrations, righthanded and lefthanded perpendicular to the lines of force. The period of the first mentioned vibration remains unchanged, those of the last are modified, one being accelerated and the other retarded. The doublets seen along the axis of the field, the triplets seen across it are in this manner simply explained and also the observed polarisation-phenomena. Besides we must expect according to the theory that the outer components of the triplet are of equal intensity and likewise the two circularly polarized components of the doublet. Eye observations as well as the negatives taken by myself and others have always confirmed till now this most simple symmetrical distribution of intensities. The question arises cannot the external magnetic forces, sufficient to direct the molecular

currents assumed in the ionic theory of magnetism¹), favour the circular vibrations more than those along the lines of force²). If this be assumed we are also compelled to admit that the revolving of the ions takes place more in a given direction than in the contrary. Hence then there must be a difference of intensity between the two outer components of the triplet and between the two components of the doublet. Although the ordinary magnetism of the highly-magnetic substances has probably disappeared in the spark, it seems rather natural to examine in the first place iron, nickel and cobalt in search of a phenomenon in which the „molecular currents” of AMPÈRE (or that part of these currents, which is produced by the motion of the light-ions) would manifest themselves optically. However it seems to me by no means decided beforehand, that other substances would not exhibit something of this kind. I have however investigated in the first place iron.

The first results obtained were much promising. In the field used several of the iron lines exhibited on the negatives a more intense component at the less refrangible side of the spectrum. Further inquiry has however shown that this seemingly positive result seems to be of no value. I will give the results of my experiments only in extract. Before describing them, it may be remarked, that, if a directing influence, as mentioned, exists, we must expect that the component at the less refrangible side must be intensified and in the case of the triplet as well as in that of the doublet. The sign of the charge of the ions cannot have any influence upon this result.

2. Negatives were taken in the spectra of the third and second orders obtained by means of a ROWLAND grating (radius 10 ft., 14438 lines to the inch). More accurately was studied the part of the spectrum between 3000 and 4000 A. U., when viewed in the two principal directions across and along the lines of force. The vast majority of the iron lines were, with the field used, resolved into doublets, triplets, quartets etc. perhaps only three or four lines seemed unaffected. Now I found in the case of a few lines inequality between the outer components of a triplet across and of the corresponding

¹⁾ cf. RICHARZ. Wied. Ann. **52**, p. 385, 410. 1894.

²⁾ cf. LORENTZ. Versl. Ak. Amsterdam, October '97, p. 213. [It was pointed out by LORENTZ in the article referred to, that the phenomena observed by EGOROFF and GEORGIEWSKY can be explained, without any hypothesis of the kind mentioned by the absorption, which the rays from the posterior part of the flame undergo in the anterior part.]

doublet along the lines of force. On the negative the component at the red side of the spectrum was darker, independently of a commutation of the current. Of course the difference of intensity is dependant upon the time of exposition. Upon some of the negatives the difference was for a special line perhaps 50 or 100 percent.

However it was plain enough, that the outer components of the triplets and also the two components of the doublets were, in the case of the *strong* iron lines, of equal intensity. Now in the case of feebler lines, let L be one of them, perturbations will be possible due to the overlapping of one of the components of a „normal” triplet or doublet and a feeble line, say but slightly affected by magnetism. The latter line can 1^o be present near L in the same spectrum; or 2^o belong to a spectrum of another order as the line \bar{L} ; or 3^o by the very presence of the field a special line may become relatively to other lines more intense or a new line may be originated. By taking negatives with different fields it will of course be possible to evade difficulties from these three causes, at any rate, if the supposed line is thin. We can however by taking also negatives in absence of the field exclude 1, and by taking negatives in spectra of different order or by cutting off any interfering spectrum in using absorbents 2. Having done this, it appeared that also case 3 sometimes occurs; the intensity of the iron lines relatively to the air lines varies considerably and the mutual intensity of the iron lines appreciably. New lines appear, at least lines absent on negatives taken with the field off, became distinctly visible, while yet the intensity of the field was insufficient to resolve the lines in triplets etc.

The last mentioned perturbation is of course most treacherous. Using however fields of varying intensities, I could avoid perturbation 3. Excluding however 1, 2, 3, only triplets, doublets etc. remained, which, I think, can only be called quite symmetrical. Hence till now there is no evidence for a directing influence of the magnetic field on the orbits of the light-ions.¹⁾

¹⁾ cf. PRESTON, Phil. Mag. Vol. 45, p. 333. 1898.

Royal Academy of Sciences. Amsterdam.

PROCEEDINGS OF THE MEETING

of Saturday September 24th 1898.

(Translated from: Verslag van de gewone vergadering der Wis- en Natuurkundige Afdeeling van Zaterdag 24 September 1898 Dl VII)

CONTENTS: „Congealing points and points of transition in mixed crystals of two substances”. By Prof. H. W. BAKHUIS ROOZEBOOM, p. 101. — „On the influence of salt-solutions on the volume of animal cells, being at the same time a contribution to our knowledge of their structure”. By Dr. H. J. HAMBURGER, p. 103.

The following papers were read:

Chemistry. — „Congealing points and points of transition in mixed crystals of two substances”. By Prof. H. W. BAKHUIS ROOZEBOOM.

In 1891 the author has laid down a theory on the building of mixed crystals from solutions. This theory was applied to equilibrium in systems of three substances.

Up to now no general rules had however been given for the building of mixed crystals from fused mixtures of two substances.

The investigations in this direction had either been limited to the influence of small quantities of the second substance on the fusing-point of the first (VAN 'T HOFF) or, as far as this had been extended to greater concentrations, it had been done without any theoretical guide and had led to false conclusions (KÜSTER).

It is possible to plan a survey of the different cases of equilibrium, which may present themselves between the fused mass and the mixed crystals by making use of the condition of equilibrium that the thermodynamic potential be a minimum.

Now VAN RIJN VAN ALKEMADE has already stated formerly, how in case of a continuous series of solid mixtures the potential

as a function of the concentration is represented by a similar curve as that representing the potential values of mixtures of two liquids. The problem therefore is to find out what cases may present themselves when these two curves are made to alter their relative positions, in passing from higher temperatures to lower.

The result of this investigation is, that where a continuous mixture-series exists, three cases are possible.

Case 1. The congealing points of mixtures of liquids of the substances *A* and *B*, fall gradually from the congealing point of *B* to that of *A*, in proportion as the composition of the fused liquid is shifted in the direction of *A*. (in case of *B*'s being the substance having the highest fusing-point).

Case 2. The line of the congealing points shows a maximum.

Case 3. The line of the congealing points shows a minimum.

In all the three cases there is the following connection between the concentration of the fused and the solid mixtures:

The solid mixture contains, in comparison to the fused, a higher percentage of that ingredient, at the increase of which the congealing-temperature rises. In the maximum and in the minimum the two concentrations are equal.

Important conclusions for the process of congealing and fusing and for the fractionation of mixed crystals may be deducted from this.

If the mixture-series in the solid state is not continuous, it may be deducted that two cases are possible.

Case 4. The line of the congealing points shows a break in a transition temperature, situated between the fusing points of the two components. At this point occurs a hiatus in the composition of the mixed crystals.

Case 5. The line of the congealing points shows two descending branches, meeting in a minimum point, below which temperature every fusion congeals to a conglomerate of mixed crystals with two different concentrations

There may still appear further complications, should there exist more than one solid modification in either of the components or in both. Following the same lines that have led to the discovery of the connection between temperature, concentration of solid mixture and concentration of liquid, it is equally possible to find in what manner the transition takes place from one solid state into the other with different mixing-proportions both in case of homogeneous and non-homogeneous mixtures. The number of peculiar cases is here extraordinary, owing to the fact that in both components the corresponding conditions may follow each other in different manners

and the transition-temperatures may lie at different distances from each other.

The method given enables us to deduct the phenomena in each particular case and promises to be an important expedient in solving the complicated phenomena that appear in the congealing and the consequent changes in alloys.

Physiology. — *On the influence of salt-solutions on the volume of animal cells, being at the same time a contribution to our knowledge of their structure.* By Dr. H. J. HAMBURGER.

My investigations concerning the connection which exists between the power of salts to attract water and their power to extract colouring matter out of the red blood-corpuscles, led to the hypothesis that for every salt a solution might be found in which the corpuscles would retain the same volume that they have in their own serum. This supposition proved to be correct and it could moreover be shown that those solutions which left the volume unchanged represented a precisely equal hygroscopic power.

The experiments referred to, made in 1884 and suggested by the classic analysis of the phenomena of turgescence in plant-cells by our countryman, HUGO DE VRIES, have been to me as well as to others the point of departure for a series of researches which among physiologists and pathologists have awakened a constantly increasing interest in the new doctrine of osmotic pressure. To this result the theory of electrolytic dissociation of VAN 'T HOFF and ARRHENIUS has of late contributed. And in combination with each other these two theories are on the way to clear up many a dark point in the domain of physiology. One might have supposed that in the course of 14 years other animal cells would have been investigated in the same way as have been the red blood-corpuscles. This, however, has not been the case. For various purposes indeed the hygroscopic power of fluids, such as blood, lymph, serous fluid, milk, has been studied; to the serous and mucous membranes, indeed, fluids of different osmotic pressure have been offered for absorption, in order to deduce from the change in this osmotic pressure data for the knowledge of the resorption process; but the influence of salt solutions on any other than the red blood-cells has not yet been investigated. And yet for more than one reason this is to be desired. In the first place in order to control the inferences that have been drawn from the observations made with red blood-corpuscles, and to decide many differences of opinion which

exist on them; but also with a view to the questions for the solution of which the red blood-cells are not serviceable, because they are known to lose their colouring matter very easily.

For years then it had been my desire to extend my researches in the above direction, yet, extremely simple as my plan was in principle, I was checked by the difficulty of obtaining a sufficient quantity of isolated cells, which, with the means, then at my disposal, would have enabled me to determine their volume.

This difficulty has now been overcome by the use I can make of little tubes which I constructed last year to compare the volume of bacteria in two cultures, and which admit of experiments with small quantities¹⁾. We have, as yet, investigated four sorts of cells, white and red corpuscles of different animals, spermatozoa of the frog and the intestinal epithelium of the horse and the pig.

We shall treat of the intestinal epithelium on some future occasion.

Phenomena of a peculiar nature, which have manifested themselves while studying it, make a further research necessary.

White blood-corpuscles.

These were obtained by leaving horse-blood, in a closed bottle defibrinated with pieces of glass. The red corpuscles then for the greater part precipitate, while the white all remain suspended in the fluid above. Their number is comparatively small. Now in order to obtain a fluid abounding in white blood-corpuscles, $\frac{1}{2}$ L. of turbid fluid was centrifugalized, 470 cc. of the clear serum eliminated and the precipitate equally divided in the remaining fluid. Of this fluid about $\frac{1}{4}$ c.c. was measured off and placed in test-tubes with 15 c.c. of different salt solutions. After repeated mixing, and after half an hour or a little longer 5 cc. or more were placed in the above mentioned little funnel-shaped tubes and centrifugalized till (the rapidity of revolution of the centrifuge not varying) the level remained constant for fifteen minutes.

Fluids.	Volumes of precipitate.
Na Cl solution 0.7 pCt.	46.25
Serum (isotonical with Na Cl 0.9 pCt.)	41
Na Cl solution 1 pCt.	39.25
Na Cl solution 1.5 pCt.	33.5

¹⁾ Report of the meeting of the Royal Academy of Sciences, April 21st 1897.

A regular diminution of volume is seen.

Meanwhile among the whiteblood-corpuscles there were a number of red ones. In the case under consideration 569 red ones were counted to 109 white, and the question now was whether, and to what extent the red ones must be considered responsible for this result. In order to get an idea of the share that each had contributed to the volume, let us for a moment suppose that the diameter of the white corpuscles amounts to double that of the red ones, and moreover that the latter are not discs but globules. In this case the red corpuscles would represent a volume of 569, and the white 872. This calculation tells undoubtedly to the advantage of the red corpuscles, seeing that the diameter of the white is more than double that of the red, while the shape of the latter is a dented disc and not a sphere.

In order now to inquire what share in the contraction and expansion must be assigned to the influence of the red corpuscles, the above experiment was made with red corpuscles only. If we arrange the figures obtained, and the percentage alterations in volume proceeding from them, in a table, which at the same time contains the results just obtained, a striking agreement is evident.

Fluids.	Volume of Precipitate of red corpuscles.	Volume of white and red together. See former table.	Percentage volume-increase, calculated with respect to the volume in serum.	
			Red blood corpuscles.	Mixture of red and white.
Na Cl-sol. of 0.7%.	43.5	46.25	+ 18%	+ 12.8%
Serum (isot. with Na Cl. 0.9%.)	38.5	41	-	-
Na Cl-sol. 1%.	36.75	39.25	- 4.54%	- 4.2%
Na Cl-sol. 1.5%.	31.75	33.5	- 17.5%	- 18.3%

From this table it appears that the alterations in volume of red corpuscles are exactly similar to those exhibited by the mixture of white and red, whence it follows that the red and white blood cells, *caeteris paribus*, expand and contract in an equal measure.

This remarkable result, which, as will presently appear, was confirmed by many other experiments, was made the point of departure for the investigation of which an account now follows.

Though, as may appear from the introduction, it was originally my intention to study the influence of the solution of different salts on the volume of cells, I have hitherto occupied myself only with

Na Cl solutions and mixtures of serum and water, considering them quite sufficient to answer the question which at the outset interested me most. Whence arises the equality of expansion and contraction of red and white corpuscles? A question which immediately led to another, viz., why are the cells expanded by hypotonic and contracted by hyperisotonic solutions?

Now we may with SCHWARZ consider the contents of the cell (exclusive of the nucleus) as a homogeneous mass, or with REMAK, KUPFFER, FLEMMING, BUTSCHLI and many others, as a protoplasmic mass intermingled with fluid. What must happen then in the former case, if in a Na Cl solution of 1.8 % we immerse a cell of which the homogeneous contents correspond in hygroscopic power to a Na Cl solution of 0.9 %? This mass will then contract to one half. If it is placed in a NaCl-solution of 1½ % the expansion must amount to

$$\frac{1.5 - 0.9}{0.9} \times 100 = 66\%. \text{ If on the other hand, we place it}$$

in a NaCl-solution of 0.7 % the expansion must then amount to

$$\frac{0.9 - 0.7}{0.9} \times 100 = 22\%. \text{ What appeared though? That the con-}$$

traction as well as the expansion amounts to much less, viz., respectively 17.5 and 13 % (cf. former table). This leads us to the conclusion that in the red and white corpuscles there must be a substance, which has either no part in the hygroscopic power, or a smaller one than the other ingredient, and the question then presents itself to us, how are we to suppose these substances to be arranged.

Of the different hypotheses which in this respect have been formulated for cells in general, there is only one which has appeared satisfactory to us. And this is the hypothesis of BUTSCHLI¹⁾.

According to this hypothesis the cells consist of a protoplasmic net the closed meshes of which lie close to each other, and the contents of which is a fluid. From the fact that the percentage of the contents of the cells in hygroscopic matter, notwithstanding that salt-solutions of different degrees of strength are brought in contact with it, remains the same, we may suppose that the protoplasmic net is not permeable to salts, but only to water²⁾. We may further suppose that only the fluid contents represent the hygroscopic power of the cell.

¹⁾ Cf. BUTSCHLI, Untersuchungen über mikroskopische Schaume und das Protoplasma.

²⁾ If the blood-corpuscles are considered permeable to salts, a change in the isotonic relations must then take place, for by this only is the immutability of the water-attracting force of the contents of the meshes guaranteed. For what follows, however, it is indifferent which of these two conjectures is correct. The one mentioned in the text is certainly the simplest.

It is thus only the fluid contents which produce the expansion and contraction of the cell by hypotonic and hyperisotonic solutions.

If this conjecture is correct, then in the amount of expansion and contraction of the cells there must lie a measure for the comparative volume of the two ingredients, and the result must then depend to a great extent on the concentration of the salts employed.

Under this impression we began to effect a number of determinations of the volumes of the network-substance in white blood corpuscles.

The white corpuscles used for this purpose were almost entirely freed from the red by simply leaving the fluid rich in leucocytes which was obtained by the method above described at rest for an hour. The red corpuscles were then precipitated and in the fluid above were only the white.

Let us discuss the first series of experiments somewhat more in detail. Equal quantities of white corpuscles are mixed with Na Cl solutions of 0.7, 0.94 (isotonic with the serum) and 1.5%. After centrifugalizing equal quantities of these solutions, the volume of the white corpuscles respectively amounts to 41, 35.5 and 29.5. In order to calculate the volume of the protoplasmatic net by means of these two last figures, we may reason as follows: Suppose p to be the volume of the protoplasm of the network substance, then the volumes of intra-cellular fluid amounts respectively to $(35.5-p)$ $(29.5-p)$. Seeing we must take for granted that the percentage of hygroscopic matter in the intra-cellular fluid remains the same, the following equation must hold.

$$(35.5-p) \cdot 0.94 = (29.5-p) \cdot 1.5 \\ p = 19.4$$

From the following figures obtained with the NaCl solutions 0.7 and 1.5% the equation results

$$(41-p) \cdot 0.7 = (29.5-p) \cdot 1.5 \\ p = 19.5$$

It appears, therefore, that the quantity of protoplasmic network in both cases is nearly equal.

The mean is 19.45.

If we now take into consideration that the total volume of the white blood-corpuscles in the solution of 0.94 which is isotonic with serum, amounted to 35.5, we then find a percentage of

$$\frac{35.5 - 19.45}{35.5} \times 100 = 54.5\%,$$

taken up by the protoplasmic network.

The network-volume of the same white blood corpuscles was further investigated with the aid of mixtures of serum and different quantities of water.

	Volume of white corpuscles		
(1) in undiluted serum			35.75
(2) in serum mixed with	20 %	water	39
3 " "	40 "	"	41.75
4 " "	50 "	"	43.5

From (1 and 2) we obtain the equation :

$$120(35.75-p) = 100(39-p)$$

$$p = 19.5$$

From (1) and (3) we obtain the equation :

$$140(35.75-p) = 100(41.75-p)$$

$$p = 20.6$$

From (1) and (4) we obtain the equation.

$$150(35.75-p) = 100(43.5-p)$$

$$p = 20.3$$

It will be seen, that the figures which are found for p agree very well with each other. The mean is $\frac{19.5 + 20.6 + 20.3}{3} = 20.04$,

i. e. calculated at $35.75 : \frac{20.04}{35.75} \times 100 = 56\%$, a number that corresponds well with that obtained by the NaCl solutions.

The observation might be made, that the numbers 54.5 or 56 cannot give the exact volume of the protoplasmic network as by centrifugalizing only the precipitate and not the real volume of blood-corpuscles is determined. I have, however, convinced myself by experiments, that this has no perceptible influence on the percentage of the network substance.

This may be clearly observed *inter alia* when by means of accelerated velocity of the centrifugal machine we compress the precipitate a little more.

The percentage of p is hardly or not at all changed by it. It must, moreover be borne in mind, (I) that for one experiment all fluids are centrifugalized at the same time and (II) that the figures of the last experiment, 39, 41, 75, and 43.5 are a little greater than corresponds with the real volume of the cells, because there is still fluid among the cells, the same is the case with the number 35.75 with respect to which the percentage has been calculated.

A second observation may be made viz., that in the calculation we

have overlooked the influence of the volume of solid substances dissolved in the intra-cellular fluid. The figure that has been obtained for the volume of the protoplasmic net might perhaps by this have to be a little modified.

It appears difficult to me to state exactly what degree of accuracy has been attained by our determinations. For our object, however, the absolute values are of but little moment. Meanwhile, as will be observed, the correspondence between the ciphers obtained is striking: I should be surprised if they deviated much from the exact number.

We now subjoin a table, comprising some of the experiments just described, and in which the percentage volume of p is calculated in the way that has been indicated.

I	II.	III	IV.	V.
Fluids	Volume observed of white blood-corpuscles	Volume of the protoplasmic network (p)	Mean volume of the protoplasmic network	Mean percentage of protoplasmic network in the white corpuscles
a. Na Cl sol. 0 7%	49.25	calculated from		
b. " " 0.93%	42.75	23.9	a and c	
c. " " 1.5%	35.75	24.3	b " c	56.3%
a. Na Cl sol. 0 5%	40.25	(19.4)	a " c	(64.6%)
b. " " 0 7%	35	16.7	b " d	
" " 0 94%	30.5	16.4	c " d	16.55 54.3%
d. " " 1 5%	25.25			
a. Na Cl sol. 0 25%	30.5	20.46	a " d	(20.46) (98%)
b. " " 0.5%	29.25	(15.8)	b " d	(15.8) (67.9%)
c. " " 0 7%	26.50	12.4	c " e	
d. " " 0 9%	23.25	12.6	d " e	12.5 53.7%
e. " " 1 5%	19			
a. Na Cl sol. 0 25%	37.75	(18.72)	a " d	(18.72) (73%)
b. " " 0 5%	30.25	(16.18)	b " d	(16.18) (63.2%)
c. " " 0 7%	27.25	13.2	c " e	
d. " " 0.9%	24	13.5	d " e	13.35 55.6%
e. " " 1 5%	19.75			
a. serum.	34.75			
b. serum + 20% water	37.75	19.75	a " b	
c. serum + 40% water	40.75	19.70	a " c	19.73 56.7%
d. serum + 50% water	42.25	19.74	a " d	
a. serum.	33			
b. serum + 20% water	36	18	a " b	
c. serum + 40% water	39	18	a " c	17.83 54%
d. serum + 50% water	40.75	17.5	a " d	

From this table it follows:

1. That the volume of the net-work varies between 56.7 and 53.7 pCt. of the whole cell-volume.

2. That, where a solution of NaCl of 0.5 pCt. is employed, a solution which with horseblood causes colouring-matter to issue from a portion of the red corpuscles, the figure for the net-work is found to be greater than where solutions of 0.7 and 1.5 are applied.

What applies to NaCl 0.5, appears to be in a still greater measure the case, when a NaCl-solution of 0.25 pCt. is employed. In the table the numbers bearing on this are indicated in small ciphers.

This result may be thus explained:

By a very weak salt solution the volume of the intra-cellular fluid increases so considerably that, in consequence of the great tension, the protoplasm becomes permeable and allows the contents to pass through it. Consequently the expansion cannot be so considerable as *according to the calculation* a NaCl-solution of 0.5 or 0.25 would have to produce, and the equation must, therefore, yield too great a *p*. According to this view of the matter there is a perfect agreement with what we observe in the red corpuscles; there the issue of the fluid contents of the cell is rendered visible by means of free haemoglobin.

As to the way in which protoplasma and intra-cellular fluid are arranged in the white corpuscles, we do not learn much from the experiments communicated. We may indeed explain the phenomena by considering the cell to consist of a protoplasmic net of closed meshes; but we may also imagine it to be built up of a closed husk, within which is a fluid divided among protoplasmic strands, which do not form closed meshes. In the latter case, however, we must take it for granted that the external husk is only permeable to water (cf. note on page 106).

In order now to make a choice between the two hypotheses, I measured off two equal quantities of white corpuscles, and for an hour vigorously shook one quantity with a large (number) of sharp pieces of glass. By this means a considerable quantity of white corpuscles were bruised and torn.

When now equal portions of the bruised and not-bruised white corpuscles were treated with different salt solutions, the contraction and expansion appeared for both sorts *caeteris paribus* almost the same.

The result obtained, as far as I can see, is not to be otherwise explained than by taking it for granted, that every average part of the white corpuscle, again of itself, consists of a closed net of meshes.

From simple histological experiments into which I do not propose to enter now, we learn the same.

Red Corpuscles.

The same experiments that we made with white corpuscles (omitting of course the bruising experiment) we also made with the red.

It may suffice to give again some of these experiments in a table.

HORSE BLOOD.

I.	II.	III.	IV.	V.
Fluids.	Volume observed of red corpuscles	Volume of the protoplasmic network.	Mean volume of the protoplasmic network.	Mean percentage of protoplasmic network in red corpuscles.
a. Na-Cl sol. 0.7%	41.75	calculated from a and c	19.7	
b. " " 0.94%	36	b " c	19.9	55%
c. " " 1.5%				
a. Na Cl sol. 0.7%	43.5	a " c	20.6	
b. " " 0.94%	37.5	b " c	19.9	53.3%
c. " " 1.5%	31			
a. Na Cl sol. 0.7%	42	a " c	19.7	
b. " " 0.9%	37	b " c	20.6	54.5%
c. " " 1.5%	30.25			
a. serum.	34.5	a " b	18.25	
b. serum + 20% water.	37.75	a " c	18.8	53.6%
c. " " 40% "	40.75	a " d	18.5	
d. " " 50% "	42.50			
a. serum.	35.75	a " b	19.5	
b. serum + 20% water.	39	a " c	20.1	55%
c. " " 40% "	42	a " d	20.24	
d. " " 50% "	43.5			
a. serum.	36	a " b	19.75	
b. serum + 20% water.	39.25	a " c	19.75	55%
c. " " 40% "	42.5	a " d	20	
d. " " 50% "	44			

Here too it appears, that in the different experiments the percentage relation between protoplasm and intracellular fluid shows but little deviation, and corresponds in a most striking manner with what was found in the white corpuscles of the same animal.

We were now interested in ascertaining what results the blood-corpuscles of a rabbit would yield.

We took some blood from the ear and defibrinated it. After carefully filtering off the fibrine, it was mixed with salt solutions in the same way as the corpuscles of the horse. In the following table are found some of the results obtained. The number of white corpuscles after defibrinating is, however, so small, that we are almost justified in considering the result as proceeding exclusively from the red corpuscles.

R A B B I T - B L O O D .

Fluids.	Volume of corpuscles observed.	Volume of the protoplasmic network.	Mean volume of the protoplasmic network.	Mean percentage of the protoplasmic network in corpuscles.
a. Na Cl sol. 0.7%	43	calculated from a and c	20.2	
b. " " 0.85% (isot. with blood serum).	39	a " d	19.6	19.9 51%
c. Na Cl sol. 1.2%	35.5	b " d	19.3	
d. " " 1.5%	30.5			
a. Na Cl sol. 0.7%	44.25	a " c	19.6	
b. " " 0.87% (isot. with blood serum).	40	a " d	19.9	19.5 48.7%
c. Na Cl sol. 1.2%	34	b " d	19.1	
d. " " 1.5%	31.25			
a. Na Cl sol. 0.7%	40.5	a " c	17.7	
" " 0.86% (isot. with blood serum).	36.25	a " d	18	17.93 49.4%
Na Cl sol. 1.2%	31	b " d	18.1	
d. " " 1.5%	28.5			

These experiments indicate a smaller volume for the protoplasmatic network than was found in the horse.

In the third place we communicate a few determinations with the blood-corpuscles of the frog.

FROG-BLOOD.

Fluids.	Volume of red blood-corpuses observed.	Volume of the protoplasmic network.	Mean volume of the protoplasmic network.	Mean percentage of protoplasmic network in red corpuscles.
a. Na Cl sol. 0.35%	44.5	calculated from a and d	28.7	
" " 0.6%	38	b + d	29.25	28.78
c. " " 0.7%	36.75	a + c	28.9	
d. " " 1%	34.5			
a. Na Cl sol. 0.35%	41.75	a + d	27.9	
b. " " 0.6%	36	b + d	27.8	27.5
c. " " 0.7%	34.25	a + c	26.8	
d. " " 1%	32.75			
a. Na Cl sol. 0.35%	45.5	a + e	27	
b. " " 0.6%	37.5	b + d	27.5	27
c. " " 0.7%	36	a + c	26.5	
d. " " 1%	33.5			
a. Na Cl sol. 0.35%	40.5	a + d	25.14	
b. " " 0.6%	34.25	b + d	24.83	25.16
c. " " 0.7%	33	a + c	25.5	73 4%
d. " " 1%	30.5			

If, on one hand, we compare the protoplasmic network in the horse and rabbit with that of the frog on the other, we are struck by the fact, that in the frog the volume of network is so considerable.

On further consideration, however, this point agrees with what we observed in 1886. It then appeared, that while we may dilute horse-blood serum with from 60 to 70% of water before colouring-matter issues, frog-blood serum will bear 200% and more of water. This would not be possible, if there was as much intracellular fluid in these blood corpuscles as in those of the horse's, for the expansion would have then become much too strong.

Spermatozoa of the Frog.

In conclusion I will describe a few experiments, which served

provisionally to ascertain whether cells, the nucleus of which constitutes the main mass, would conduct themselves like those we had examined. An excellent subject happened to present itself viz. immature spermatozoa of the frog; only a few stirred.

Fluids.	Volume observed of the red blood corpuscles.	Volume of the protoplasmic network.	Mean volume of the protoplasmic network,	Mean percentage of protoplasmic network in the spermatozoa.
a. Na Cl sol. 0.35%	69	a and d	48.2	
" " 0.6%	61	b " d	47.25	48.13
c. " " 0.7%	59	a " c	49	
d. " " 1%	55.5			
a. Na Cl sol. 0.35%	70	a " d	42.3	
" " 0.6%	58.5	b " d	42.2	42.83
" " 0.7%	57	a " c	44	
" " 1%	52			
a. Na Cl sol. 0.35%	62	a " d	44.3	
" " 0.6%	55.5	b " d	43	44.1
" " 0.7%	53.5	a " c	45	
" " 1%	50.5			
a. Na Cl sol. 0.35%	72	a " d	44.3	
b. " " 0.6%	60.5	b " d	44.2	44.5
c. " " 0.7%	58.5	a " c	45	
d. " " 1%	54			

We see therefore, that the spermatozoa give nearly the same cipher as the blood-corpuscles of the frog.

In how far a correspondence will prove to exist between the quantitative proportion of protoplasma and intracellular fluid in different cells of one and the same individual, we shall have to learn from further researches.

CONCLUSION.

1. Besides the red blood corpuscles, the white and the spermatozoa

also exhibit contraction by hyperisotonic and expansion by hypotonic solutions.

2. The observation, that the amount of this expansion and contraction is much less than it would be if the cells in question consisted of a homogeneous mass, leads to the conclusion, that the cells must consist of two substances, one representing the hygroscopic power of the cell, the other, which has no share, or only a slight one in the hygroscopic power.

3. By the quantitative determination of the expansion and contraction which the cells undergo under the influence of NaCl-solutions of different concentrations, or of serum, mixed with different quantities of water we have a means of fixing the percentage proportion between the two constituents of the cells.

From the experiments hitherto made it has appeared, that in the white blood corpuscles of the horse the volume of protoplasm-substance amounts to

56.7% — 53.7%

For the red corpuscles of the horse	55% — 53%
" " " of the rabbit	51% — 48.7%
" " " of the frog	76.4% — 72%
" " spermatozoa of the frog	79.4% — 73.2%

If we injure the white bloodcorpuscles by mechanical means, the expansion and contraction under the influence of salt solutions appears to be almost unchanged. The results obtained are with difficulty otherwise to be explained but by supposing, that these cells consist of a network of protoplasm, the closed meshes of which contain a fluid that exclusively represents the hygroscopic power of the cell.

Consequently BÜTSCHLI's theory of the „Wabenstructur” of the cells receives, at least for the leucocytes, from physiological experiments a powerful support, which seems not to be superfluous in face of the objections of an histiological nature which have been urged against the arguments in its favour.

Royal Academy of Sciences. Amsterdam.

PROCEEDINGS OF THE MEETING

of Saturday October 29th 1898.

(Translated from: Verslag van de gewone vergadering der Wis- en Natuurkundige Afdeeling van Zaterdag 29 October 1898 Dl. VII).

CONTENTS: Report on the 2nd International Catalogue Conference. By Prof. D. J. KORTEWEG, p. 117. — „Attention and respiration”. By Prof. C. WINKLER, p. 121. (With 5 plates.) — „A simple deduction of the characteristic equation for substances with extended and composite molecules”. By Prof. J. D. VAN DER WAALS, p. 138. — „The rate of substitution of a nitrogroup by an oxalkyl”. By Prof. C. A. LOBRY DE BRUYN and Dr. A. STEGER, p. 144. — „On some anomalies in the system of MENDELEEFF”. By Prof. TH. H. BEHRENS, p. 148. — On the action of methyl alcohol on the imides of bibasic acids”. By Prof. S. HOOGEWERFF and Dr. W. A. VAN DORP, p. 151. — „Description of an open manometer of reduced height”. By Prof. H. KAMERLINGH ONNES, p. 154. — „Cup-shaped red bloodcorpuscles (Chromocytetes)”. By Dr. M. C. DEKHUYZEN (Communicated by Dr. P. P. C. HOEK), p. 154. — „Some remarks upon the 14-monthly motion of the Pole of the Earth and upon the length of its period”. By Dr. E. F. VAN DE SANDE BAKHUYZEN (Communicated by Prof. H. G. VAN DE SANDE BAKHUYZEN), p. 157. — „The influence of pressure on the critical temperature of complete mixture”. By Mr. N. J. VAN DER LEE (Communicated by Prof. J. D. VAN DER WAALS), p. 158.

The following papers were read:

International Catalogue. — Prof. D. J. KORTEWEG reads the following Report:

The interest, which many members of our Academy have shown in the issue of the second International Catalogue Conference, held in Londen from Oct. 11th to 13th, as well as the circumstance that our Academy has declared itself willing to contribute towards the part, which the Netherlands are to take in the work, being well executed (see the Proceedings of the ordinary meeting of Nov. 28th 1896), have induced me here to give a short account of what has been done since the holding of the first conference, in July 1896.

To some extent this account is to be considered as a sequel to my communication, published in the Proceedings of our meeting of Sept. 26th 1896. As I did then I shall now again confine myself as much as possible to those points that may be considered to be of more lasting interest to science.

The former conference had addressed itself to the Royal Society requesting that this Society might appoint a committee for studying and reporting all questions regarding the Catalogue that still remained unsettled.

In compliance with this request a copious report, dated March 30th 1898, was sent to the different governments and to the delegates of the former conference. These last were requested to state their personal remarks in an unofficial manner.

This report, which we will call the Report of the Royal Society-Committee, contained in the *first place* further proposals regarding the international organization to be introduced. One of these concerned the instituting of an International Council and of International Conventions, besides the Central Bureau in London and the National or, as they are now called, Regional Bureaux, the establishment of which had already been resolved upon by the first conference and which are to do the regular work of composing the Catalogue.

The International Council, formed by delegates of the Regional Bureaux, one for each, was to meet at least once in every three years in London and was to constitute the governing body of the Catalogue. It has however to submit to the decisions of the International Conventions, which shall be held in 1905, in 1910 and after that every tenth year, the members being appointed by the governments participating in the organization.

A further proposal tended to institute for each science an International Committee of Referees, consisting of five members, appointed by the International Council. It is to this Committee that the director of the Central Bureau shall have to apply on the arising of difficulties in the application of the classification-schedule, drawn up for that particular science. Changes of and additions to the schedule, which the Central Bureau deems desirable, are submitted to its approval, while it has the right independently to introduce such changes and additions.

In the *second place* the costs of the enterprise were estimated in this report.

This was done on three different suppositions, *firstly*, that only

a Book-Catalogue shall be issued; *secondly*, that moreover a primary Slip-Catalogue shall be edited, with this restriction however that but one slip has to be issued for each publication (book, paper etc.); *thirdly*, that a secondary Slip-Catalogue shall be composed in which each paper has to be represented by one slip for every subject treated therein, so far as these subjects give rise to different entries in the Book-Catalogue; every effort being made however to prevent an excessive extension of the number of slips pertaining to one paper.

As a result of the committee's estimates and calculations it appeared that on the *first* supposition the receipts probably will cover the expenses, if not at once at least after a certain period; that this may be the case even on the *second* supposition; but that it can hardly be expected on the *third* supposition.

In the *third place* the report of the Royal Society-Committee contained classification-schedules, worked out for most of the sciences that have to be treated in the Catalogue.

In these schedules, the construction of which differs considerably for the different sciences, according to the particular demands of each science, ample room has been left everywhere for changes and extensions, that may be desirable in future. The registration-symbols and numbers only play a subordinate part. They are technically indispensable to the work of the Central Bureau and for arranging the slips of the Slip-Catalogue. Acquaintance therewith however may be considered as wholly or to a great extent superfluous to him who uses the Catalogue.

When too many entries should come under the same heading, a further sub-division may be attained by the introduction of "significant words" which may afterwards, if desirable, be transformed into new headings. In this way a natural growth of the classification will be arrived at, keeping up with the development of science.

Taking into account this in my opinion very suitable construction of the book-catalogue and the circumstance, that the instituting of Committees of Referees seems to me a sufficient guarantee that the needs and demands of the real scientific workers in the different sciences will be decisive, I complied with the above-mentioned request of the Royal Society-Committee by stating that my personal opinion was in favour of the project in general. At the same time I subjected the mathematical schedule to a closer examination and added some remarks on the classification-schedules of more general tendency.

In the second conference now held, this Report of the Royal Society-Committee was made the base of the deliberations, in order to come as much as possible to a decision on the subjects contained therein.

Amongst the conclusions so arrived at I have to mention in the *first place* those concerning the international organization.

The establishment of an International Council and of International Conventions, constituted as proposed by the Royal Society-Committee, that is to say as described above, was accepted by the conference.

The instituting of the International Committees of Referees on the other hand was referred for consideration to the International Council when constituted.

In the *second place* resolutions were passed on the arrangement of the catalogue.

Notwithstanding the greater costs that will be incurred by it, the maintaining of the slip-catalogue, besides the book-catalogue, was urgently insisted on from many sides. The resolution taken by the former conference of editing the catalogue in *both* forms was unanimously confirmed. It was however deemed necessary to make the finantial prospects of the enterprise, which cannot at present be rightly estimated, decide on the question whether only the primary slip-catalogue or also the secondary one will have to be issued.

This last course was thought very desirable in itself, as it offers the only means of collecting and keeping together the complete slips for each particular branch of science. The hope was expressed that a suitable arrangement of the secundary slips might greatly reduce the large difference of cost which, according to the report of the Royal Society-Committee, would exist between the two forms of the slip-catalogue.

As to the classification-schedules in the *third place*, it was resolved that separate schedules shall be composed for the following sciences:

Mathematics; Astronomy; Meteorology; Physics; Cristallography; Chemistry; Mineralogy; Geology (including Petrology); Geography (only the Mathematical and the Physical part); Paleontology; Anatomy; Zoology; Botany; Physiology (including Pharmacology and Experimental Pathology); Bacteriology; Psychology; Anthropology.

For the rest the assembly confined itself to draw up some very general rules for the construction of these schedules, the further development of which is left to a provisional International Committee, which probably shall appoint experts for every branch to assist in the working out. In many cases certainly the provisional classification-schedule, added to the Report of the Royal Society-Committee, may serve as a base.

Appointed at first to judge of and to fix the classification-schedules, the Provisional International Committee has at the same time been invested with the extensive power of taking resolutions on different questions left undecided by the conference, regarding the construction of the catalogue. This committee will have to frame a report, not later than July 31st 1899; which will be incorporated in the decisions of the conference.

It cannot be denied that the task of this International Committee is a heavy one. In order to render it somewhat easier and to give to all opinions an opportunity of making themselves heard, the delegates have been charged with the formation of local committees, each in his own country, to consider the questions left undecided and to report there on to the International Committee within six months.

In my opinion these committees will indeed be able to do useful work. For if it might appear that there was but little difference of opinion on matters of importance, the decisions of the International Committee would certainly be facilitated. The clear exposition of the different questions in the report of the Royal Society-Committee raises a well founded expectation that such concordances will take place.

I venture to express the hope, that the Committee of our Academy, which has already formerly occupied itself with the International Catalogue, will by its cooperation enable me to fulfil this part of my charge.

Conformable to the above the President promises again to convocate the Committee which has on former occasions given advice on the International Catalogue.

Physiology. — Prof. C. WINKLER on: „*Attention and respiration*”.

§ 1. *Introduction.*

The present physiology of the cortex of the brain justly attaches great importance to the idea of association. Since it has been proved that different functions may be ascribed to different parts of the cortex of the brain, it has become necessary on anatomical grounds to suppose connections between those parts, moreover to assume on clinical grounds, that in psychical processes these connections are used.

The physiology of the cortex of the brain is compelled to accept the theory of so-called parallelism between physiological and psychical processes — for without this theory all physiology as a foundation of psychology is impossible — it is therefore required that a special physiological modification in the central nervous system correspond to every special psychical process.. At once however, a difficulty arises.

Through self-observation, psychology knows a succession of different conscious states, but no more. It teaches nothing concerning the way, in which this succession occurs. Physiology, therefore, has to explain the way, in which this succession takes place.

For this it uses the idea of association. It assumes that a physiological modification in one group of cells can propagate along fibres to another and bring about a new modification there. If it is true, that in general only modifications in cells may be accompanied by conscious parallel processes, and not the processes propagating in the fibres, the fact that self-observation knows succession and no more, is not strange. But the idea of association has a wider application.

It is its task to point out what special physiological modification corresponds to every special conscious condition.

The characteristics of the former may be determined in different ways, either by the place occupied by the cell-group which is modified, or by the combination of two or more co-acting cell groups. By means of association this combination is possible in numberless variations.

To physiology taken in this sense, which aims at combining a succession of conscious conditions with an association of physiological parallel processes, the name of association-physiology has been given. It is of course, rigorously a science of matter. It does not suffer itself to be led astray in its views by the existence of conscious parallel series. It states their existence, but keeps silent about their nature. It treats conscious life as the life of an automaton, who perceives scarcely anything of the processes going on in him, but who reacts to some, by far the fewest in number, with the corresponding conscious parallel series.

At all times a difficulty has presented itself to the association-physiology. It cannot be contradicted that self-observation, in our wanting to make movements, in our being attentive and in many other conscious processes gives the impression, as if we were active agents in the conscious processes. Whence this feeling of being active? How is it to be explained that self-observation misleads the subject

about the part it plays, though to the association-physiology the subject appears as a passive, successively perceiving and imaginating subject?

A better knowledge of the psychomotor area of the cortex has solved this question fairly satisfactorily. A number of clinical observations have shown that perception and remembrance of a once performed movement of the muscles as well as the movement itself, depend upon the psychomotor area remaining intact.

A number of physiological experiments have proved, that every modification of a definite centre in this part of the brain — e.g. one caused by an electric current, — flows to a special group of muscles and brings about a special combination of movements.

If the supposition is admitted that the modification brought about in the cortex of the brain is no other than the one originating under physiological conditions, the latter, whose conscious parallel is known to our self-observation as remembrance of the same combination of muscular contraction which is to be performed, will give rise to the error of the subject. The subject sees that on the remembrance of such a combination of movements, the movements themselves invariably follow and concludes that the former causes the latter.

The physiological modification, however, remains the cause of the muscular contraction. Its conscious parallel series, unexplained, perhaps unexplainable, does nothing. The subject, as long as it is taught by self-observation only, fancies that it is active, that it possesses a feeling of effort.

When we speak of a will to move, we mean the remembrance of a movement of the muscles, which is to be performed, whose physiological parallel series brings about this same movement, while the subject fancies that the remembrance is the cause.

All remembrances of movements of the muscles have this character of activity in common and it is also found in many compound psychical processes.

When many impressions from sources of light bring about visual perceptions, there is generally but one which we retain, or as WILHELM WUNDT expresses it, of which we have the apperception, compared with the many sensations of which we have the perception. We direct our attention to that one actively, as it seems to us. It has a chance of being distinctly remembered, while the others pass by unnoticed, are forgotten.

Here too, a feeling of being active. If we did not possess this gift of apperception, if we could not, even in appearance, detain an image actively, our thinking would be a whirl of successive images (recollections), as may perhaps occur in the brain of a maniac.

Direction of our thoughts in one sense, would be impossible.

It is the question if, in this exertion of the attention which appears so undoubtedly active to the self observation, the remembrance of movements do not play a similar part, as was the case with the will to move.

Can the subject actively, by the association of psychomotor cell-groups with the modifications which are accompanied by conscious remembrances in the observing regions of the cortex be made to fancy, that he fixes his remembrance?

To answer this question, it is necessary to know beforehand, whether there are contractions of the muscles, and if so what contractions, through which the man who fixes his attention is different from himself, as he is when he is not attentive.

For every modification of the psychomotor area must necessarily flow to the periferic muscles. Even if it is not very strong, it must bring about some modification in the corresponding group of muscles, though it be only a beginning of contraction, an increase of the tension of the muscles.

It will appear — and this I want to show in my exposition --- that the attentive man distinguishes himself from one not attentive in various respects by changes of a motor nature and that this remarkable combination of movements may be reproduced from one single point by electric stimulation of the cortex of the brain of a dog.

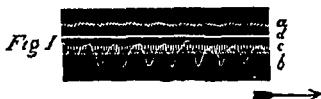
§ II. *The variations in the volume of the brain during psychical processes.*

Before entering upon my real subject, I must dwell for a moment on the modifications, which occur in the brain when at rest or during conscious function. Many years ago Mosso has proved that the volume of the brain varies continually. Sometimes people have a defect in the bones of the cranium. Then the brain, covered only by the membranes on that place, is fit for observation. If the movements of the brain in half-sleep are registered — movements visible even to the naked eye — it appears that the brain regularly varies in volume according to the varying quantity of blood it contains. We see then :

1^o. Oscillations caused by pulsation, because every contraction of the heart urges blood in the scull.

2^o. Oscillations caused by respiration, because at every expiration the blood can flow less easily from the brain.

3^o. Special oscillations of volume, which are longer and occur at irregular periods; their cause is less clear.



Oscillations of the volume of the brain caused by pulsation and respiration in half-sleep.

- a. tracing of the volume of the brain.
- b. tracing of the respiration. The descending line is that of the inspiration.
- c. tuning fork. 2 vibrations per second.
- d. abscis.

(LAURENS. June 18th 1898. Defect of the scull caused by tuberculosis of the os parietale.)

Figure I is an example of the two first mentioned, in fig. II and III the last mentioned are very clear.

Whenever a sensation of sound draws his attention, whenever the person performs an intellectual process, e.g. the solution of a sum, the hyperaemia of the brain increases; it decreases as soon as the exertion stops. The long last-mentioned oscillations continue for some time afterwards, to be of much greater amplitude and of much longer duration than before the exertion took place (see fig. III). On the other hand the oscillations caused by respiration become less visible during the exertion; after the exertion, however, they become very distinct.

At last the oscillation caused by pulsation changes too. It becomes quicker. Moreover the long oscillations are influenced to a certain degree by

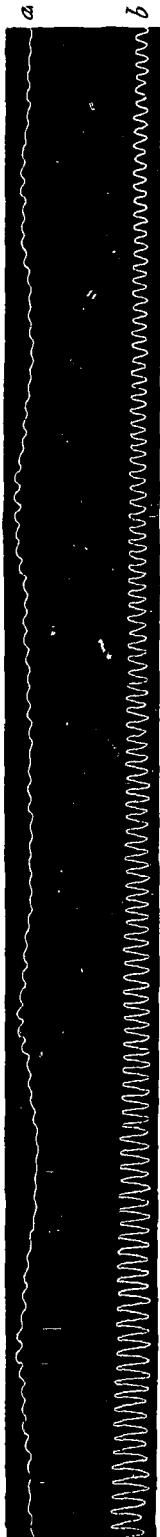


Fig. II.

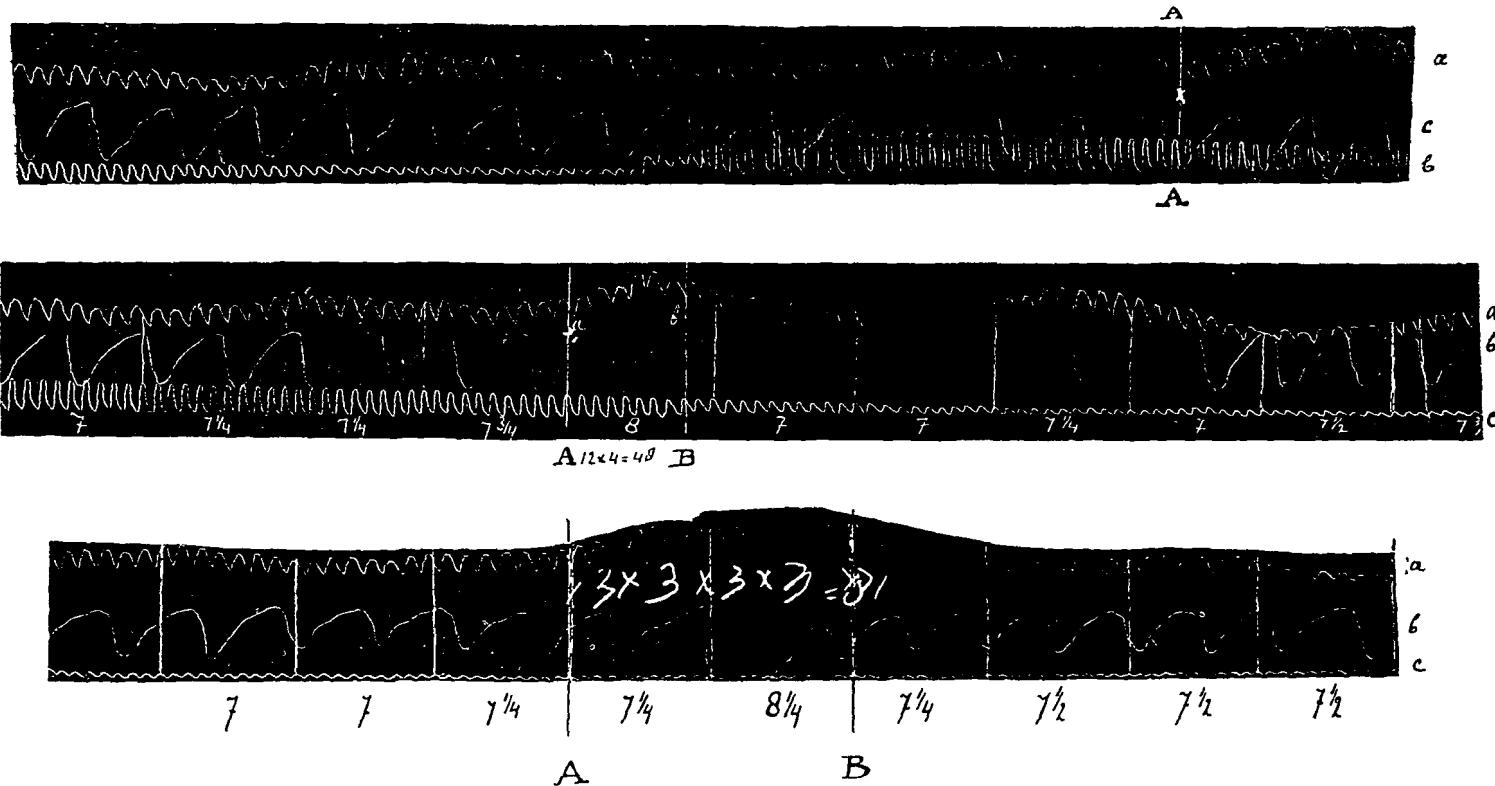
Oscillations caused by pulsation and respiration, and oscillations of long duration, the last very great ones, in the volume of the brain during half-sleep.

- a. tracing of the volume of the brain.
- b. tuning fork. 2 vibrations per second.

(GRAUSZ. Febr. 2nd. 1898. Defect of the scull in the os parietale after amputation of the Ganglion Gasseri.)

Fig. III

(126)



Oscillations in the volume of the brain during intellectual labour. (By GRAUSZ. See fig. II.)

Fig. III. 1. At A there is a knock at the door. Fig. III. 3. from $A-B$ a sum $12 \times 4 = 48$ is being solved.

Fig. III. 3. from $A-B$ a sum $3 \times 3 \times 3 \times 3 = 81$ is being solved.

In all tracings α = that of the volume of the brain, b = that of the respiration, c = that of the tuning-fork with 2 vibrations p. sec.

the oscillation caused by pulsation. At the moment when the curve of such a long oscillation rises high above the abscis, the duration of the oscillation of the pulse is long¹⁾. The increase of the volume of the brain is preceded by acceleration of the pulse, as appears from the graphical representation in fig. IV and from the figures in the note.

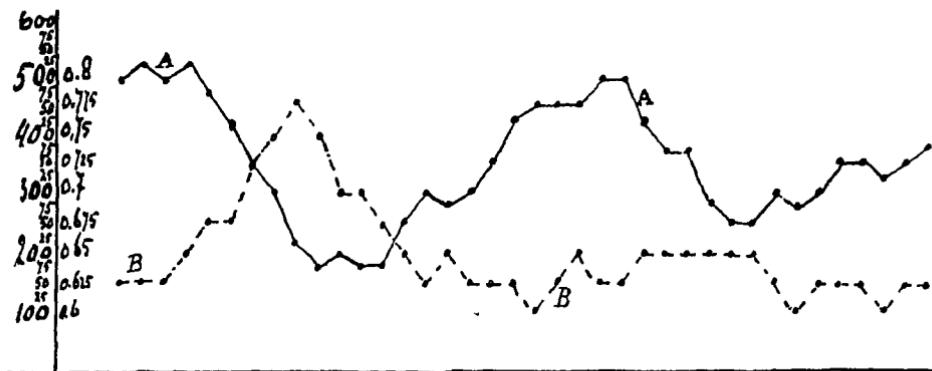


Fig. IV. Graphical representation of the duration of the pulsations in seconds, compared to the elevation of the tracing of the volume of the brain above the abscis at every pulsation.

A. Height above the abscis of the volume of the brains in mM.

B. Duration of the pulse in seconds, - - - - - (GRAUZS, see fig. II.)

So it is clear, that in registering the pulsation of an artery, we must find the same acceleration and retardation, which returns at long irregular intervals and is not dependent on the respiration.

¹⁾ If we measure the height above the abscis of the long oscillation, when the cylinder rotates quickly, and the duration of every oscillation of the pulse which corresponds to that height, we find :

Height above abscis in mM.	Duration of pulsation in seconds.	Height above abscis in mM.	Duration of pulsation in seconds.	Height above abscis.	Duration of pulsation in seconds.	Height above abscis.	Duration of pulsation in seconds.
5.00	0.62	4.25	0.65	1.75	0.675	4.50	0.65
5.25	0.62	3.50	0.725	2.50	0.65	4.50	0.625
5.00	0.62	3.00	0.75	3.	0.625	5.00	0.625
5.25	0.65	2.25	0.775	2.75	0.65	5.00	0.65
4.75	0.675	1.75	0.77	3.00	0.625	4.25	0.65
		2.00	0.77	3.50	0.625	3.75	0.65
		1.75	0.77	4.25	0.625	3.75	0.65
				4.50	0.6	3.50	0.65etc.

If we take into consideration that even these single facts prove, that every increase of the volume of the brain is attended by quick pulsations, we shall find that a great deal may be learned from the study of the pulsation of the artery, when the brain is at work.

This will be my first investigation.

*§ 3. The influence of the exertion of attention on the heart,
the respiration and the extensor muscles of the hand
and the neck.*

If we examine the influence which the exertion of attention has on the heart and consider it separately for a moment, remarkable facts will be found.

First a few words about the arrangement of such experiments.

As an example of a strong exertion of attention, which is very slightly if at all attended by feelings of pain, the solving of a sum is taken. The intellectual labour which is not attended by feelings of pain or pleasure as suggested by the French authors is taken as starting-point. The person used for the experiments is laid on a couch. When he himself is perfectly calm and all around is quiet, the pulse and the respiration are registered. It has been arranged that a number, generally of two figures, shall be called out to him at a given moment, and that he shall continue to multiply this number with itself, till a signal tells him to stop. He is to calculate without using expedients. He works therefore with visual representations, with remembrances of visual representations of numbers. Generally the experiment lasts \pm half a minute.

The result, whether right or not right, is not pronounced by the calculator; it must be dismissed at once from his mind. For many experiments have proved, that the correctness or incorrectness of the result is of no importance to my purpose; the rest or unrest after the labour is.

In such experiments the pulse and the respiration may be registered.

The person to be experimented on lies calmly, the cylinder rotates quickly. Three levers note down. The pulsation of the arteria radialis, the respiration and a tuningfork, which makes ten complete vibrations a second, are registered with air-transport. The latter enables us to read the time accurately down to the twentieth parts of seconds, independent of the irregularities of the clockwork of the cylinder. The tuning-fork is kept in continual vibration by an electro-magnet.

In this quiet half-sleep we see what follows:

Towards the end of the respiration, which has become calm, the pulsations are of a long duration. When the inspiration begins, they are shorter. Towards the end of the inspiration and during the beginning of the expiration the pulsations are at the quickest.

When the expiration has reached its maximum, they are very long; the same thing begins over again just before or after the beginning of the inspiration.

It is not necessary to dwell on the influence of the respiration on the duration of the pulse and the pressure of the blood, which is fairly well-known. It appears, however, also, that the duration of the pulse varies at irregular intervals. Independently of the respiration, which never loses its influence, the pulsations become quicker or slower at irregular periods. These particularities must not be overlooked.

When we have registered long enough to have a sufficiently trustworthy starting-curve (see fig. VII) in half-sleep, a number is called out to the person. Even if he does not calculate, this call is sufficient to bring about a slight, passing acceleration of the pulse. If however, he calculates, the acceleration sets in at once. It increases, according to the increase of exertion. When after half a minute the signal "stop" is given, the acceleration of the pulse continues for some seconds. After that, a shorter or longer amphiox stadium follows, in which there is neither acceleration nor retardation. Finally

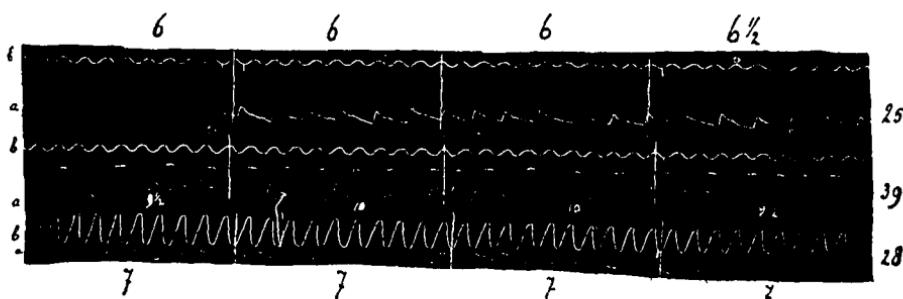


Fig. V. Three series of pulsations one above the other.

In all three: *a.* is the pulsation of the arteria radialis.

b. is the tuning-fork of 2 vibrations per second.

The lowest series, with 28 pulsations in 20 seconds before

The middle series, with 29 pulsations in 20 seconds during

The highest series, with 25 pulsations in 20 seconds after

} the sum.

(J. DE VYE, April 1898. Reduced to $\frac{1}{3}$ of its size by means of photography.)

and sometimes for a very long time, a compensatory retardation follows¹⁾ (see also fig V and VI).

All this occurs by oscillations. The influence of the respiration

¹⁾ Here is an example in figures: Experiment March 1898, J. DE VRITS, Pulse and respiration noted down during quick rotation. Duration of the pulse in seconds.

0.875	0.825	0.7	0.565	0.5	0.625	0.775	0.875	0.925	0.9
0.8	0.8	0.75	.0575	0.25	0.625	0.825	0.85	0.9	0.9
0.85	0.8	0.75	0.6	0.5	0.625	0.8	0.85	0.925	0.925
0.8	0.8	0.7	0.55	0.525	0.625	0.85	0.875	0.95	0.975
0.8	0.775	0.7	0.55	0.525	0.65	0.875	0.875	0.925	0.975
0.85	0.75	0.7	0.525	0.55	0.625	0.825	0.85	0.9	0.9
0.85	0.75	0.65	0.55	0.525	0.625	0.825	0.875	0.9	0.925
0.85	0.775	0.65	0.525	0.55	0.65	0.825	0.9	0.9	0.95
0.875	0.775	0.675	0.525	0.525	0.675	0.875	0.975	0.9	0.925
0.85	0.775	0.675	0.525	0.55	0.7	0.875	0.9	0.950	0.95
0.875	0.775	0.675	0.5	0.525	0.725	0.875	0.875	0.925	0.95
0.900	0.775	0.7	0.5	0.55	0.75	0.85	0.9	0.925	0.925
0.875	0.8	0.675	0.525	0.575	0.75	0.9	0.875	0.9	0.925
0.85	0.8	0.675	0.5	0.55	0.85	0.875	0.875	0.9	0.925
0.85	0.75	0.675	0.5	0.575	0.825	0.85	0.875	0.9	0.925
0.85	0.75	0.675	0.5	0.575	0.85	0.85	0.925	0.9	0.9
0.85	0.75	0.625	0.5	0.575	0.85	0.85	0.925	0.935	0.95
0.85	0.75	0.625	0.5	0.6	0.85	0.85	0.9	0.9	0.975
0.875	0.7	0.625	0.5	0.575	0.8	0.925	0.925	0.875	0.9
0.8	0.7	0.625	0.5	0.575	0.8	0.925	0.925	0.875	0.9
0.825	0.675	0.625	0.475	0.6	0.8	0.925	0.925	0.9	0.9
0.85	0.65	0.625	0.475	0.6	0.775	0.9	0.925	0.925	0.925
0.825	0.65	0.6	0.475	0.6	0.825	0.950		0.9	
0.8	0.622	0.575	0.475	0.6	0.8	0.9		0.925	
		0.6	0.475	0.625	0.8			0.95	
			0.5	0.65				1.—	
				0.625				0.925	

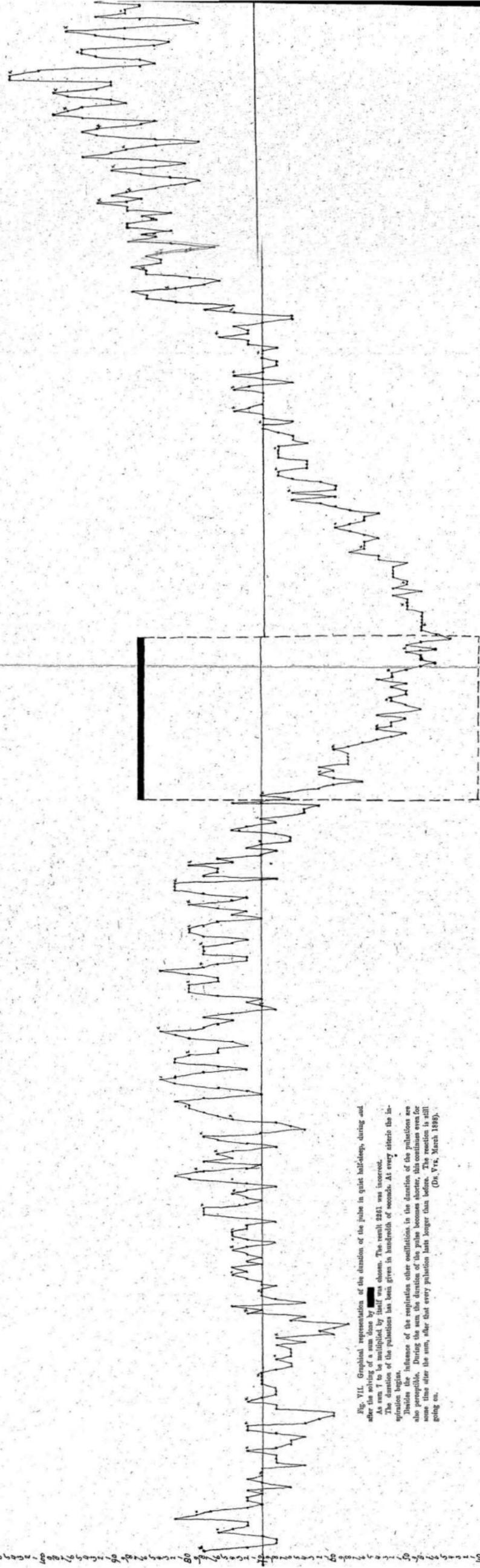


FIG. VII. Graphical representation of the duration of the pulse in quiet half-sleep during and after the solving of a mathematical problem by [redacted]. As sum 7 to be multiplied by itself was chosen. The result 291 was incorrect. The duration of the pulsations has been given in hundredths of seconds. At every instant the inspiration begins. Besides the influence of the respiration other oscillations in the duration of the pulsations are also perceptible. During the sum the duration of the pulse becomes shorter, this continues even for some time after the sum, after that every pulsation lasts longer than before. The reaction is still going on.

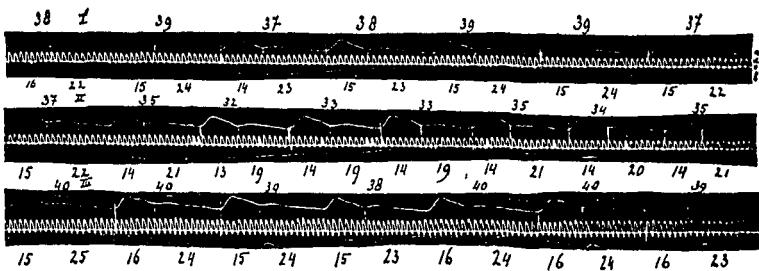


Fig. VI. Three series of pulsations one above the other during quick rotation of the cylinder.

In all three *a* = the pulsation of the arteria radialis, *b* = the respiration
c = the tuning-fork of 10 vibrations per seconds, *d* = the abscis.

The highest I has been taken before —, the middle II during —, the lowest III after the sum.

The figures give the duration of the pulse in $\frac{1}{20}$ of seconds (half vibrations of the tuning-fork). Those along the upper border give the duration of pulsation. Those along the lower border give the division of the length of a pulse by the dicrotism.

(Prof. SALTEM, March 1898.) Reduced to $\frac{1}{2}$ of its size by means of photography.

is more clearly noticeable after the sum than before, and also the longer oscillations return. (See the graphical representation fig. VII for the different oscillations, the acceleration, the amphibole stadium and the compensatory retardation as consequences of exertion).

The acceleration of the pulse under such circumstances has, however, been known since THANHOFER. The compensatory retardation has not had due acknowledgment. And yet, compare fig. V, it may be very considerable.

But this is not all that happens. During the sum the blood-pressure in the radial artery seems to increase, at least the curve rises, and at the same time the form of the pulse-curve changes somewhat. Though the shortening of the duration principally influences the diastole of the heart, yet the dicrotism moves a little nearer to the top when the pulse is short, than when it is long.

At least in an absolute sense. Comparatively, however, the dicrotism has moved farther from the top. Compare fig. VI.

Summa summarum. During the exertion of the attention the pulse is accelerated. After that it becomes slower. The acceleration sets in at once, at least if the other influences on the duration of the pulse are taken into account.

Let us now turn to the respiration. This is registered by the

pneumograph of Marey. The descending line represents the inspiration, the ascending line the expiration in every curve which has been obtained in that way. The arrangement of the experiment is the same as before.

During the sum, the respiration becomes quicker and more superficial, after the sum slower and more extensive (fig. VIII—XIII).

The first expiration can be lengthened, but it reaches rarely the height of the preceding exhalations. The shorter duration of the next respirations, however, is principally dependent on the shorter duration of the exhalation¹⁾, while the lengthening of the exhalation after the sum is characteristic (fig. IX and X). It exists even then, when the duration of the respiration is little or not lengthened, because a slight shortening of the inspiration continues after the sum.

A tendency to deep inspiration with limitation and specially shortening of the exhalations is characteristic of what happens during the sum. Not with everybody, though. Sometimes the deeper inspi-

¹⁾ The counting fig. 8, 9, 10 shows this clearly:

Fig. 9.

Before the sum.	During the sum.		After the sum.		Before the sum.	During the sum.		After the sum.		Duration of exhalation. Duration of inhalation.	
	Duration of exhalation.	Duration of inhalation.	Duration of exhalation.	Duration of inhalation.		Duration of exhalation.	Duration of inhalation.	Duration of exhalation.	Duration of inhalation.		
1.3sec.	2.5sec.	0.6sec.	1.9sec.	1.1sec.	3.8sec	1.2sec.	2.4sec.	0.9sec.	1.7sec.	0.9sec. 2.6sec.	
1.4 "	2.6 "	1. "	2.2 "	1.2 "	3.6 "	0.9 "	2.4 "	0.8 "	1.8 "	1. " 2.9 "	
1.3 "	3. "	1.1 "	1.8 "	1.3 "	3.3 "	1.1 "	2.6 "	0.8 "	2.2 "	1. " 2.8 "	
1.3 "	3.2 "	1. "	1.9 "	1.3 "	3.6 "	1.1 "	2.5 "	0.8 "	1.7 "	0.9 " 2.6 "	
		1.2 "	1.9 "	1.5 "	3.4 "	1.1 "	2.2 "	0.9 "	1.2 "	1. " 2.9 "	
		1. "	2.2 "	1.2 "	3.9 "			0.9 "	1.5 "	1. " 2.9 "	
								0.8 "	1.7 "	1.2 " 2.6 "	
								0.6 "	1.4 "	0.9 " 2.4 "	
								0.6 "	1. "		
								0.6 "	2.1 "		
On an average		On an average		On an average		On an average		On an average		On an average	
1.3sec.	2.8sec.	1. sec.	2. sec.	1.3sec.	3.6sec.	1. sec.	2.4sec.	0.8sec.	1.6sec.	0.9sec. 2.7sec.	

Fig. VIII.-XIII. Changes of the respiration during the sum from *a*-*b*.

In all figures: 1 = the pulsation of the arteria radialis, 2 = the absciss, 3 = the tuning-fork of 5 vibrations per second. 4 = the respiration (the descending line is the inspiration).

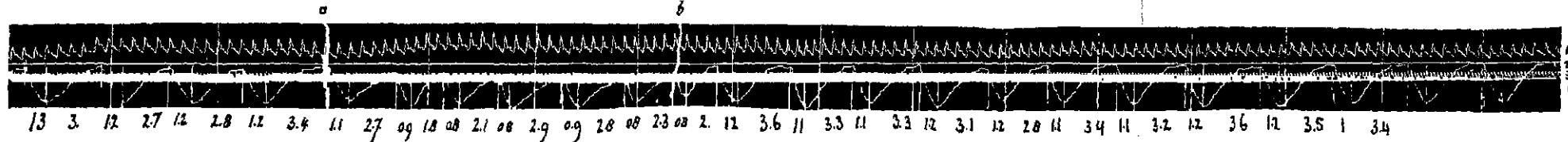


Fig. II

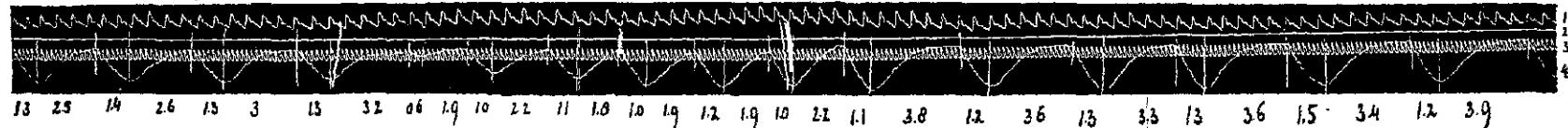


Fig. Y

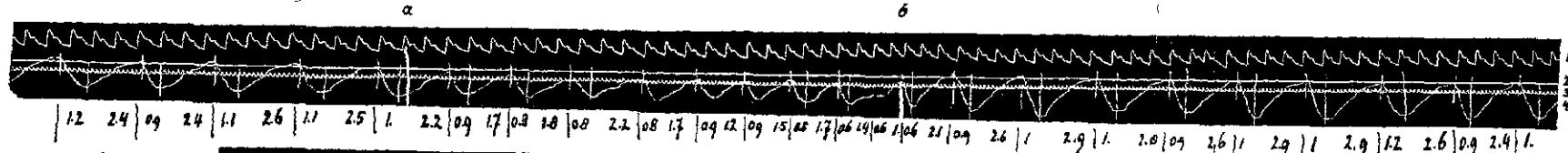


Fig. XI

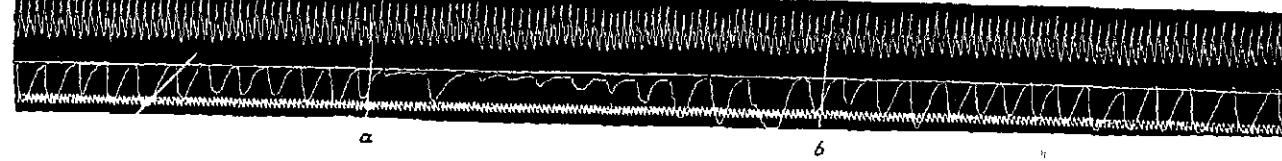


Fig. VIII. During the sum, respiration quick, expiration restrained, inspiration becomes deeper. After the sum the expiration is lengthened.

The ciphers give the duration of the in. and expiration in seconds. Sum 76×76 . (WINKLER, July 1898).

Fig. IX. During the sum the respiration becomes quicker, after preceding long expiration it becomes less extensive, the inspiration becomes deeper. After the sum the expiration is lengthened. Ciphers as in fig. VIII. Sum 59 x 59. (WIARD BECKMAN, July 1898).

Fig. X. During the sum the respiration becomes very quick, the expiration less extensive, the inspiration deeper. Then the expiration is somewhat lengthened. Ciphers as before. Sum 47×47 . (SLEYN, July 1895).

Fig. XI. During the sum, the expiration, which is long at first, is continually interrupted by quicker inspirations. It becomes less and less extensive and makes place for deeper inspiration. After the sum lengthened expiration.
(VAN DER PLAT, July 1898).

Fig. XII. During the sum, nothing but exceedingly quick and superficial respiration. Then slightly longer expiration. Ciphers as above Sum 88 x 83.
(Dr. RUTTINGA, Sept. 1898).

Fig. III.

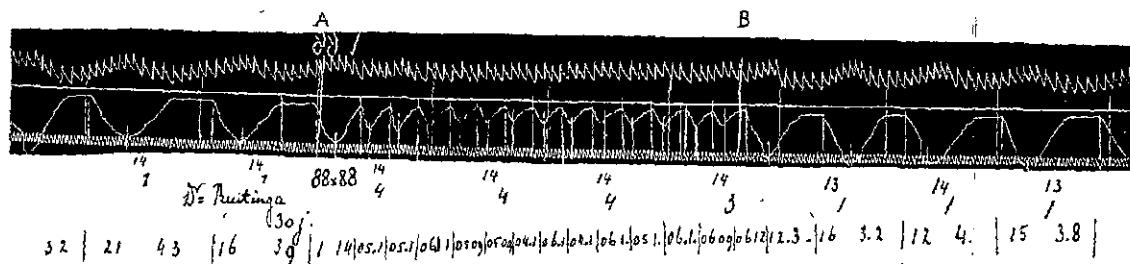
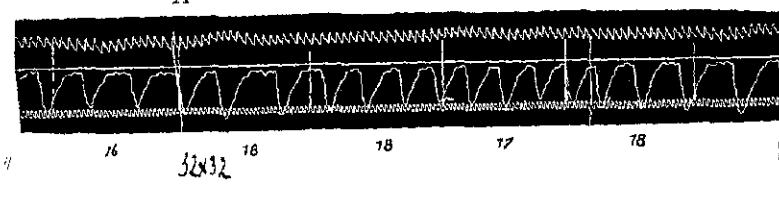


Fig. XIII



ration is wanting and gives place to an enormous acceleration of the superficial respiration (see fig. XII). Sometimes there is a form intermediate between the two (see fig. XIII).

Summa summarum. During the exertion the respiration, specially the expiration, is shorter and more superficial; both inspiration and expiration are diminished in the beginning, but give gradually way to a stronger position of inspiration of the thorax. Then a compensatory retardation of respiration follows, specially of the expiration, with an inclination towards more extensive expiration and longer intervals.

During such a sum, however, the lumen of the periferic blood-vessels changes also. HALLION and COMTE have invented a simple instrument to register the volume of the fingers, the so-called capillary pulse of the fingers. A caoutchouc cylinder is placed firmly along the fingers in the hollow of the hand, while the whole hand is covered by a solid glove. Every change in the volume of the fingers under the influence of the working of the heart or the respiration, is imparted to the caoutchouc and can be registered.

And now, just as BINET and COURTIER have already pointed out, it appeared also to me, that sometimes after a preceding extension, the volume of the fingers decreases during the sum (see fig. XV).

BINET and COURTIER think rightly, that this is partly the consequence of a spasm of the blood-vessels in the fingers. In fact, when we try to find out with Mosso's sphygmomanometer, as they also did, how it is with the blood-pressure in the capillaria, the result is, that the pressure of the blood increases in the capillaria during the sum.

If according to the particular arrangement of this instrument we start from a very slight opposing pressure (in casu 30 mM. Hg.), a curve is traced during the sum, which does not only ascend, but shows increasing deviations (see fig. XVI α); and if we start from the optimum of the opposing pressure (in casu for myself \pm 50 mM. Hg.) the tracing rises, while the deviations decrease (fig. XVI β).

So the volume of the finger becomes smaller, the pressure in its small blood-vessels increases. There is undoubtedly a spasm of the vessels in the periphery. Besides the spasm of the vessels another influence works on the volume of the fingers. The deep inspiration empties the blood-vessels and has certainly some influence on the descending of the curve which the volume of the fingers traces during the sum. In the third place it is not impossible that in the arrangement of the experiment as done by HALLION and

COMTE and BINET and COURTIER, a slight extension-movement of the fingers causes a release of the cylinder. If this were the case, a descending of the tracing registered in that way, would be found.

It was natural to register also the extensor muscles of the hand and the fingers. This may be very fitly done with an air-cushion. In this case, the local capillary pulse is of course noted down, but every local expansion of the muscles will occur as a rising in the curve.

Such a rising takes place. The influence of the spasm of the vessels, which exists in the periphery, as we saw before, should be noticeable here, and the tracing should descend. But on the contrary the tracing very considerably ascends as fig. XVII and XIX show.

The objection, that an air-cushion does not work as a volume-writer, but as a sphygmomanometer, is not justified. This is also to be seen on the curve traced. The deviations remain as large as before in spite of the enormous rise of the curve during the sum (fig. XVII and XIX).

When an air-cushion is placed at the same time on the flexor-muscles of the hand and the fingers, and is made to trace the slight deviations, we see hardly a change, at the utmost an indication of a slight falling of the line, which is to be regarded as a consequence of the spasm of the vessels.

It is then unvariably a volume-pulse which is noted down, and if we place an air-cushion on the muscles of the calf (fig. XIX), exactly the same volume-curve is obtained, as we know since BINET and COURTIER for the fingers, and which is represented during the sum in fig. XV. There is only one explanation left: the extensor muscles of hand and fingers are contracted to some degree, or rather, for the movement does not manifest itself in a change of place visible to the eye, the tonus of the extensor muscles of hand and fingers has become greater. In consequence the curve which has been traced during the sum by the air-cushion placed over these muscles rises.

If finally we place the air-cushion on the neck, an observation like the one on the extensor muscles of the hand can be made. The curve, which is here of course very much influenced by the respiration, and which is registered by the air cushion as rising by inspiration, but falling by expiration, rises during the sum. Sometimes the head is even visibly thrown backwards. (see fig. XX).

Summa summarum. The tonus of the extensor muscles of the hand and neck is augmented.

If we take all this together, the question arises, if all these

Fig. XV—XIX. Changes in the extremities during the sum.

XIX.



Fig. XVII.

Fig. XVIII.

96

Fig. XVII and XVIII. Tracings of the local volume registered with small air-cushions over the extensor muscles (a) and over the flexor muscles (b) of the lower part of the arm. c. abscis. d. tuning-fork with 2 vibrations per second.

During the sum curve a rises, curve b not or it goes down. The rise is dependent on the increase in tension of the extensors, which does not take place in the flexors or in a far smaller degree. (LANGELAAN, Sept. 1898).

Fig. XV.

Fig. XVIIa.

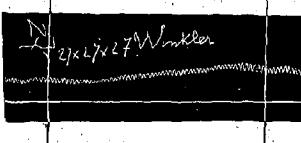


Fig. XVIIb.

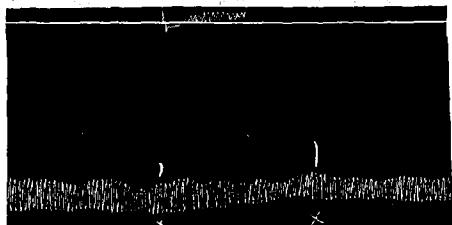


Fig. XIX. Tracing of the local volume registered with air-cushions above the extensor muscles of the lower part of the arm and above the muscles of the calf.

- A that on the extensor muscles of the arm,
- B that on the muscles of the calf.

During the sum, the air-cushion on the muscles of the calf notes down the usual volume-curve, as is represented in fig. XV for the fingers. The curve on the extensor muscles rises as a consequence of the tension of the muscles which takes place.

(WILARD BECKMAN, Sept. 1898).

Fig. XX.

Fig. XXa.

96

Fig. XXb.

64

96

Fig. XX (WINKLER, Sept. 1898). Fig. XXa (WILARD BECKMAN, Sept. 1898). Fig. XXb (LANGELAAN, Sept. 1898). Tracings registered with immovable air-cushions on the muscles of the neck. The tracing rises at every inspiration, because the head threatens to be thrown a little backwards through the tension of these muscles. This happens in a far greater measure during the sum.

Fig. XV. Change of the volume of the fingers registered with the moulchouair cylinder of HALLION and COMTE.

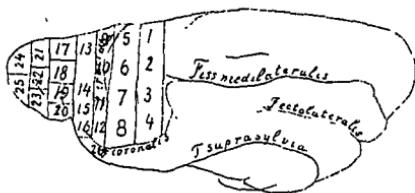
During the sum (multiplication: 17 and 19) this curve goes down after a short slight rising. The volume of the fingers decreases accordingly (WINKLER, June 1898).

Fig. XVI. Change of the pressure of the blood in the capillaries registered with Mossé's sphygmomanometer. In fig. XVIa the opposing pressure is ± 30 mM. Hg.

During the sum the tracing rises and the deviations increase. In fig. XVII the opposing pressure is ± 50 mM. Hg., that is the optimum, with which the greatest deviations are obtained. During the sum the tracing rises, but the deviations become smaller. So the blood-pressure increases during the sum. (WINKLER, Sept. 1898).

Prof. C. WINKLER: „Attention and Respiration.”

Fig. XX.



a

Fig. XXI.

b

Side-view of the scheme of the cortex of the brain of a dog. The ciphers indicate the squares, which were successively stimulated with the induction current. When 15 and 16 are stimulated we see deeper inspiration or less extensive expiration, or quicker superficial respiration.

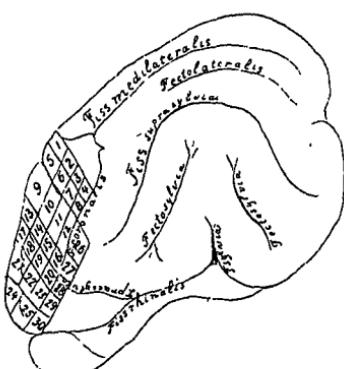


Fig. XXII.

Side-view of the scheme of dog's brain. (as above).



Side-view of the brain of a dog. If the place, indicated by a circle, is stimulated with an induction current, the inspiration becomes deeper, the respiration quicker, the expiration less extensive and the pulsation quicker. After the stimulation the expiration is lengthened, the pulsation slower than before. By stimulation with a moderately strong induction-current we find stretch-movements of the fore leg and lifting up of the head.

Fig. XXIII.



Front-view of the same brain of a dog. The circle agrees with point 16 of the schemes above.

changes cannot be regarded as consequences of changes of respiration.

In fact, leaving the periferic contraction of the vessels out of account, there is much to be said in favour of this conception. Inspiration and acceleration of the pulse, expiration and retardation of the pulse belong together and it is scarcely possible that the tendency to inspiration should not be accompanied by acceleration of pulsation during the sum, whereas the long respirations after it must be attended by retardation. As to the stretching of the muscles of hand and neck, every one can observe in himself, that a strong inspiration is attended by these movements.

So my exposition comes to this, that the popular saying that one who listens very attentively, listens with suspended breath is not a very incorrect expression. I know, however, very well, that the movements here described, are not the only things, which happen during the exertion of attention.

§ IV. All the movements described here, may be brought about experimentally by stimulating a certain area of the cortex of dog's brains by means of an induction current.

When we try to find out whether the movements mentioned above can be brought about experimentally, it is obvious, that we must turn to the cortex of the frontal lobe of the brain. We are brought to this on clinical grounds. Patients suffering of the cortex of the frontal lobe on both sides show an inability of detaining an image; they are often confused maniacs; they are so no less by their inability of remembering events. Both phenomena point to a marked disturbance in the power of apperception.

I began therefore a methodical investigation with Mr. WIARDI BECKMAN who will discuss the literature¹⁾ on this subject and a number of details of the experiments. The question was, whether the functions of respiration could be modified by stimulation of the cortex of the frontal lobe of a dog's brain.

To this purpose the cortex was stimulated with a secondary induction current with electrodes, at a distance of two mM. from one another.

Four Leclanché-cells feed the induction coil, a sledge of DU-BOIS REYMOND, and the current can be scarcely felt on the tip of the

¹⁾ I leave it to Mr. WIARDI BECKMAN to point out the agreement and the differences of our results with the investigations of HRTZIG, MUNK, KRAUSE, SEMON, HORSLEY and SPENCER.

tongue when the secondary coil was 11 cM. from covering the primary.

A Desprez-signal indicates the moment of stimulation on the cylinder, while the respiration is registered with the pneumograph of Marey and the arteria femoralis with an instrument borrowed from the tonometer of Talma.

At eight o'clock in the morning the dog gets from 30—50 mGr. morphia, depending on its size. In narcose with ether the frontal lobe is exposed as far as the dura mater, according to the well-known method.

After a long time of rest and when the respiration is no longer disturbed by the morphia, the experiment begins about one or two in the afternoon.

The surface of the brain, representations of which will be found in fig. XXI—XXIV, is divided into a number of squares, so far as it is required for our purpose. The size of these squares are determined by the topography of the cortex.

After having determined the different centres on the cortex by stimulating a well-known area of the motor region, the animal breathing calmly, it is first found, that there are a number of centres from which it is impossible to bring about any change whatever in the respiration.

It appears however, that constant changes may be brought about regularly after stimulation of a few points indicated by 15 and 16 in the scheme. This point lies in the transition from the gyrus sigmoideus (prae-cruciatus) to the frontal lobe and in the second lateral gyrus near the place, where the fissura coronalis approaches the fissura praesylvia closely. As soon as we stimulate with an exceedingly weak current e.g. with 11 cM. distance between the coils, some quick inspirations are seen to be brought about as in fig. XXIV. If we stimulate with a current somewhat stronger, the inspiration becomes somewhat quicker and is accompanied by acceleration of pulsation (see fig. XXV). A moderately strong stimulation, e.g. of 9 cM. distance, causes a stronger spasm of inspiration with strong acceleration of pulsation, whereas the current also irradiates, and causes stretch-movements of the muscles of arm and neck (see fig. XXVI), which were distinctly visible, so that it was unnecessary to register them. In fact the respiration (inspiration) centre mentioned, lies between the centre for stretching the muscles of the hand and that for lifting the neck.

This acceleration of respiration with deeper inspirations, eventually restraining and shortening of the expiration is always followed by

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Fig. XXIV—XXVIII. Changes of respiration and pulsation during the stimulation of point 16 (see fig. XX—XXIV) of the cortex of the brain of dogs.

In all curves the abscissæ and the time have been registered. Time by a tuning-fork with 2 vibrations per secund.

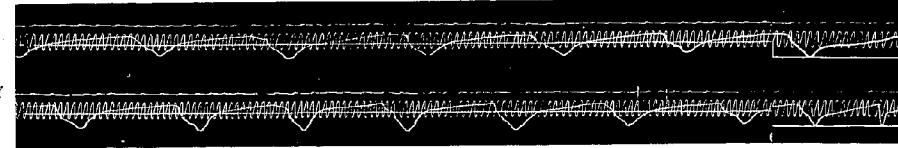


Fig. XXIV

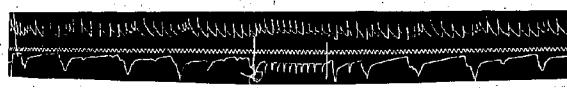


Fig. XXV

Fig. XXIV. Respiration and pulsation of a dog registered (July, 15th 1898). At *d* and *e* = point 15 and 16 the cortex is stimulated with a weak induction current (distance of the coils 11 cm.). During the stimulation acceleration of the respiration, specially shortening of the expiration, which continues for some time after the stimulation. Then strong lengthening of the expiration.

Fig. XXV. Respiration and pulsation of a dog (July 12th 1898). At *e* = point 16 the cortex is stimulated. Quick and superficial respiration during, long expiration after the stimulation.

Fig. XXVI and XXVII. As before of a dog (July 9th 1898). At III stimulation of point 16 with a moderately strong current (distance of the coils 8 à 9 cm.). During the stimulation tendency to deep inspiration and acceleration of the pulse. Then lengthening of expiration and retardation of the pulse.

Fig. XXVIII. Respiration (*a*) and pulsation (*b*) of a dog (July 27th 1898). At 16 stimulation with the induction current during the time indicated by the signal. In every phasis of the respiration, both in inspiration, and expiration, deep inspiration and acceleration of the pulse, as well as quicker respiration takes place.

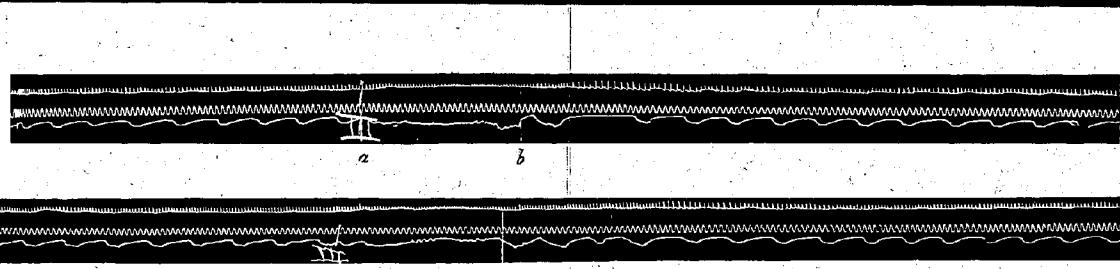


Fig. XXVI

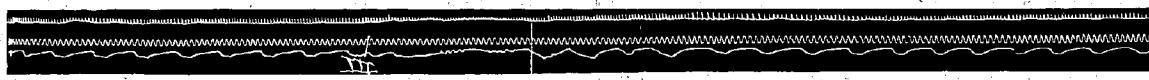


Fig. XXVII



Fig. XXVIII

long exspirations, nay even by the expiration-position of the thorax, and at the same time a distinct retardation of the pulse is seen to follow as a consequence of that stimulation. The influence on the respiration of electric stimulation of this point 16 is perfectly constant, and can be brought about in all phases of the respiration, as fig. XXVIII proves. There is always stronger inspiration etc.

Summa summarum. By stimulation of a very small region in the frontal lobe of a dog's brain, acceleration of respiration, tendency to deep inspiration, acceleration of pulsation, contraction of the extensor-muscles of hand and neck may be brought about, followed by retardation of pulsation and long expiration.

The movements expressing attention are consequently localized on a comparatively small area of the cortex of a dog's brain and may be demonstrated experimentally.

§ V. Conclusion.

Let us now return to the theory of the parallelism between physiological and psychical events.

I have shown in a series of arguments, that any man, fixing his attention, performs a constant series of movements. I do not mean to say that these are the only movements, through which the attentive man differs from himself, as he is in a not attentive state. I am convinced that there is a great deal to be done in this direction. But the movements described here form a sharply defined group. I believe that they all depend on a modified respiration.

Self-observation teaches us, that we turn our attention actively to a perception. We are justified in assuming that representations of movements possess an active character for the self-observation. We find attentively examined representations also connected with the representation of movement, and in consequence provided with the same character.

The physiological process parallel to attention, from the point of view of the association-physiology may be explained by the existing complexes of movements. A special complex of movements may be produced by means of an electric current from the cortex of a dog's brain. It is on a larger scale exactly the same combination of movements which is found in the attentive man.

When a physiological change whose parallel series is e.g. a visual representation, propagates and brings about a new change in the psycho-motor inspiration centre, there is for the self-observation, a

visual representation + a representation of a respiration-movement, performed in a special way. Self-observation calls this complex the apperceived or the attentively observed visual representation.

Though psychology cannot be treated now experimentally, yet the theory of physiological parallel processes justifies the opinion that attention and representations of movements originated by special respiration movements, are closely connected. Closer investigations on monkey's brains will be necessary. The other movements expressing attention will have to be submitted to a systematical investigation in the same way, to throw further light on the problem treated in this paper.

Physics. — „*Simple deduction of the characteristic equation for substances with extended and composite molecules*“. By Prof. J. D. VAN DER WAALS.

If the quantity of substance inclosed in a certain volume is considered as consisting of material points, which may also be done with extended molecules, composed of atoms, the equation

$$\sum \frac{1}{2} m V^2 = \frac{1}{4} \frac{d^2 \sum m r^2}{dt^2} - \frac{1}{2} \sum (Xx + Yy + Zz) \dots \dots (a)$$

holds, provided that the quantities occurring in this equation be applied to all material points.

If groups of these material points are united to separate systems, as is the case with molecules, which cannot be considered as one single point, the equation mentioned above, becomes:

$$\begin{aligned} \sum \frac{1}{2} m V_z^2 + \sum \sum \frac{1}{2} \mu V_r^2 &= \frac{1}{4} \frac{d^2 \sum m r_z^2}{dt^2} + \frac{1}{4} \frac{d^2 \sum \sum \mu r_r^2}{dt^2} - \\ &- \frac{1}{2} \sum (Xx_z + Yy_z + Zz_z) - \frac{1}{2} \sum \sum (Xx_r + Yy_r + Zz_r) \dots (b) \end{aligned}$$

in which the index z relates to the centres of gravity of the systems, and the value r indicates the value of a quantity relatively to the centre of inertia.

For the stationary condition of the centres of gravity as well as of the systems themselves, this equation is simplified to:

$$\begin{aligned} \sum \frac{1}{2} m V_z^2 + \sum \sum \frac{1}{2} \mu V_r^2 &= - \frac{1}{2} \sum (Xx_z + Yy_z + Zz_z) - \\ &- \frac{1}{2} \sum \sum (Xx_r + Yy_r + Zz_r) \dots \dots \dots (c) \end{aligned}$$

The condition which is required for considering a group of points as a system is, that these points keep always together, whatever may happen and that the quantity $\Sigma \Sigma \mu r_r^2$ keeps constant.

For the term $- \frac{1}{2} \Sigma (Xx_z + Yy_z + Zz_z)$ we may write $\frac{3}{2} (N + N_1) v$, so that the latter equation may be written:

$$\begin{aligned} \Sigma \frac{1}{2} m V_z^2 + \Sigma \Sigma \frac{1}{2} \mu V_r^2 &= \frac{3}{2} (N + N_1) v - \\ &- \frac{1}{2} \Sigma \Sigma (Xx_r + Yy_r + Zz_r) (d) \end{aligned}$$

In these equations collisions taking place between material points, cannot furnish a value, as in every point where a collision takes place, there are two forces of opposed direction, which, working at the same point, destroy each other. The forces in the term $\frac{1}{2} \Sigma \Sigma (Xx_r + Yy_r + Zz_r)$ are simply the attractive forces between the points of the system and possibly also the attractive forces which are exercised on a system by the surrounding ones.

It is true that in transforming $- \frac{1}{2} \Sigma (Xx_z + Yy_z + Zz_z)$ to $\frac{3}{2} (N + N_1) v$ it has been assumed for these latter forces, that for a system, which does not lie near the surface ΣX is equal to 0, but from this does not follow that ΣX_x is equal to 0.

If to the moving systems themselves the virial equation is applied, we get the equation

$$\Sigma \Sigma \frac{1}{2} \mu V_r^2 = - \frac{1}{2} \Sigma \Sigma (X'x_r + Y'y_r + Z'z_r) . . . (e)$$

provided that in X' , Y' and Z' all forces, also those which exist on the surfaces as pressures, are taken into account. These systems move in a space, in which the pressure is $N + N_1$ per unity of surface, and if we were justified in considering the pressure as really exercised on the surface of every system, the value furnished in the second member of the equation would be equal to $\frac{3}{2} (N + N_1) b_1$, if we represent the volume of all the systems together by b_1 .

As this pressure, however, is transferred on every system by the collisions with the other systems, in calculating this value, we must consider that pressure as exercised at a distance twice as great, so on the surface of a volume, whose lineal dimension is twice that of the system; at least for spherical systems. Of the value obtained in this way, the half is to be taken, because a pressure exercised by the first system on the second is at the same time a pressure, which is exercised by the second on the first. The equation (e) becomes then if we put $b = 4 b_1$

$$\Sigma \Sigma \frac{1}{2} \mu V_r^2 = \frac{3}{2} (N + N_1) b - \frac{1}{2} \Sigma \Sigma (Xr_r + Yr_r + Zr_r) . (f)$$

If (f) is subtracted from (d) the well-known equation is obtained

$$\frac{3}{2} (N + N_1) (v - b) = \frac{1}{2} \Sigma m V_z^2.$$

The equation (f) may be considered to contain the condition for the stationary state of the molecules themselves. In the form given it is, however only applicable, if the molecule is supposed to be composed of material points, which do not form again separate systems. If the latter is the case, the equilibrium of every separate system will give rise to a new equation, which, however, will not change the equation $\frac{3}{2} (N + N_1) (v - b) = \frac{1}{2} \Sigma m V_z^2$.

For a mixture consisting of $n_1 + n_2$ molecules, we find the virial-value of the surface-pressure of all the molecules together through the observation, that the amount of the pressure on the unity of surface for the two kinds of molecules is proportionate to the numbers which are found in unity of volume and therefore also proportionate to n_1 and n_2 . For collisions with a molecule of the first kind, a surface-pressure amounting to $\frac{n_1}{n_1 + n_2} (N + N_1)$ must be assumed, and for collisions with molecules of the second kind a surface-pressure of $\frac{n_2}{n_1 + n_2} (N + N_1)$.

We find for the quantity with which $\frac{3}{2} (N + N_1)$ is to be multiplied in order to indicate the value of the virial of the pressure, which is exercised on the surfaces of the moving systems, the same value as Mr. LORENTZ (Wied. Ann. 1881, Bd. XII, Heft 1) has found, viz.:

$$b = \frac{\frac{2}{3} \pi (\sigma_1^3 n_1^2 + \sigma_2^3 n_2^2 + 2 \sigma^3 n_1 n_2)}{n_1 + n_2}$$

It is easy to deduce, by the preceding way of obtaining the characteristic equation that the value of b is equal to 4 times the volume of the molecules only in case of infinite rarefaction, and that it must be smaller in case of less great rarefaction; it is not even difficult in that case, to give a first approximation of the way, in which b depends on the volume of the substance. By the calculation of the equation (f) we find the value of the virial of the pressure on the moving systems to amount to half the value of the virial of a pressure $N + N_1$, exercised on as many surfaces as there are systems,

but the systems are supposed to be limited by a spherical surface, described with a radius, which has twice the length of the radius of the systems themselves. Let us call these larger spheres: distance-spheres.

All these distance-spheres are supposed to lie quite outside one another and to have no points in common. As the volume of all these spheres together is 8 times as great as the volume of the molecules, the case that all these spheres lie outside one another is by no means possible, if the volume is smaller than $2b$.

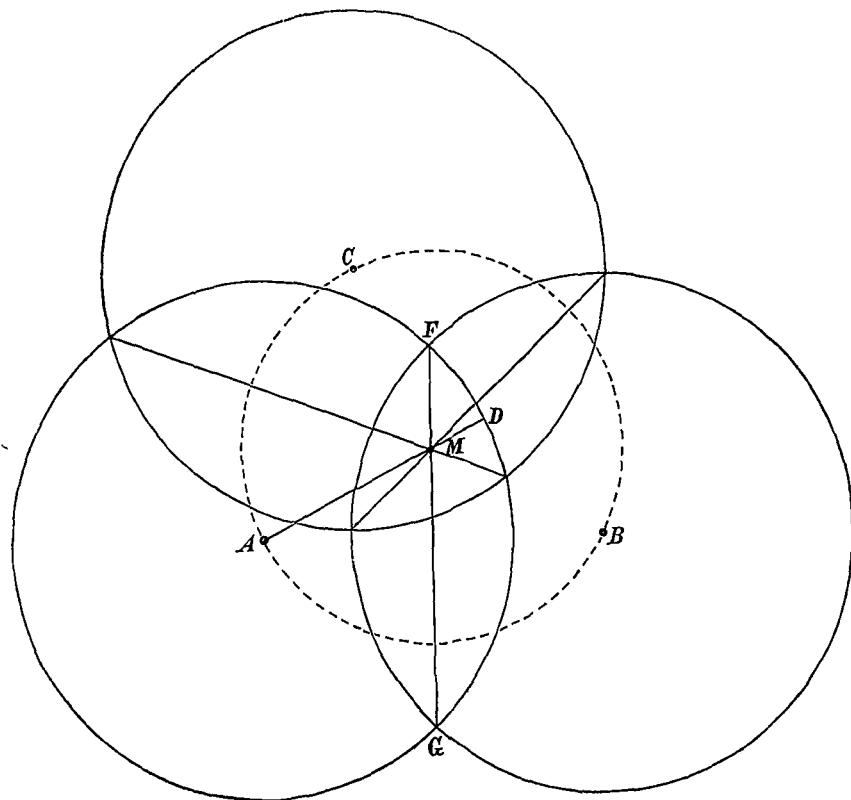
But even if the volume is so large, that the distance-spheres would lie quite outside one another, if the molecules are supposed to be spread in the space at regular distances, a great number of distance-spheres are sure to cover one another in consequence of the fact they are spread quite irregularly. Now the question is, in how far the computation of the value of the virial of the pressure $N + N_1$ is to be modified in consequence of this fact. If we have some molecules, lying in such a way that the distance-spheres intersect, we have not two entire spherical surfaces on which pressure is exercised but a surface consisting of two parts of spherical surfaces. The pressure within the space enclosed by them, is the same, as if it consisted of two separate parts, but the value of the virial of the pressure for the two molecules together amounts to twice $\frac{3}{2}(N + N_1)(B - S)$, if B is the volume of a distance-sphere and S the volume of the segment which is cut off from a distance-sphere by the plane of their intersection. In other words, we must take into account only that part of the distance-sphere that reaches up to the plane of the intersection, instead of the whole distance-sphere.

We come, accordingly, to the same result which I had obtained in another way before (Verslag Kon. Ak. van Wetenschappen Amsterdam, 31 October 1896).

A second approximation is also mentioned there, and though the determination of the value of that correction leads to such long calculations, that as yet I have not brought them to an end, yet I will make some remarks on the way in which this value might be obtained.

If A , B and C are taken for the instantaneous position of the three centres of the distance-spheres and M for the centre of the circumscribed circle, the mean value of the volume limited by the surface of the distance-sphere A and the two planes FM and AMD , will represent the second correction.

If we put $AM = \alpha$ and $\angle AMG = C$, and the radius of the distance-sphere = R , the value of the volume FMD will be



$$\begin{aligned}
 I = & \frac{2}{3} R^3 Bg \operatorname{tg} \left[\operatorname{tg} C \frac{\sqrt{(R^2 - \alpha^2)}}{R} \right] - \\
 & - \alpha \sin C \left(R^2 - \frac{\alpha^2 \sin^2 C}{3} \right) Bg \operatorname{tg} \frac{\sqrt{(R^2 - \alpha^2)}}{\alpha \cos C} + \\
 & + \frac{\alpha \sin C \alpha \cos C \sqrt{(R^2 - \alpha^2)}}{3}.
 \end{aligned}$$

If C moves on the circle ABC , the centre of which is M , α and C remain the same and consequently I keeps its value also.

Let provisionally, the distance of A and B remain invariable and let C move arbitrarily, then M moves along the line FG . If we call the height of M above AB equal to h , I may be considered as function of h by observing that $\alpha^2 = h^2 + \frac{r^2}{4}$ (r the distance AB) and

$\sin C = \frac{r}{2\alpha}$. If the whole figure is turned round line AB , and if

the whole space, in which C may be found is divided into volume-elements ΔV , we have to determine

$$\int \frac{N}{v} I \Delta V.$$

As I is known as function of h , ΔV must also be given as dependent on h .

If we represent the angle which CM forms with FG by φ , the annular volume-element, in which C lies, is to be represented by

$$2\pi d\varphi dh (h + \alpha \cos \varphi)^2.$$

If we take φ between 0 and the value which it has, when C lies on the distance-sphere of A , twice the value of this integral is to be taken.

As the value of φ is quite determined by h , when C lies on the sphere of A , the integration must be done with respect to h , and the limits are to be determined, between which h is to be taken.

The highest value of h is of course $\sqrt{\left(R^2 - \frac{r^2}{4}\right)}$; the lowest value may be found from

$$h + \sqrt{\left(h^2 + \frac{r^2}{4}\right)} = \sqrt{\left(R^2 - \frac{r^2}{4}\right)}$$

or

$$h = \frac{R^2 - \frac{r^2}{2}}{2\sqrt{\left(R^2 - \frac{r^2}{4}\right)}}$$

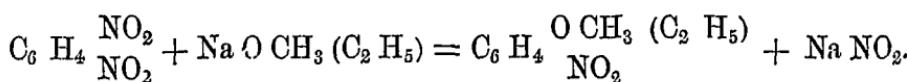
The lowest value, however, cannot descend below $-\sqrt{\left(R^2 - \frac{r^2}{4}\right)}$

which would be the case if $r > R\sqrt{3}$. This is the cause that the integration, must be done in two tempo's, and that we have to calculate

$$\int_R^{RV3} dr \int_{RV3}^{2R} dh + \int_{RV3}^{2R} dr \int_{-V\sqrt{\left(R^2 - \frac{r^2}{4}\right)}}^{V\sqrt{\left(R^2 - \frac{r^2}{4}\right)}} dh .$$

Chemistry. — „*The rate of substitution of a nitrogroup by an oxalkyl*“.
By Prof. C. A. LOBRY DE BRUYN. With reference to DR. A. STEGER's dissertation.

Up to this moment no investigation had taken place of the velocity of substitution of the atoms or atomgroups, immediately connected with the benzene molecule by other groups; yet such an investigation was of importance for the knowledge of the qualities of the benzene-nucleus and the influence of the different places on each other. A reaction which was excellently fit for this purpose I had formerly met in a comparative study on the three dinitrobenzenes viz. the substitution of one of the nitrogroups in ortho- and para-dinitrobenzene by oxymethyl or oxyethyl¹⁾ according to the equation:



A preliminary experiment had already shown me that the para-dinitrobenzene at 0° is more speedily substituted with sodium-methylate than ortho-dinitrobenzene.

Now Mr. STEGER has studied quantitatively the four above mentioned reactions at temperatures of 25°, 35° and 45°; besides the influence of concentration, that of the presence of a sodiumion and that of the gradual adjunction of water were also examined. The results of this study, published by Mr. STEGER as a dissertation, are of general interest, to which I think I may draw attention.

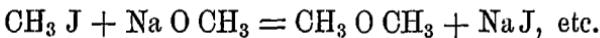
In the first place then it is now proved that the velocity of substitution is greater with paradinitrobenzene than with ortho-dinitrobenzene; for both the transposition takes place more speedily with sodium-methylate than with sodium-methyleate, a phenomenon that agrees with the observations of others. The conduct of ortho- and paradinitrobenzene is not the same for the two alkylates, a difference that may be brought back to the influence of the two alcohols that served as solvents. The influence of the temperature however, being different for the two dinitrobenzenes, was equal for each of them towards the two alcoholates.

The question as to the influence of the concentration on the velocity of reaction was important with a view to an investigation by HECHT, CONRAD and BRUCKNER²⁾ on the velocity of aetherformation,

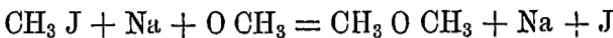
¹⁾ Recueil 13 (1894) 101, 106.

²⁾ Z. f. phys. Ch. 5 (1890) 289.

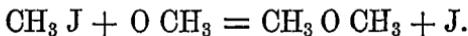
consequently of reactions which were perfectly comparable to those mentioned here; for they run on the equation:



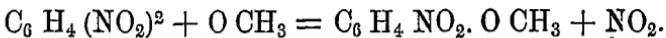
Said investigators had proved that in the process of the aether-formation the reactionconstant increases with the dilution. To us it is evident (which in 1890 could not yet be the case with H., C. and BR.) that we must here think of the influence of the electrolytic dissociation increasing in proportion to the dilution, that is to the increase of the oxalkylions¹⁾ and to write the following equation :



or, the sodiumion not taking part in the reaction



The reaction studied by Mr. STEGER would then be represented by



But then it came out, remarkably enough, that in opposition to the process of aetherformation the constant of the last named reaction was not changed by the concentration.

The question that now naturally offered itself as a consequence of this result was this. If a regular influence of the dilution, as required by the electrolytic dissociationtheory does exist in the aether-formation and does not exist in the substitution of the dinitrobenzenes, what is then in both reactions the influence of the addition of a substance with the same ion? If a sodium salt is added, the increase of the concentration of the sodiumions will keep back that of the oxalkylions and consequently cause the reactionconstant to fall. By H., C. and BR. this question had been studied cursorily and unsatisfactorily only.

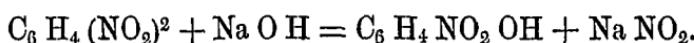
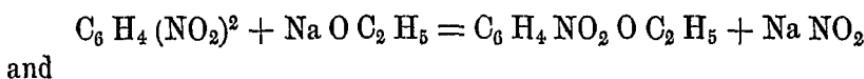
When investigating the velocity of the aetherformation while adding increasing quantities of Na J, it was now shown, that the reactionconstant, in accordance with the iontheory, decreases in re-

¹⁾ Neither $\text{C}_6\text{H}_4\text{J}$ nor $\text{C}_6\text{H}_4(\text{NO}_2)^2$ conduct the electric current in alcoholic solutions.

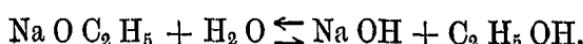
gular gradations. On the other hand however the experiment showed that for the transposition of orthodinitrobenzene a change in the velocityconstant by the addition of a sodium salt is not noticeable. Consequently we arrive at the conclusion, improbable in itself, that of two for the rest perfectly homogeneous reactions, one is under the influence of the electrolytic dissociation while the other is not. More acceptable seems to be the supposition that the theory of the electrolytic dissociation cannot be considered as an element of explanation in all reactions in solutions containing electrolytes and that the transpositions between substances, which both or one of which behave as an electrolyte may still depend upon other causes than the nature and concentration of the ions.

To the problems to which we are led by this result, one of the foremost is: what change the conductiveness of alkylatesolutions undergoes by dilution.

The study of the reaction we treat of here, has still been extended for orthodinitrobenzene and this has also afforded some results which are not unimportant. Some observations of an earlier date had shown that the addition of very small quantities of water sometimes greatly alters the qualities of alcohols. Consequently we resolved to equally investigate that influence on methylic and ethylic alcohol. The result was unexpected in many respects. In the first place we saw the reactionconstants continued to exist, even in mixtures of alcohol and water with up to 50 pCt. water. Now it is generally supposed that sodium-alcoholate with only a little water is already transformed, at least partly, in natron and alcohol. If natron had been present we might have expected the formation of nitrophenolate and consequently the two following reactions taking place simultaneously:



In the latter case a second mol. natron should have formed the sodiumnitrophenolate and consequently the concentration of the natron should be equally diminished. In case of the two reactions running simultaneously an equal velocity might not be expected. With this difference in velocity the equilibrium between NaOH and NaOC₂H₅ had to be equally altered and so, besides the three mentioned reactions there, should appear a fourth:



If all these reactions should take place simultaneously, there could be no question of a constant. And yet, as mentioned above, there were found constants for the two alcohols, even for alcohols diluted with an equal weight of water. The explanation proved to be easy. In opposition to the generally adopted idea, it was shown that the sodium in methylic alcohol diluted with 50 pCt. of water was mostly bound to oxalkyl. The same may also be proved by a quantitative estimation of the sodiumnitrophenolate formed, which was possible along the colorimetric way owing to the yellow colour of that salt. The quantities formed of it, proved to be very small and amounted only to a few percent for 50 pCt. and 60 pCt. ethylic alcohol.

There also came out another peculiarity viz. this, that the addition of water causes the reactionconstant in methylic alcohol to rise from 0.0169 to 0.0249, in ethylic alcohol to fall from 0.0261 to 0.0104. For a percentage of \pm 12 pCt of water the constant of the two alcohols is the same. It was now highly to be regretted that the constantly decreasing solubility of orthodinitrobenzene in the diluted alcohols did not allow to continue the experiment down to water. The two curves that may be constructed with the constants and the percentage of water must necessarily meet in one point, the one that would indicate the reactionconstant for aqueous natron, that is in absence of the alcohols. Yet we were able to extend our tests so far that we have shown for the still more diluted alcohols how much nitroanisol and nitrophenetol and how much nitrophenol were formed when boiling dinitrobenzene with natron that contained 40, 20 and 10 pCt. of the alcohols. The result of these tests — see below — shows that even in the strongly diluted methylic alcohols the formation of the methoxyl-compound forms the principal reaction, while in ethylic alcohol a considerable quantity of nitrophenetol is created nevertheless.

Methylic alcohol	b. pt.	time	dinitrobenzene	oxalkyl-comp.	
40%	81°	8 h.	3.03 gr.	2.7 gr.	\pm 96%
20 >	89°	>	3.01 >	2.4 >	\pm 88 >
10 >	94°	16 >.	3. >	2.0 >	\pm 70 >
Ethylic alcohol					
40%	86°	8 h.	3.2 >	2. >	\pm 63 >
20 >	92°	16 >.	3.3 >	1.15 >	\pm 35 >

In continuing these experiments it will in the first place be our purpose to make out if in the aether formations, taking place in alcohols diluted with water, the same conduct appears as in the transformations of the dinitrobenzenes treated here. Further we shall try to find substances the transformations of which adapt themselves to a quantitative study, both in alcohol and water and in any mixture of these two solvents, with simultaneous estimation of one of the products.

Chemistry. — „On some anomalies in the system of MENDELEEFF”.

By Prof. TH. H. BEHRENS.

In his fourth paper on isomorphism („Beiträge zur Kenntnis des Isomorphismus”, IV, in „Zeitschr. f. physik. Chemie, VIII, 1) Mr. J. W. RETGERS has denied the existence of isomorphism between tellurates and sulfates and has given to tellurium a place between Ru and Os ($\text{Fe} = 56$, $\text{Ru} = 104$, $\text{Te} = 128$, $\text{Os} = 195$). His views are founded on two facts: on the absence of compound crystals, composed of $\text{K}_2 \text{SO}_4$ and $\text{K}_2 \text{TeO}_4$ and on the existence of isomorphism between chlorotellurates, chloroplatinates and chlorostannates. In the second place the anomaly in the differences of the atomic weights of Sb, Te and J is mentioned and likewise the analogy between $\text{H}_2 \text{Te}$ and $\text{H}_2 \text{S}$, but the principal weight is attached to the conclusion deduced from isomorphism. His first assertion is drawn from experiments with $\text{K}_2 \text{MnO}_4$, a compound, forming green crystals with $\text{K}_2 \text{SO}_4$, but not with $\text{K}_2 \text{TeO}_4$. It has been confirmed by experiments with $\text{Ag}_2 \text{CrO}_4$, by which compound growing crystals of $\text{Ag}_2 \text{SO}_4$ are stained from amber yellow to a fiery red, while it does not enter into crystallizing $\text{Ag}_2 \text{TeO}_4$.

Where Mr. RETGERS has made use of isomorphism between chlorotellurites (he writes: chlorotellurates) and chloroplatinates, to substantiate his second assertion, his argumentation is less stringent, as he has not given experiments of his own. The base of his second assertion must therefore be thoroughly tested.

No compound crystals were obtained with TeCl_4 and PtCl_4 , the solubilities of the two compounds $\text{K}_2 \text{PtCl}_6$ and $\text{K}_2 \text{TeCl}_6$ differing too much. With $\text{Cs}_2 \text{SnCl}_6$ and $\text{Cs}_2 \text{TeCl}_6$ and with $\text{Cs}_2 \text{OsCl}_6$ and $\text{Cs}_2 \text{TeCl}_6$ better results were obtained. Beautiful compound crystals were produced by adding CsCl to mixed solutions of TeCl_4 and IrCl_4 , strongly acidulated with hydrochloric acid. In this case the compound crystals were light brown, while the one of the components is yellow, the other red-brown.

To ascertain, if Te comes more near to Sn or to Ir and Os experiments with several acids were tried, with a somewhat starting result. Tellurium dioxide is dissolved by KHC_2O_4 , forming well developed microscopical crystals of a double oxalate, resembling crystals of potassium-zirconium-oxalate in form, dimensions and manner of formation.

Here the question arose: How far may the isomorphism of the compound K_2SnCl_6 with analogous compounds of other tetrachlorides extend? In the first place the behaviour of $PbCl_4$ was examined. With strong hydrochloric acid, cooled by ice, PbO_2 gives a yellow solution, from which $CsCl$ precipitates dark yellow octahedrons, while in a solution, mixed with $SnCl_4$, bigger crystals of light yellow colour are formed. From mixed solutions of $SnCl_4$ and $PtCl_4$ no compound crystals could be produced, while mixed solutions of $SnCl_4$ and $IrCl_4$ gave a complete series, varying in colour from dark red to a faint reddish yellow. Crystallization of pure chloroiridate is prevented by means of a strong dose of hydrochloric acid and heating, before adding rubidium chloride. At ordinary temperature only a few compound crystals are formed, and these are faintly tinged. Fractionating by repeated additions of $RbCl$ is to be recommended; it facilitates the preparation of a complete series. Mixed solutions of $SnCl_4$ and $OsCl_4$ are not so easy to manage, besides the variation of colour (from colourless to light brown) is not so striking. Finally experiments were made with $MnCl_4$, prepared by dissolving MnO_2 in cooled hydrochloric acid. From this solution $CsCl$ precipitated dark red octahedrons, resembling crystals of the compound K_2IrCl_6 , while from solutions, mixed with $SnCl_4$, compound crystals of all shades between a dark red and a faint reddish yellow were obtained, perfectly imitating the series produced from mixed solutions of $SnCl_4$ and $IrCl_4$.

These last experiments are particularly suggestive. The isomorphism of $KMnO_4$ and $KClO_4$ is the principal argument for placing Mn near Cl in the seventh group of MENDELEJEFF; on account of the isomorphism between K_2MnO_4 and K_2SO_4 it might be placed in the sixth group; reasoning from the experiments with the tetrachlorides of Mn and Sn would bring it into the fourth group; from this it must be removed to the third group if you start from the isomorphism between common alum and manganum-alum; finally, if you fix your attention to the crystalline forms of manganous oxalate and of the double phosphate of ammonium and manganum, Mn must have its place in the second group, next to Mg. The compounds of chromium exhibit a similar protean behaviour.

With a view to isomorphism chromium might be placed in the eventh, the sixth, the third, and on account of chrom-ammonium in the eighth group. For tellurium you have choice between three groups. If the principal weight is accorded to the analogy between H_2Te and H_2S , it must be placed in the sixth group; if you follow Mr. RETGERS, in putting the isomorphism of K_2TeCl_6 with K_2OsCl_6 and K_2IrCl_6 upon the foreground, a place must be found for it in the eighth group; if the attention is fixed on the somorphism of Cs_2TeCl_6 with Cs_2SnCl_6 and on the analogy of potassium-tellurium-oxalate with potassium-zirconium-oxalate, it may be put into the fourth group. Meanwhile it may not be overlooked, that Te owes its place in the sixth group solely to the analogy of H_2Te and H_2S ; that, to fit into this group, its atomic weight ought to range below 126 ($Sb = 120$, $Te = 123$ (?), $J = 126.5$) and that the atomic weight of Te stands likewise in the way of a migration to the fourth and to the eighth group. In the fourth group no place is open next Sn for an element having an atomic weight about 120 ($Sn = 117.4$, $Ce = 141$), and if Te should be placed in the eighth group, between Ru and Os, search would have to be made for seventeen new elements in this group. The difference between the atomic weights of two acknowledged members of this group, taken in a vertical column, amounts to 45—48; between Ru and Te it would be half as great ($Ru = 104$, $Te = 126—128$), thus: three unknown elements, filling the gap between the first and second file, eight between the second and third ($Ru = 104$, $Os = 195$) and six below the third ($Pt = 195$, $U = 240$). Not impossible, but certainly not probable. Probably, if Mr. RETGERS had been acquainted with the isomorphism of Cs_2MnCl_6 and Cs_2SnCl_6 , he would have stopped at the statement, that isomorphism between tellurates and sulfates does not exist, and would not have proceeded to seek a place for tellurium in the eighth group with a difference of 22 between two consecutive atomic weights instead af 45.

The weight of isomorphism in chemical speculations must be further reduced than has been done by Mr. RETGERS. The hypothesis, that isomorphism among a group of compounds does point to isomorphism of the elements which these compounds have in common, is abandoned by most chemists, but isomorphism between sulfates and chromates, between phosphates, arsenates and vanadates, stated without restriction, is even objectionable. In alum trivalent chromium can take the place of aluminium, while sulfuric acid cannot be exchanged for chromic acid, and yet this exchange can

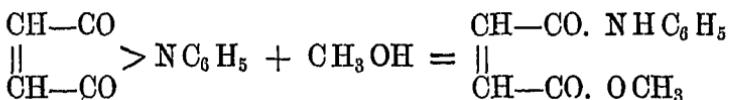
easily be made in sulfates of univalent and bivalent metals, without altering their crystallization. In double arseniate of ammonium and calcium phosphoric and vanadic acid cannot fill the place of arsenic acid, while in lead-apatite the three acids can be exchanged without any visible change in the form and structure of the crystals. Here isomorphism is restricted to small groups of compounds, whose limits are narrow and sharply defined; in other cases the limits of isomorphous groups are very wide, wider than generally admitted. Take for example the double arsenate of ammonium and calcium, $(\text{NH}_4)_2 \text{Ca As O}_4 + 6 \text{H}_2\text{O}$. In this compound Ca can be exchanged for Ba, Sr, Pb, Mg, Mn, Fe, Co, Ni, Zn, Cd and Cu. Nearly the same extension of change is found in triple acetates of the type: $\text{Na Mg} (\text{UO}_2)_3 (\text{C}_2 \text{H}_3 \text{O}_2)_9 + 9 \text{H}_2\text{O}$. They form rhombohedral crystals, imitating the regular (tetrahedric) crystals of the compound $\text{Na} (\text{UO}_2) (\text{C}_2 \text{H}_3 \text{O}_2)_3$. Speculation on a possible connexion between the wide range of isomorphism in complicated double and triple compounds with their constitution leaves the impression, that their form is ruled and fixed by constantly recurring nuclei of great volume, or, with other words: that isomorphism and morphotropism have a cause in common. The nucleus of the triple acetates is the compound $\text{Na} (\text{UO}_2) (\text{C}_2 \text{H}_3 \text{O}_2)_3$. Morphotropism plays here a prominent part, accompanying isomorphism and producing a striking imitation of regular forms in rhombohedral crystals. Obviously a connexion exists between these phenomena and the combination of isomorphism and morphotropism, found in some families of minerals, e.g. the pyroxenes, where exchange of Mg for Ca, of Ca for Zn or Mn does change the system of crystallization, while the general shape or habitus of the crystals remains unchanged.

Chemistry. — „*On the action of methylic alcohol on the imides of bibasic acids*”. By Prof. S. HOOGEWERFF also in the name of Dr. W. A. VAN DORP.

In April of this year we have placed in the Recueil des Travaux Chimiques des Pays-Bas et de la Belgique a short account, which shows that, by heating the imides of bibasic acids with methylic alcohol containing muriatic acid, the methylic ethers of the amido-acids, corresponding to the imides, are formed in some cases.

At least this was observed with the phenylimides of succinic acid and maleic acid, also with those, substituted in the radical of the acid. For example the phenylimide of maleic acid with methylic alco-

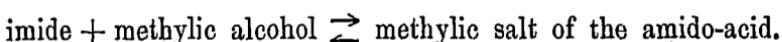
hol containing muriatic acid forms the ethereal salt according to the equation



On a closer study of this reaction we have found that the co-operation of the muriatic acid may be promotive in some cases to the formation of the methylic salt, but is not required for it.

Several imides investigated by us are transformed partly in the ethereal salts of amido-acids, when heated with absolute methylic alcohol without the addition of methylic alcohol containing hydrochloric acid. It is best to work in sealed glass tubes at temperatures between 150°—200° C. and sometimes higher still; but methylic salt formation also takes place when the imides are for a long time boiled with methylic alcohol.

We could show besides in some cases that these ethereal salts, when heated with methylic alcohol, are partly transformed again in imides and methylic alcohol. Perhaps these reactions are reversible:



This point will be investigated.

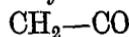
BERTHELOT and PÉAN DE ST. GILLES in their classic papers on the formation of ethers from acids and alcohols say, that the maximal quantity of ethereal salt that can be formed is but slightly dependent on the temperature. This seems equally to be the case, at least sometimes, in the case of the formation of the ethereal salts of amido-acids from imides.

We have prepared the ethereal salts of the corresponding amido-acids from the following imides by heating with methylic alcohol: succinimide, succinphenylimide, succinparanitrobenzylimide, maleinphenylimide, phtalphenylimide.

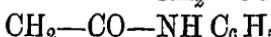
Succinimide $\begin{array}{c} \text{CH}_2-\text{CO} \\ | \\ \text{CH}_2-\text{CO} \end{array} > \text{NH}$, when heated for three hours in sealed tubes with the octuple weight of absolute methylic alcohol at a temperature of 170° C., gives the methylic salt of the succino-amido-acid $\begin{array}{c} \text{CH}_2-\text{CO}-\text{NH}_2 \\ | \\ \text{CH}_2-\text{CO OCH}_3 \end{array}$ in large quantity.

We prepared this ethereal salt, which we did not find mentioned

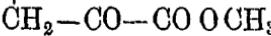
in literature, also by letting methylic iodide act upon the silver salt of succinoamido-acid, and convinced ourselves that both compounds are identical. If the imide is boiled with methylic alcohol, ethereal salt formation takes equally place, though slowly.



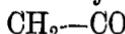
The transformation of the succinphenylimide $\begin{array}{c} \text{CH}_2-\text{CO} \\ | \\ \text{CH}_2-\text{CO}-\text{NH} \end{array} > \text{N C}_6\text{H}_5$ in



the ethereal salt of the amido-acid $\begin{array}{c} \text{CH}_2-\text{CO} \\ | \\ \text{CH}_2-\text{CO}-\text{CO O CH}_3 \end{array}$, which takes

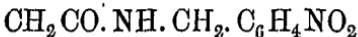


place easily if methylic alcohol containing muriatic acid is used, seems to be more difficult when this alcohol alone is employed. We had to heat up to 240° C. in order to obtain a satisfactory result. We consider our work with this imide as not yet finished.



If the succinparanitrobenzylimide $\begin{array}{c} \text{CH}_2-\text{CO} \\ | \\ \text{CH}_2-\text{CO} \end{array} > \text{N CH}_2\text{C}_6\text{H}_4\text{NO}_2$ is

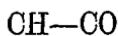
heated with a septuple quantity of methylic alcohol for some hours



at 170° C., a small quantity of the ester $\begin{array}{c} \text{CH}_2\text{CO} \\ | \\ \text{CH}_2\text{CO O CH}_3 \end{array}$

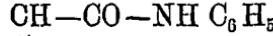


is formed. Experiments on ethereal salt formation at lower temperatures were not made in this case. On the other hand this methylic salt, when heated to 170° C. with methylic alcohol, is for the greater part transformed in the imide.



The maleinphenylimide $\begin{array}{c} \text{CH}-\text{CO} \\ || \\ \text{CH}-\text{CO} \end{array} > \text{N C}_6\text{H}_5$ (1 part), when heated

with methylic alcohol (7 parts) at 170° C., is transformed partly in



the methylic phenylamidomaleinate $\begin{array}{c} \text{CH}-\text{CO} \\ || \\ \text{CH}-\text{COO CH}_3 \end{array}$. The same

reaction takes place already, though slowly, when the phenylimide is boiled with methylic alcohol.



The phtalphenylimide $\text{C}_6\text{H}_4\begin{array}{c} \text{CO} \\ \diagdown \\ \text{CO} \end{array} > \text{N C}_6\text{H}_5$, when heated with

methylic alcohol on a waterbath in a flask connected with an inverted condenser, or at a higher temperature in sealed tubes, produces small quantities of the methylic phenylamidophtalate

$\text{C}_6\text{H}_4\begin{array}{c} \text{CO}-\text{NH C}_6\text{H}_5 \\ \diagdown \\ \text{CO O CH}_3 \end{array}$. The fact of this ethereal salt being unstable

against methylic alcohol we learn at once, when trying to crystallize

it from this solvent. The warm solution soon deposits some phenylimide.

To conclude we have found in comparative experiments that the imides form the ethereal salts of amido-acids much easier with methylic alcohol, than with aethylic- or propylic alcohol. The greater ethereal saltforming faculty possessed by the former alcohol¹⁾ shows itself equally in these experiments.

We herewith tender our best thanks to Messrs. VAN BREUKELE-VEEN and VAN HAARST, who assisted us with great zeal in the present investigation.

Delft/Amsterdam, October 1898.

Physics. — „*Description of an open manometer of reduced height*“.
By Prof. H. KAMERLINGH ONNES.

(Will be published in the Proceedings of the next meeting).

Zoology. — „*Cup-shaped red bloodcorpuscles. (Chromocraters)*“.
By Dr. M. C. DEKHUYZEN. (Communicated by Dr. P. P. C. HOEK).

The red bloodcorpuscles of the lamprey (*Petromyzon fluvialis*) when examined living or after fixation, exhibit a remarkable shape, which has escaped the attention of investigators. They are bell- or cup shaped cells. Their body contains a rather deep cavity which may be called an „oral invagination“. The rather wide opening is round, but owing to the facility with which the cells change their shape, may become a split or a triangle.

A second less evident „aboral“ invagination is found at the aboral pole, in a somewhat eccentric position however. Seen from above one of the poles, the cell is somewhat oval, almost round. No wonder that such a shape is not recognized, when the blood is spread out in a thin layer, dried and then preserved.

There is scarcely an object imaginable better calculated to make the objections evident which must be alleged against the usual methods of drying for the purpose of investigating the blood.

True amoeboid properties are wanting; some of the damaged cells

¹⁾ Vid. MLNSCHUTKIN, Lieb. Ann. **195**, p. 357.

however do indeed remind us of the thorn-apple shapes of the red blood-corpuscles of mammalia and amphibia.

The nucleus is ellipsoid. It lies in the proximity of the aboral invagination, and appears to be connected here with the boundary layer of the body of the cell by means of the microcentrum, the group of centrosomes.

The retraction sometimes tapers off rather sharply. The bottom of the deep oral invagination also appears to be connected, though much more loosely, with the nucleus.

Under injurious influences the mouth may disappear and the cell become globular, or the bottom of the oral invagination be thrust outwards like a bag turned inside out, so that a (clear) vesicle is extruded from the oral ring, *or the nucleus may be ejected from the opening of the mouth*. In preparations where the boundary layer of the body of the cell presents itself as a sharply defined membrane, a ring is visible round the mouth, viz. the oral ring. Round the aboral invagination we observe in such preparations, though less frequently, an aboral ring. What kind of peripheral, oral and aboral differentiations, preformed in the living cell, lie at the bases of these images, we cannot as yet say.

These cells may be called chromocraters, a word derived, by analogy with the term chromocyt in zoology, from the Greek $\chi\omega\alpha\tau\eta\omega$, a cup.

The desirability of proposing this new name arises from the circumstances that the chromocratera proves to be a red blood-corpuscle occurring in widely divergent groups of animals. It may be a thing of importance in phylogeny and in mammalia an inheritance of great antiquity.

The common red blood-corpuscles of mammalia (rat, cavia, rabbit) pass through a stage, in which they are nucleated chromocraters, and that as mature erythroblasts or normoblasts, when the nucleus is already pycnotically degenerated, which cells eject their nucleus through the oral invagination.

They then become cup-shaped erythrocytodes (known already to RINDFLEISCH¹⁾ and HOWELL²⁾): e.g. enucleated chromocraters with deep oral and less deep aboral invaginations.

In the full-grown red blood-corpuscles of man viz. in the tips of

¹⁾ RINDFLEISCH, Ueb. Knochenmark und Blutbildung. Archiv f. mikrosk. Anatomie. XVII. 1880.

²⁾ HOWELL, The life history of the formed elements of the blood, especially the red bloodcorpuscles; Journal of Morphology. IV. 1890.

the fingers it may also be shown that the two invaginations of the bi-concave discs are not morphologically equal. Their chromocrateric nature may be rendered visible by „fination” of the out-flowing blood in osmic acid. (according to an observation by Mr. H. W. BLÖTE, assistant at the Leyden Laboratory of Physiology).

We will by no means assert that all nuclei are eliminated by ejection; there seems to be no reason to question the statements made by trustworthy writers of cases in which the nucleus is destroyed by intracellular degeneration.

In the freshly fixed thighbone-marrow of a *Cavia* in an advanced stage of pregnancy, a round corpuscle, corresponding in all respects to the pycnotic nucleus of mature normoblasts, was observed in nearly all leukocytes next to the polymorphous nucleus.

Moreover the ejection of the nucleus already figured by RINDFLEISCH, may be easily observed e.g. in the thighbone-marrow of a rabbit three weeks old.

Now chromocraters, corresponding in shape to that described of the Lamprey, were also observed by me in *Phoxichilidium femoratum*, a *Pycnogonid* abounding in the port of Nieuwediep. Also just the same typical exterior of the aboral invagination, sometimes sharply tapering and pointing to the adjacent nucleus. *The ejection of the nucleus through the oral opening of a somewhat damaged cell has also been observed.*

I myself have observed chromocraters only in mammalia, lampreys, and the species of Pycnogonid just mentioned. Now we find in zoological literature descriptions and figures by different writers, who probably have seen chromocraters but owing to the difficulty of obtaining living specimens of the sometimens scarce material, their statements could not as yet be controlled or verified.

Unknown to each other the writers observe with surprise and more or less incidentally, that they have seen bloodcells which were cupshaped.

DOHRN in his *Monograph of the Pantopoda of the Gulf of Naples* says, that in nearly all Pycnogonids he had seen bloodcells which he called, „Ballons”. „Sieht men sie im Blut circuliren, so erscheinen sie gefaltet, wie ovale Ballons aus Seidenpapier die nicht mit Luft voll erfüllt sind.”

GRIESBACH¹⁾ has figured the haemoglobin-containing cells of *Pectunculus glycimeris*, a Lamellibranch Mollusc. „Sie sehen mützen-

¹⁾ GRIESBACH, Beiträge zur Histologie des Blutes. Arch. f. mikroskop. Anatomie XXXVII. 1891.

förmig aus, sie lassen sich vergleichen mit einem eingedrückten Gummiball, sie ähneln dem Hut eines Pilzes und durch die eingedrückte Stelle sieht man deutlich den Kern hindurchschimmern."

From the figure I think I may conclude that we can here have to do with hardly anything else than with chromocraters, at least with cells which have a great many properties in common with those we have described.

EISIG in his *Monographie der Capitelliden* has figured at least dish-shaped red bloodcells in Chaetopod worms (*Notomastus*). Nor should we omit to mention the figures and communications of CUÉNOT¹⁾ concerning *Cucumaria Planci*, an Echinoderm, and of *Sipunculus* and *Phascalosoma* (Gephyreans).

That the same very characteristic species of cell, possessing a shape, the appropriateness of which is at least very questionable, the significance enigmatic (unless the calyxiform bloodcells of the Gephyreans be congenial with chromocraters) should occur among such widely divergent groups of animals as Pyenogonids, Petromyzontes, Molluses (perhaps) and Mammalia, justifies, in my opinion the conclusion, that the chromocrater is a heritage from the common ancestors of the above mentioned groups of animals, that is from *worms*. In the ontogenesis of the red blood-corpuscles of mammalia the ancestral calyxiform nucleated blood-cell again appears for a short duration.

This investigation was carried on in the Physiological Laboratory of Leyden and at the zoological station at the Helder.

By the communications of GIGLIO TOS²⁾ our attention was directed to the Lamprey.

Astronomy. — „Some remarks upon the 14-monthly motion of the Pole of the Earth and upon the length of its period”. By Dr. E. F. VAN DE SANDE BAKHUYZEN (Communicated by Prof. H. G. VAN DE SANDE BAKHUYZEN).

(Will be published in the Proceedings of the next meeting.)

¹⁾ CUÉNOT, Etudes sur le sang etc. Arch. d. Zool. expérimentale et générale. IX, 1891.

²⁾ E. GIGLIO TOS, Sulle cellule del sangue della lampreda. Accad. reale della scienze di Torino. 1896.

Physics. — „*The influence of pressure on the critical temperature of complete mixture*”. By Mr. N. J. VAN DER LEE. (Communicated by Prof. J. D. VAN DER WAALS.)

In 1886 (Wied. Ann. Bd. 28) Mr. ALEXEJEW published investigations, undertaken to find out the influence of temperature on the mutual solubility of two liquids which are but partly soluble in each other. A temperature proved to exist, above which mixture in all proportions takes place (critical temperature of complete mixture). The existence of this temperature had been already supposed before. In 1880 (Verh. Kon. Ak.), Prof. VAN DER WAALS had pointed out, that the pressure too, must play a part in this phenomenon, and he had found that in a mixture of ether and water, the meniscus, parting the two phases, becomes flatter by higher pressure. Though this might be considered as a proof that the two phases approached each other as far as composition is concerned, complete mixture is not attained. Mr. ALEXEJEW seems to have been induced through this treatise, to examine also the influence of pressure for the mixtures investigated by him, but with a negative result. (I have not been able to consult the description of these last experiments. I know them only from citations). By the theory of the surface ψ (J. D. VAN DER WAALS: „Théorie moléculaire d'une substance composée de deux matières différentes. Arch. Néerl. T. 24 or Versl. Kon. Ak. 23 Febr. 1889) the influence of the pressure on the mutual solubility was also examined. In 1894 Mr. J. DE KOWALSKY published the description of his investigations (C. R. T. CXIX p. 512) concerning this theory. No influence of the pressure was found — not even by very high pressures — except for a triple mixture of aethylalcohol, isobutylalcohol and water. A pressure of 900 atm. made the liquid homogeneous at a temperature of about 3° below the temperature of complete mixture. Of late Mr. KLOBBIE (Zeitschr. f. phys. Chem. 24. 617. 1897) has found a perceptible influence of the pressure for mixtures of ether and water by a pressure of 100 atmospheres, without, however, determining its degree.

The purpose of the experiments which will be described here, was, to find something about the influence of the pressure on the mixture of liquids.

Of all the mixtures examined by Mr. ALEXEJEW the pair water and phenol seemed best fitted for these investigations. The critical temperature of complete mixture is about 67° . The phenol used was from MERCK & Co. and was tested beforehand by determining the fusing-point. It was preserved in the dark in sealed glass tubes,

each containing about the quantity necessary for one experiment. The proportion of the mixtures was regulated by putting together quantities of a given weight of the two substances. As the determinations were made for mixtures whose temperature of complete mixture was near the critical, the liquids were divided into two phases at the temperature of the room. Therefore the mixture was well shaken, so that an emulsion was obtained. A certain quantity of this was quickly poured into a capillary funnel, with which the liquid was brought into the observation-tube. This tube consisted of a straight tube of thick glass with about 3 m.M. inside diameter. It was fused together at one end, widened in one place just as the CAILLETET-tubes are, and cemented in a brass mounting. After a sufficient quantity of the mixture was poured into this tube, the stirring-contrivance was applied to the liquid, consisting of a magnetised needle, round which a closely fitting glass capillary was slided, which was then fused together on both ends and provided with glass balls to prevent its sticking to the glass walls. To bring this bar in motion an electro-magnetic stir-apparatus was used, like the one described by A. VAN ELDIK (Versl. Kon. Ak. 1897). After the stir-apparatus had been put in, the air-bubbles were removed as much as possible, and then the mercury was put in the tube. For this a not too great quantity was first carefully put in at the top of the tube. If care was taken that the mercury had but very little velocity, it kept sticking high in the tube, in consequence of the capillarity. After that it was easy to make it glide along a capillary down to a few cM. above the surface of the mixture. Now mercury could be poured in by means of a capillary-funnel, in which air-bubbles could be avoided. At last the whole mercury fibre could be lowered, till the lower end reached the surface of the mixture. After the tube had been completely filled with mercury, it could be turned upside down and placed in a steel vessel, which was in connection with the forcing-pump. Within this steel vessel a glass vessel filled with mercury, was placed in such a way, that the lower end of the glass tube was below the surface of the mercury. The space was further filled with glycerine, and the glass vessel closed with a brass nut. To have it hermetically closed, rubber rings are used.

The pressure could be kept constant with this instrument for a considerable time and could be read by a metal-manometer, a new one specially used for these experiments. This manometer had been tested, before it was received by us. As the influence of the pressure proved to be very slight, little differences of pressure were not paid

attention to, and it was not necessary to use a more accurate instrument. This slight influence, which had been expected beforehand from the theory and in connection with the experiments before-mentioned, made it necessary to make the observations near the temperature of complete mixture. Here the difficulty presented itself, that the heat which would be developed by compression, might be the cause that the temperature, which had first been below the temperature of complete mixture, would rise above it. In this case mixture would not be the direct consequence of the pressure. Therefore it was of great importance to make an arrangement by means of which it would be possible to keep the temperature round the glass-tube constant for a considerable time: for in this way it would be possible to remove the disturbing influence mentioned before. The attempts to get a suitable thermostat, led to the following arrangement agreeing in many respects with the one described by W. WATSON (Phil. Mag. Vol. 44 July 1897): a glass cylinder, 30 cM. high and with a diameter of 5,5 cM., is closed on both ends with caoutchouc stoppers. These stoppers are pierced in the middle, so that a glass cylinder of the same height as the former and with a diameter of 2,6 cM., can be placed concentrically with the other.

The annular cylindric vessel obtained in this way, is placed so as to have its axis vertical. Two holes made in the higher stopper, give an opportunity of adjusting two glass tubes. One of these is closed with a tap and serves to bring the liquid into the vessel. The other is (the glass tubes being in close contact) connected by means of a rubber tube with another glass tube placed vertically, which leads through a cooling-apparatus, and is bent horizontally just above it. It leads further to a T shaped piece to which are fastened 1°. an open manometer 2°. another T shaped piece. One arm of the latter T shaped piece has a rubber tube with a squeezer. This squeezer is so constructed that by means of a screw the admission of air can be nicely regulated or entirely stopped. The other arm leads to two large closed bottles, having together about 40 L. contents, and from there to a rubber tube (with a squeezer), which brings about the connection with a waterairpump. In this way the vessel is connected with a large space, in which the pressure can be lowered. The regulating of this pressure, read by means of the open manometer, can be easily done by the two squeezers mentioned. Then a small quantity of mercury is first brought into the vessel, enough to cover the bottom, and to preserve the caoutchouc in this way, and on it a quantity of aethyl-alcohol, which can be made to boil under various pressures. The

heat required was furnished by an alternating current, easily obtained by the installation of Electra which is found in the laboratory. This current is led through a spiral, wound round the inner cylinder of the boiling-vessel, the ends of which have been sealed through the glass of the outer cylinder. A necessary condition for the remaining constant of the temperature proved to be that the spiral remained entirely merged in the liquid.

The glass forcing-tube containing the mixture was then placed nearly in the axis of the inner cylinder with a thermometer and the tube of the electro-magnetic stir-apparatus near it. The space was then closed with wads on both sides. The thermometer was quite inside that part of the inner-cylinder, where the temperature had proved to be the same throughout. Therefore correction for the sticking out of the mercury was not necessary. Within the same space the mixture which was to be examined and the tin tube were placed.

The mixture was now brought to about the temperature of complete mixture, which took about an hour. Then the temperature was made to rise a little under continual stirring; after a moment's rest it was made to rise again etc., till the cloudiness (the proof of the presence of two phases), had quite disappeared. Then the temperature was read. After that the temperature at which the cloudiness reappeared was determined in the same way by cooling the mixture. By repeating these observations a few times, the temperature of complete mixture could be very accurately determined. (The thermometer was verified at the Reichsanstalt). In the same way the temperature of complete mixture by greater pressure was determined. The results of these observations follow:

$22\% \quad x = 0.05$

pressure:	norm.	30	60	90	120	150	180	atm.
temp. of compl. mixt.:		66.7	66.7	66.9	67	67.1	67.2	67.3

$34\% \quad x = 0.09$

pressure:	norm.	30	60	90	105	130	atm.
temp. of compl. mixt.:		67.6	67.6	67.8	67.8	68	68.1

$38\% \quad x = 0.11$

pressure:	norm.	60	90	atm.,
temp. of compl. mixt.:		67.3	67.5	67.7

47% $x = 0.15$

pressure:	norm.	60	90	120	atm.
temp. of compl. mixt.:		64.8	65.1	65.3	65.3

49% $x = 0.16$

pressure:	norm.	30	60	90	120	150	180	atm.
temp. of compl. mixt.:		65	65	65.1	65.2	65.3	65.5	65.6

55% $x = 0.18$

pressure:	norm.	30	60	90	atm.
temp. of compl. mixt.:		61.2	61.2	61.3	61.3

(The percentage of phenol has been given here. In the theory of the surface ψ we suppose that the quantity of one of the liquids is $M_1(1-x)$ and of the other M_2x , in which M_1 and M_2 represent the molecular weights. Here $M_2 = C_6H_5OH$.)

These observations prove, that increase of pressure causes the temperature to rise. Considered in connection with the theory of the surface ψ , they show that the longitudinal plait (second plait) has a plait-point on the side of the greater volumes and that at that plait-point it turns its hollow side towards the x -axis. As moreover, it is possible to prove in general theoretically, that rise of temperature makes the plait move in such a way that the projection of the connodal line on the XV plane falls within that of a preceding connodal line, it follows from this, that the longitudinal plaits (second plaits) will be moved to the side of the x -axis if the temperature rises. At a certain temperature the two connodal lines will intersect. Then we have three co-existing phases. When the temperature rises, the two points of intersection approach each other, in other words, the phases begin to resemble each other more and more. At last the two curves touch; there is only one liquid phase left, co-existing with the gas-phase. This temperature is called the temperature of complete mixture. This, however, is but one of the many critical temperatures of complete mixture, for at a higher temperature it is possible, by increasing the pressure, to get two liquid phases, which can become perfectly equal in composition and molecular volume. In this case also, there is a critical temperature. So the critical temperature rises through increase of pressure in the mixture examined.

If once more the case that two connodal lines intersect is examined, it is easily seen, that the two intersection-points are nodes of the longitudinal plait (second plait). When the temperature rises they approach each other, and when the curves touch, the point of contact may be considered as two nodes having fallen together. From this follows, that when two connodal curves touch, the point of contact is a plait-point of the longitudinal plait (second plait). The spinodal curve of the longitudinal plait will therefore also touch the spinodal curve of the transverse plait (first plait). A consequence of this is, that the differential coefficient $\frac{dp}{dx_1}$ (see T. M. p. 15) must be 0 in this case which points to a maximum or minimum of the line $p = f_1(x_1)$, that is the curve, which represents the relation between the composition and the pressure of the saturated vapour of the mixture.

To test this conclusion experimentally, some pressures of vapour were measured according to the method denoted by LEHFELDT (Phil. Mag. July 1898), the results follow:

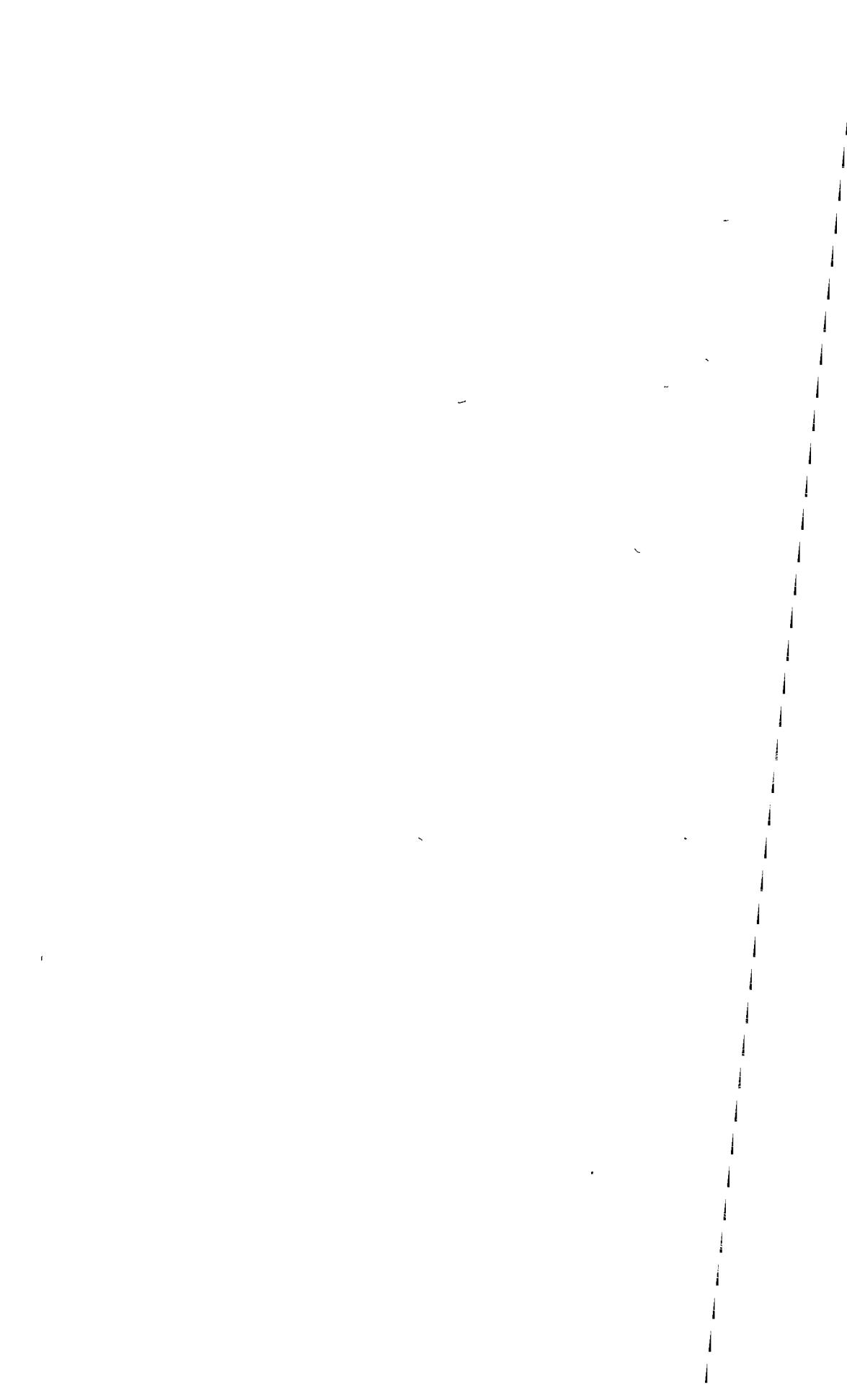
4.8% T.	$x = 0.01$ P. in m.M.	10.1% T.	$x = 0.02$ P. in m.M.	18.9% T.	$x = 0.04$ P. in m.M.
72.4	263	77.2	321	72.2	260
73.9	280	77.3	323	72.3	262
74.9	291	77.5	325	72.9	269
75.1	294	81.9	388	73.7	277
76.1	306	85	438	75.5	299
77.4	323			75.7	301
78.5	338			76.5	312
79.9	357			77.7	328
81.6	382			77.9	331
83	403			78.9	344
83.8	416			79.6	353
85.4	443			80.5	367
86.6	464			81.6	384
87	471			83	405
				84.4	428

33.6% T.	$x = 0.09$ P. in m.M.	50.9% T.	$x = 0.17$ P. in m.M.
71.2	251	71.2	251
72.2	261	71.4	253
73.1	272	72.4	264
74.1	284	73.7	277
74.4	288	74.6	289
75.5	300	76	306
76.7	315	77.2	321
76.9	318	78.3	337
77	319	79.3	351
77.7	328	81.3	378
78.7	343	81.8	386
79.4	352	82.8	401
79.9	360	83.8	417
80.9	375	85.6	448
81.6	385	86.6	462
82.6	400	86.7	468
84.2	426	86.8	470
85	440		
77.2% T.	$x = 0.39$ P. in m.M.	84.0% T	$x = 0.5$ P in m.M.
73.3	253	75.9	212
73.5	254	76.3	215
74.5	262	77.5	229
75.7	276	77.9	234
76.5	282	79.7	253
77.7	296	81.6	274
78.9	311	82.8	287
79.3	315	85.8	323
80.6	330	89	381
81	336	89.1	382
81.6	344	89.4	389
83.2	367		
84.4	387		
86.2	419		
86.4	422		
87.6	444		

From these figures the following values were found by interpolation for the curves $p = f_1(x_1)$. (The pressures for pure water are taken from the tables of LANDOLT. A slight correction ought to be made in the figures mentioned, which is, however of no influence on the general course of the pressure curves).

$\%$	x	75°	77°	79°	81°	83°	85°
0	0	289	314	340	369	400	433
5	0.01	293	318	344	373	403	436
10	0.02	294	319	345	374	405	438
19	0.04	294	319	345	374	405	438
34	0.09	294	319	346	375	406	440
51	0.17	294	319	345	374	404	438
77	0.39	268	289	310	336	364	397
84	0.5	201	223	245	267	290	311

Though its exact place cannot be determined from these figures, yet it appears that there is a maximum, which corresponds nearly with value 34 pCt. ($x = 0.1$), so nearly with the composition by the critical temperature of complete mixture of three phases.



KONINKLIJKE AKADEMIE VAN WETENSCHAPPEN
TE AMSTERDAM.

PROCEEDINGS OF THE MEETING

of Saturday November 26th 1898.

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(Translated from: *Ver slag van de gewone vergadering der Wis- en Natuurkundige Afdeeling van Zaterdag 26 November 1898 Dl. VII.*)

CONTENTS: „Haemato poiesis in the placenta of Tarsius and other mammals”. By Prof. A. A. W. HUBRECHT, p. 167. — „On a Contagium vivum fluidum causing the Spot-disease of the Tobacco-leaves”. By Prof. M. W. BEIJERINCK, p. 170. — „On congealing- and melting-phenomena in substances showing tautomerism”. By Prof. H. W. BAKHUIS ROOZEBOOM, p. 176. — „Variation of volume and of pressure in mixing”. By Prof. J. D. VAN DER WAALS, p. 179. — „Equilibriums in systems of three components. Change of the mixing-temperature of binary mixtures by the addition of a third component”. By Mr. F. A. II. SCHREINEMAKERS (Communicated by Prof. J. M. VAN BEMMEL), p. 191. — „On the accurate determination of the molecular weight of gases from their density”. By Prof. J. D. VAN DER WAALS, p. 198. — „Some remarks upon the 14-monthly motion of the Pole of the Earth and upon the length of its period”. By Dr. E. F. VAN DE SANDE BAKHUYZEN (Communicated by Prof. H. G. VAN DE SANDE BAKHUYZEN) p. 201. — „A standard open manometer of reduced height with transference of pressure by means of compressed gas”. By Prof. H. KAMBLINGH ONNES, p. 213. (With one plate.)

The following papers were read:

Zoology. — „Haemato poiesis in the placenta of Tarsius and other mammals.” By Prof. A. A. W. HUBRECHT.

The various authors who have investigated in the course of the last thirty years the first origin of the mammalian red blood-corpuscles have come to conclusions that are far from unanimous. This can in part be ascribed to the wish to look upon the red blood-corpuscles without nucleus of the full-grown mammalia as morphological elements, that are equivalent with the nucleated red corpuscles of the lower vertebrates and of mammalian embryos.

Against this view SCHÄFER, SEDGWICK-MINOT and RANVIER (whose experimental proof has however a year ago been disposed of by VOSMAER) protested. The two first-named look upon the non-nucleated mammalian blood-corpuscles as plastids, that are formed in cells in an analogous way as are the chlorophyll-granules in vegetable cells. The majority of the remaining investigators consider the non-nucleated mammalian blood-corpuscles as cells from which the nucleus has either been extruded (RINFLEISCH, VAN DER STRICHT, BIZZOZERO, SAXER, KOSTANECKI, HOWELL, MONDINO), or in which the nucleus gradually disappears within the bloodcell (KÖLLIKER, NEUMANN, SANFELICE, SPULER, LÖWIT, ELIASBERG, FREIBERG, GRUNBERG, ISRAEL, PAPPENHEIM). DISSE, summarizing the results obtained up to 1895 writes as follows: „Eine sichere Entscheidung der Frage nach dem Modus der Entkernung der rothen Blutzellen erscheint einstweilen unmöglich, da die directe Beobachtung des Vorganges der Entkernung im strömenden Blut unthunlich ist.”

On comparing the maternal and the embryonic blood-corpuscles as they circulate in each other's immediate vicinity in any section of the preserved placenta of various mammals in various stages of development we are struck by two facts. Firstly the nuclei of the embryonic blood-corpuscles differ in many respects from the nuclei of the very earliest bloodcells that arise in the area vasculosa. Secondly it is the first-named „nuclei” and it is not the corpuscle that encloses them, which resemble both in size and very often in staining properties the non-nucleated corpuscles of the mother, so that the question imposes itself whether, if indeed the nucleated embryonic mammalian blood-corpuscles change into non-nucleated corpuscles by extrusion of the nucleus, it might not much rather be this so-called nucleus (which differs notably from a normal nucleus) which will correspond to the definite non-nucleated corpuscle, than the vesicle from which it has been expelled.

The observation of quite a different series of phenomena in the placenta of Tarsius spectrum leads to a confirmation of this hypothesis. They render it probable that during the development of the Tarsius-placenta part of the cell-material which is actively concerned in this development, becomes converted into blood-corpuscles that are set free in the circulating maternal blood which bathes it. These bloodcorpuscles, entirely corresponding to those which we encounter everywhere in the maternal bloodvessels, do not take their origin out of the cytoplasma but out of the nucleoplasma and do not consist of chromatin so characteristic for the nucleus, but rather and principally of nucleolar matter which plays a part in many cell-nuclei

by the side of the chromatin. Haematopoiesis occurs in various ways in the *Tarsius*-placenta. Every now and then we notice a nucleolar body (the nuclear membrane surrounding it becoming partially indistinct and then disappearing) being set free and mixing up with circulating blood-corpuscles from which it cannot possibly be distinguished. Besides this simpler mode of origin we find another in which large-sized, so-called giantcells with lobulated and gemmating nuclei play a part. Numerous nuclear fragments are set free from it, the nucleus itself vanishing in the process. The fragments are of equal size, behave in a corresponding way towards the most different staining reagents and might be designated as „haematogonia”. All the intermediate stages between these haematogonia and normal blood-corpuscles were observed and similarly their development out of the enlarged nucleus and not out of the cellplasm could be demonstrated. It are not only maternal but also embryonic trophoblastcells which partake in this haematopoiesis under similar phenomena of proliferation; the blood-corpuscles thus formed are, also caught up by the maternal blood and circulate with it.

A destructive significance cannot reasonably be given to the giantcells in the *Tarsius*-placenta: they are decidedly constructive elements, which furnish not only blood-corpuscles, but also the walls of bloodlacunae. This double part is often played by solid strands of cells in lower vertebrates.

It deserves attention that the participation of giantcells with characteristic proliferating nuclei in the formation of blood in the bone-marrow, the liver and the spleen of mammals was expressly recognized by NEUMANN, KÖLLIKER, PEREMESCHKO, KUBORN, SAXER, ELIASBERG, FREIBERG a. o. Many of them look upon the proliferation of the nuclei of these giantcells (which are perfectly distinct from those other giantcells, the osteoclasts which occur in their immediate vicinity in the bone-marrow) as the first step in the formation of blood-corpuscles, although none of them has expressed the opinion that these latter should not be looked upon as cells but as nuclear derivates. As soon as we do this, on account of what we have observed in *Tarsius*, light is also thrown on the development of the fullgrown non-nucleated corpuscles out of embryonic nucleated ones, a phenomenon which as above indicated is undoubtedly comparable to it.

Similar haematopoietic processes are noticed in the placenta of *Tupaja*, which differ in detail but agree in general outlines with what has here been described for *Tarsius*.

Whether blood is also formed in the placenta of other mammals must be carefully looked into. Corpuscles are figured, mixed up with maternal blood-corpuscles, by NOLF for the bat's placenta, by MAXIMOW for that of the rabbit, by SIEGENBEEK VAN HEUKELOM for that of man, with which I feel inclined to identify my „haematogonia” and of which the first and last-named author decidedly state that they are distinguished by certain characters from polynuclear leucocytes.

Neither of them, however, refers what he has observed to haematopoeisis.

In point of fact MASQUELIN and SWAEN (1880) and FROMMEL (1888) have already stated that blood is formed in the placenta respectively in the rabbit and in the bat. Their observations have up to now convinced but few and do not correspond in their details with my own. What I have myself observed in the rabbit, the hedgehog, the shrew and the mole has never emboldened me to conclude to the existence of haematopoietic processes in the placenta: it was not until I had examined the Tarsius-placenta in which the phenomena are so extraordinarily lucid that I was forced to draw the conclusions of which a rapid sketch was given above, but which is in no way meant to be a generalisation. Ungulates and Lemurs, certain Edentates (and probably also the Cetacea) undoubtedly miss a similar haematopoiesis. Its strong development in Tarsius is perhaps connected with the unfavourable relation in which the small and delicate mother finds itself placed with respect to the comparatively large foetus, while moreover each parturition is generally immediately followed by a new pregnancy, a circumstance which however exhausting its effect may be upon the mother is decidedly most favourable to the collector of embryological material.

This short account will soon be followed by a full description with plates and figures, which will appear in the Report of the Zoological Congress that was held at Cambridge in 1898.

A discussion followed in which Prof. MAC GILLAVRY and Prof. HUBRECHT took part.

Botany. — „*On a Contagium vivum fluidum causing the Spot-disease of the Tobacco-leaves*”. By Prof. M. W. BEIJERINCK.

The spot-disease of the tobacco plant, also called mosaic-disease, consists in a discolouration of the chlorophyll, spreading in little spots over the leaf and afterwards succeeded by the partly or entirely dying away of the tissue which originally composed the spots. Com-

monly the discoloration first manifests itself near the nerves of the leaf by a considerable increase of chlorophyll; afterwards the inter-spaces between the dark green spots are affected by a bleaching-process, which mostly does not go farther than to give a yellow hue to the sickened parts, but which in some cases, causes variegation. The dark green spots grow in the beginning more intensely than the rest of the leaf, thus becoming blown-up protuberances rising from the upper surface of the leaf. This, however, occurs oftener with artificial infection experiments than on the tobacco fields, where the diseased leaves remain flat. The third phase of the disease consists in the locally dying of little spots irregularly spread over the leaf; they get soon brown, are very brittle and even the culling of the leaf may change them into holes. They make the leaves valueless as wrappers of cigars.

Professor ADOLF MAYER pointed out in 1886 that this disease is contagious. He pressed the sap out of diseased plants, filled with it little capillary tubes, put them into healthy plants and after two or three weeks he found these to be likewise attacked by the disease.

In 1887 I endeavoured to solve the question whether any parasite might be found as cause of the disease. It was clear that, if this should really be the case, there could only be thought of bacteria, for microscopic observations had not indicated the least traces of microbes. The bacteriologic culture-methods proved that aerobic bacteria could not come into consideration for they failed as well in the tissue of the healthy as of the diseased plants, and the same holds good regarding the anaerobics. So, it was certain that here was an instance of a disease caused by a contagium fixum. This consideration induced me in 1897 to conduct new infection experiments in order to become better acquainted with the characteristics of the contagium. The chief results of these experiments are the following.

In the first place it was proved that the sap pressed from diseased plants and filtered through very dense porcelain was absolutely devoid of bacteria, without losing of its virulence. Attempts made to point out in the filtrate aerobics or anaerobics again did not give any result.

In order to answer the question whether the virus ought to be considered as corpuscular or as dissolved or liquid, some parenchyma of diseased plants rubbed fine was spread over agar-plates and then, left to diffusion. A virus, consisting of discrete particles, would needs remain on the surface of the agar and consequently be in the impossibility of rendering the agar virulent; a virus, really

dissolved in water would, on the contrary, be able to penetrate to a certain depth into the agar.

After about ten days' diffusion, a time which I considered sufficiently long in accordance with that wanted for the diffusion unto a considerable depth of diastase, and trypsin, the surface of the agar-plate was first cleaned with water and then with a solution of sublimate; then the upper layer was removed by means of a sharp platinum spatula. In this way the inner part of the agar might be reached without its having any contact with the particles adhering to the surface. Infection experiments performed with these deeper layers caused as well the disease as the porcelain-filtrate. So, there seems no doubt left but the contagium must be fit for diffusion and consequently considered as fluid.

The infection experiments were performed with the expressed sap by injections with the syringe of PRAVAZ. The most proper place for injecting is the stem, and in particular the youngest parts which are still in growth. The nearer the place of injection is to the terminal bud, the sooner its consequences show themselves. This is evident from the experience that only those leaves are susceptible of infection which are still in growth and in the phase of cell-multiplication, meristems being by far the most susceptible. By making use of this fact and injecting the virus cautiously quite near to active meristems, I was of late enabled, three days since the injection, already to observe the first symptoms of the disease, whilst otherwise they must be waited for two or three weeks longer. Full-grown leaves, and even leaves whose cells are still in the phase of elongation but no more in that of multiplication, are unfit for infection.

As the quantity of virus, sufficient to produce a large number of diseased leaves, is extremely small, and as the juice of these leaves will serve to infect an unlimited number of plants, it is clear that the virus must increase within the tissues. In accordance with what is said before, this increase occurs in and with the dividing cells, the full-grown tissues of the plant not allowing any such increase. This quality of the virus reminds, to a certain extent, of the action of the cecidiogenous substances, which likewise exert their influence in those parts of plants only, which are still in a state of growth and cell-multiplication. Out of the plant it seems impossible to bring the virus to increase. This conclusion must be drawn from the fact that bougie-filtrate mixed and long kept with a certain quantity of the filtrate of juice of a healthy plant, not only does not increase but even loses in virulence in the same degree as if it had been

diluted with pure water. It is not difficult to convince oneself of this, for the quantity of the virus employed is of great influence upon the symptoms and the course of the spot-disease. If a considerable quantity of virus is at once introduced into the plant, the first diseased leaves, which develop from the bud, not only show, besides the usual symptoms, a remarkable relaxation and suspense of growth, by which they remain much smaller than normal leaves, but also deep, irregular pinnate or palmate incisions in the margin in consequence of some lateral nerves remaining short. As the chlorophyll-tissue thereby develops very imperfectly and as, especially near the nerves, the formation of chlorophyll may be quite deficient, these leaves get a peculiar striped appearance and by their shape belong to the true monstrosities¹⁾. When a small quantity of the sap is used such deformations don't appear at all, so that it seems to me that an increase in virulence of the sap under the said circumstances would not have escaped me.

Consequently I consider it as certain that the virus can only be reproduced in the manner described, with and through cell-multiplication of the plant. In my opinion this fact must be related to the fluid or dissolved state of the virus, for, with regard, to a contagium fixum, were its particles ever so small that they escaped all microscopic observation, there is no plausible reason why it might not augment, like parasitical bacteria, out of the fosterplant. It does not even appear impossible that a microscopically invisible, but notwithstanding corpuscular contagium, might occasion visible colonies on culture gelatine. A fluid virus, fit for diffusion like that of the spot-disease, would penetrate into the gelatine or the agar and, if it were fit for reproduction, it would then alter the chemical nature of the nutritive substances, which might perhaps be observed by a change of colour or of refrangibility in the plates. When „sowing” the virus on malt-extract gelatine, and on plates obtained from some plant-infusion with 2% canesugar and 10% gelatine, — according to my experience excellent culture masses for bacteria commonly occurring on plants, — such alterations could in no way be observed. Moreover, though reproduction or growth of dissolved matter is not quite inconceivable, yet, it is difficult to imagine in what way such a process might be achieved. A division-process in the molecules causing their multiplication, meets, to my mind, with great difficulties; even the conception of „molecules which take food”, would seem to me vague, if not inconsistent. The increase of the contagium now, is

¹⁾ On one of my plants such a leaf had taken the form of an ascidium.

on the contrary, partly explained by the circumstance that it must first be bound to the protoplasm of the living cell itself, and so carried on into the reproduction. In any case, two enigmas seem by this fact to a certain degree to be reduced to one¹⁾.

If the ground in which the tobacco-plants grow is infected with the virus, one sees, after some time, the disease appear in the terminal bud. The time of the incubation varies very much and depends on the size of the plants. In smaller plants I remarked the first symptoms of the disease in the new-formed leaves of the terminal bud two weeks after the infection; in bigger ones after three to six weeks. Root and stem are in this case obliged to convey the virus often to considerable distances. Various observations prove that this conveyance goes, by way of exception, along the xylem, which conducts the water; usually, however, it seems to follow the so-called descending sap, and then it probably goes along the phloem.

That the first mentioned way may be followed, must be concluded from the order of succession in which the symptoms of the disease appear when a great quantity of the contagium is introduced into the stem, for in this case those parts of the young leaves get first diseased which are exposed to the strongest evaporation, such as the tops and margins which reach freely out of the bud. The conveyance of the virus along the phloem may be concluded from the following observation.

If one infects the middle-ribs of full-grown leaves, or of leaves whose cells are in a state of elongation but no more of cell-division, the leaves themselves continue healthy, but the virus turns back to the stem, thence, in the usual way, to infect the meristems of the buds and the youngest leaf-rudiments. This return of the virus from the leaf down to the stem must undoubtedly be by the way of the descending sap, that is along the phloem.

¹⁾ Perhaps, basing on these considerations, one may think of the possibility that enzymes, in a manner agreeing with that of the contagium fluidum, are reproduced in the cells, and might thus, to a certain extent, be considered as independently existing. Concerning this point I wish to observe the following. Pressed yeast, cultivated in nutrient containing diastase, takes from it a notable quantity of diastase which it is difficult completely to remove from it by washing. If, however, this yeast continues growing in a medium free from diastase, then the diastase soon totally disappears. That this might be attributed to incongruence between the protoplasm of the yeast-cell and the diastase-molecule, so that a persistent union might possibly be effected by other microbes or by means of the tissue-cells of higher organisms, is not probable, the yeast-cell being not absolutely free from diastatic substances, containing, for instance, some glucase. For the moment, therefore, I must consider such a conclusion as unavailable.

From out the root, infection is possible even with plants of two or more decimeters in height. If for this purpose woundings of the root are necessary, is not yet clear; probably roots may absorb the virus from the ground even through their unhurt surface. As infection only occurs in the buds and meristems, the number of healthy leaves found at the bottom of the plant, indicates in some way the date of the infection, in case the virus has entered through the root.

Without any loss of virulence the virus may be dried and in that state exist through the winter, for instance in the ground; part, however, gets lost then, just as is the case with many bacteria and yeast species. The leaves, too, keep their virulence when dried, so that the dust of the brittle leaves helps, no doubt, to spread the disease. Precipitating the virus with strong alcohol from its solution and drying the precipitate at 40° C., it remains virulent.

As was to be expected, the virus in moist state is rendered inactive, not only by boiling temperature, but already at 90° C. The lowest deadly temperature I have not been able to fix; I think it will be found between 70 or 80° C.

Above I alluded to the formation of characteristic leaf-monstrosities when a large dose of the virus is injected. Another, but rarer effect of artificial infection, is variegation or albinism. Hitherto I obtained this effect in too few plants, than that I should be able to point out how it may be expected with certainty; but I have some hopes that further experiments will enable to produce it at will.

That albinism, or at least one of the forms in which it appears, shows a certain relation to the leaf-spot disease, may be allowed already at a superficial view of the latter. However, until now, we are obliged to admit that an important difference exists between them as to the way of transferring the infection. In so far as may be concluded from the relatively few experiments concerning this point, infection for albinism requires a direct uniting, by means of grafting or budding of the variegated with the green plant. Infection, on the contrary, of green plants with the crushed tissue of variegated varieties, seems never to produce any results. My above mentioned plants, however, indicate that there must exist another way along which variegation may be called forth, namely by a virus existing outside of the plant.

Probably there are various other plant-diseases, which originate in a manner alike to that of the spot-disease of the tobacco. The disease of the peach-trees in America, described by ERWIN SMITH under the name of „Peach Yellows” and „Peach Rosette” (U. S.

Departm. of Agriculture, Farmers' Bulletin N°. 17, Washington 1894) are, according to the description, undoubtedly caused by a contagium fluidum, but it is still dubious whether the infection is only transferred by grafting and budding, or, — which is more probable, — also by a virus existing outside of the plant.

Chemistry. — „*On congealing- and melting-phenomena in substances showing tautomerism*”. By Prof. H. W. BAKHUIS ROOZEBOOM.

The latest discoveries on tautomerism, which have shown, that tautomeric substances in the liquid state must be considered as mixtures of two kinds of molecules of different structure, have raised the problem how to explain the complicate congealing- and melting-phenomena of such substances, in case both forms or one of them can appear in the solid state.

Some remarkable investigations on this subject have of late been made by BANCROFT and his disciples, which were a continuation of a theory of DUHEM.

At an attempt to unravel the investigations of CLAISEN on this subject, the reader had come to the same conclusions, which may be united to a perfectly clear graphic sketch.

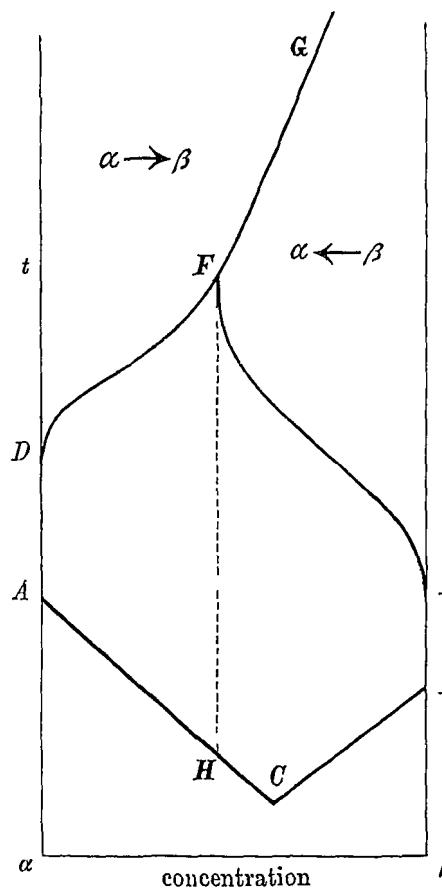
BANCROFT having already published this, there would be no reason to revert to the subject, if not all examples chosen by him, referred to cases in which all the melting- and congealing-points were found in the region of temperatures in which equilibrium is still obtained between the two forms in the liquid state.

In such a case we generally have the disadvantage of there being no certainty about the mixing-proportion of the two substances at the moment of melting or congealing. Consequently it is impossible to give quantitative representations.

To arrive at a good understanding of the phenomena, it is therefore desirable to begin with a deduction of the conduct of tautomeric substances, the congealing temperatures of which are below the temperature-limit where in the liquid state transformation between two forms is still possible.

If we call the two forms α and β , we may build up a sketch in which the mixing proportion of α and β is measured on the horizontal axis of 0—100 and the temperature on the vertical axis.

According to the supposition made above, the congealing appears in the ordinary and simplest form of the congealing of mixtures of two substances i. e. starting from the melting points A and B of the two modifications, we have two melting-lines AC and BC



meeting in the point C , below which *every* liquid mixture congeals entirely to a conglomerate of α and β crystals.

We now suppose that at higher temperatures not all mixing-proportions in the liquid state are possible, but equilibrium appears, giving at each temperature a definite mixing-proportion of α and β . We represent these mixing-proportions by the line FG which is arbitrarily drawn as rising from left to right, representing the case in which the transition

E $\alpha \rightarrow \beta$ takes place with absorption of heat.

The reverse is also possible, or it may be a vertical line if the heat is zero.

On the left of the line FG is the region in which the transformation $\alpha \rightarrow \beta$ takes place, on the right that of the transformation $\alpha \leftarrow \beta$.

Now in case both transformations do not take place at lower temperatures, the supposition naturally occurs to us — on analogy of many other phenomena of recent date — that the transition from the region of *reciprocal equilibrium* to the region of no equilibrium is formed by two regions of *one-sided equilibrium*, as may be represented by splitting up the line GF into two lines FD and FE which end below certain temperatures on each of the axes.

If I now heat the solid modifications only a little above their melting point, they will be in a condition to congeal again at the same temperatures at which they melted. But if I heat them to a higher temperature, keeping them for some time at a temperature belonging to the region of one-sided or reciprocal equilibrium, a greater or less transformation in the liquid will take place between the two modifications, varying afterwards according to the velocity of cooling. The manner of proceeding of the congelation after return to this region will depend upon all these circumstances.

From the sketch the following conclusions may be drawn:

Case I. The substance α or β is heated for a considerable time after melting at a temperature situated *above* that of point F . Then the liquid is *slowly* cooled.

No matter what the heating-temperature was, and whether we started from α or from β , the congealing now begins in point H^1), depositing solid α , until it is completed in C , depositing solid $\alpha + \beta$.

Case II. The substance α is heated for a considerable time after melting at a temperature situated *below* F , being afterwards *slowly* cooled.

The first melting point is now variable, always lying above H , and that the more so in proportion to the heating having taken place to a lower temperature. The first deposit is solid α , the congealing ends in C .

Starting from β , and in proportion to the heating having been lower than F , the first congealing point will be at first below H , depositing α , then falling to C , then rising along CB depositing β . C remains the final congealing point.

Case III. The substance α or β is heated for a considerable time after melting at a temperature *above* F , but afterwards *quickly* cooled.

The first congealing point is situated to the right of H , no matter whether α or β be the starting point, and this the more so in proportion to the heating having been greater and the cooling quicker.

The starting point of the congealing may even move past C , so that β becomes the substance first deposited, provided the line FG running sufficiently to the right. C remains the final congealing point.

Case IV. The substance α or β is heated for a considerable time after melting at a temperature *below* F being afterwards *quickly* cooled.

The results both for α and β are the same as in Case II.

Case V. If the heating does not last long enough to reach the final equilibrium in the liquid state, the same result as in Case II

¹⁾ The point H lies vertically under F , or better still under the point of FE on which a vertical tangent may be drawn to this curve.

will be obtained in case of a quick cooling — independent of the situation of the heating-temperature.

The velocity of cooling as mentioned in Case III—V, is supposed to be such that the liquid finds no occasion to alter its mixing-proportion α/β acquired at a higher temperature. Should the velocity of cooling be less, this only results in the differences between the first acquired congealing-point and point H being smaller than in a case of very quick cooling. The deviations remain however in the same direction.

It is therefore possible to determine with perfect accuracy the apparently curious phenomena of congelation, with a knowledge of the lines $G F$, $F D$ and $F E$. On the other hand it would be possible, from observation of the congealing temperatures, after the substance α or β having gone through a sharply defined way, to conclude to the situation of different points of these lines, consequently to determine their direction, if this were not possible along other ways.

The phenomena described here, may appear not only in tautomeric substances, but in all substances which in the liquid state, give two modifications that are apt to transformation.

Consequently many optic isomers that show equilibrium at higher temperatures, come under this head. There however the matter can often become complicated, it being possible that after the transformation of the *d* or the *l*-form, the racemic form deposes in the solid condition. It is however easy to take this into account.

Physics. — „Variation of volume and of pressure in mixing”.

By Prof. J. D. VAN DER WAALS.

The supposition of Mr. AMAGAT (C. R. 11 Juillet 1898), that in a mixture every gas can be considered to occupy the volume which it would occupy separately under the same pressure and at the same temperature, comes to the same thing as supposing, that mixing under a constant pressure does not cause variation of volume, and that there would be no question of either positive or negative contraction. As at great densities (of liquids), mixing is generally accompanied by contraction, the thesis, also in case of slight densities, can only be meant as an approximation. For slight densities this thesis can be tested by means of the characteristic equation of a mixture.

For a molecular quantity of a mixture, consisting of $m_1(1-x)$ and m_2x unities of mass, the equation holds:

$$pv = MRT \frac{v}{v - b_x} - \frac{a_x}{v}$$

or by approximation

$$pv = MRT - \frac{1}{v} (a_x - b_x MRT) \dots \dots \dots \quad (1)$$

For each of the components

$$pv_1 = MRT - \frac{1}{v_1} (a_1 - b_1 MRT) \dots \dots \dots \quad (2)$$

would hold for a molecular quantity, and

$$pv_2 = MRT - \frac{1}{v_2} (a_2 - b_2 MRT) \dots \dots \dots \quad (3)$$

If we put $v = v_1(1-x) + v_2x + \Delta_v$, and take into consideration that $a_x = a_1(1-x)^2 + 2a_{12}x(1-x) + a_2x^2$ and $b_x = b_1(1-x)^2 + 2b_{12}x(1-x) + b_2x^2$

we get

$$\begin{aligned} p\Delta_v = & - \frac{1}{v} (a_x - b_x MRT) + \frac{(1-x)(a_1 - b_1 MRT)}{v_1} + \\ & + \frac{x(a_2 - b_2 MRT)}{v_2} \end{aligned}$$

by subtracting the sum of $(1-x)$ times equation (2) and x times equation (3) from equation (1).

As pv as well as pv_1 and pv_2 are equal to MRT by approximation, we may put

$$MRT \Delta_v = \{a_1(1-x) + a_2x - [a_1(1-x)^2 + 2a_{12}x(1-x) + a_2x^2]\}$$

$$- MRT\{b_1(1-x) + b_2x - [b_1(1-x)^2 + 2b_{12}x(1-x) + b_2x^2]\}$$

or

$$MRT \Delta_v = -x(1-x)\{[2a_{12} - a_1 - a_2] - [2b_{12} - b_1 - b_2] MRT\}$$

or

$$\Delta_v = \alpha(1-\alpha) \left\{ \frac{a_1 + a_2 - 2a_{12}}{MRT} - (b_1 + b_2 - 2b_{12}) \right\} \dots \quad (4)$$

From this equation (4) follows: 1°. that the absolute value of the variation of volume at a given temperature is independent of the pressure, under which the mixture takes place, of course only as long as it does not surpass the limit, below which the calculations mentioned are sufficient approximations; 2°. that the maximum value of this volume variation is found for $\alpha = \frac{1}{2}$; so if the substances to be mixed have the same volume. For air, which is composed of oxygen and nitrogen, the volumecontraction will amount to no more than $\frac{16}{25}$ of the value, found when equal volumes of oxygen and nitrogen are mixed. The quantities must, of course, be chosen in such a way, that in both cases the total volume of the components is the same; 3°. that it depends on the value of the expression:

$$\frac{a_1 + a_2 - 2a_{12}}{1 + \alpha t} - (b_1 + b_2 - 2b_{12}) \dots \dots \dots \quad (5)$$

whether negative or positive contraction takes place.

As in the characteristic equation the volume, occupied by the molecular quantity under the pressure of one atmosphere and at 0° , has been taken as unity of volume, the quantity Δ_v is also expressed in that unity.

It is true that the unity of volume in the three equations (1), (2) and (3) is not absolutely the same, on account of their different degree of deviation from the law of BOYLE, but the influence of this fact may be neglected in these calculations, as the deviation it causes, is a small quantity of higher order.

If we proceed to the discussion of the expression (5), we see in the first place that $b_1 + b_2 - 2b_{12} = 0$ comes to the same thing as assuming the co-volume of a mixture equal to the sum of the co-volumes of the components.

The circumstance, that it is easier to arrange arbitrarily formed bodies, which take up together a certain volume, in a given space, when the bodies are different in size, than when they are all of the same size, makes it probable, that the co-volume of a mixture of molecules of different sizes will be smaller than 4 times the real volume. In the deduction of the characteristic equation, in which, however, the molecules are thought as spheres, this has been proved,

and $\frac{3}{2}(\mathcal{V}b_2 - \mathcal{V}b_1)^2 (\mathcal{V}b_2 + \mathcal{V}b_1)$ ¹⁾ has been found for the value of $b_1 + b_2 - 2b_{12}$, an expression which is always positive. On the other hand, this expression, which is equal to 0, when $b_1 = b_2$, and which is positive, as well when $b_1 > b_2$ as when $b_1 < b_2$, shows that for a small difference in size the value of $b_1 + b_2 - 2b_{12}$ is very small. If we might neglect it, it would only depend on the sign of $2a_{12} - a_1 - a_2$, whether mixture would cause contraction or not.

When $2a_{12} - a_1 - a_2$ is positive, mixing is favoured by the molecular forces. For if we suppose the two gases before the mixing separated by a mathematic surface, $\frac{a_1}{v_1^2}$ and $\frac{a_2}{v_2^2}$ are the forces which

oppose mixture, and $\frac{2a_{12}}{v_1 v_2}$ is the force, which draws the two substances through the bounding surface. In this case we may put $v_1 = v_2$, and the sign of $2a_{12} - a_1 - a_2$ proves to be decisive.

In general we are justified in expecting, that when mixing is favoured by the molecular forces, and when in consequence of the mixing a smaller molecular volume must be subtracted from the external volume, both circumstances cause positive contraction (negative value of Δ_v).

If $(a_1 + a_2 - 2a_{12})$ and $(b_1 + b_2 - 2b_{12})$ are both positive, a temperature exists, below which Δ_v is positive and above which Δ_v is negative, just as is the case for the deviation from the law of BOYLE for a simple substance. But in general we may expect that $-\Delta_v$ (volume contraction) will be small and that the thesis of Mr. AMAGAT will hold true with a high degree of approximation, at least in all cases, in which the properties of the components differ little. In the first place because $a_1 + a_2 - 2a_{12}$ and $b_1 + b_2 - 2b_{12}$ are both equal to 0, if the substances are the same, and we may therefore put, that when the difference is small, the value of these quantities will be small, compared to each of the terms, of which they consist, e.g. $a_1 + a_2 - 2a_{12}$ is small as compared with a_1 or a_2 , and $b_1 + b_2 - 2b_{12}$ is small as compared with b_1 or b_2 . Secondly on account of the factor $x(1-x)$; for air this factor amounts to no more than $\frac{4}{25}$.

Our equation cannot be tested at the values which Mr. AMAGAT gives for air of the ordinary temperature and which begin at a

¹⁾ Théorie Moléc. Arch. Néerl. Tom. XXIV.

pressure of 100 atm., as the equation is not sufficiently approximative, and because Mr. AMAGAT himself comes to the conclusion, that the deviation found does not exceed the possible errors of observation. It were desirable that similar experiments were made with equal volumes of substances which differ much in physical properties. In testing, the equation (4) should be replaced by another which would hold with a higher degree of approximation. If the volumes v_1 , v_2 and v show a marked difference, the term $a_1 + a_2 - 2 a_{12}$ should be replaced by:

$$v_1 v_2 \left\{ \frac{a_1}{v_1^2} + \frac{a_2}{v_2^2} - \frac{2 a_{12}}{v_1 v_2} \right\}$$

an expression, which at a feeble density may be considered as equal to $a_1 + a_2 - 2 a_{12}$, but which approaches

$$b_1 b_2 \left(\frac{a_1}{b_1^2} + \frac{a_2}{b_2^2} - 2 \frac{a_{12}}{b_1 b_2} \right).$$

if the density increases.

We have no right to expect, that the value of Δ_v will remain perfectly constant at various degrees of density (which would follow from the approximative equation), and at any rate the reservation is to be made, that always either two gas-phases or two liquid-phases are to be mixed. We may however expect, that the value of Δ_v will keep within certain finite limits, and that therefore, that which may seem large as compared to two small volumes (liquid volumes), may be neglected if compared with very large gas volumes.

Mr. KUENEN (Dissertation 1892, Leyden) has made observations, from which the quantity Δ_v may be determined for mixtures of CO_2 and CH_3Cl and has found it positive. It is to be regretted, that he has not tried to determine the value of Δ_v , but that he gives the increase of pressure, which is to be applied, in order to reduce the volume of the mixture to the sum of the volumes of the components. Intricate calculations are necessary for finding our result confirmed, namely that Δ_v is not 0, but that it has a value of the same order of greatness all through the course of the isotherme. If he had restricted himself to the determination of Δ_v , he would undoubtedly have come to the conclusion, that Δ_v is quasi-constant, and he might have given an approximative rule, which I now feel obliged to ascribe to Mr. AMAGAT, though AMAGAT's rule $\Delta_v = 0$ must be replaced by Δ_v is nearly constant.

I shall not carry out the intricate calculations, which would be necessary to calculate Δ_v from Mr. KUENEN's observations of the increase of pressures, but I shall restrict myself to an approximation, sufficient to conclude that the different values of Δ_v in Mr. KUENEN's experiments must have been quantities of the same order.

From

$$p = \frac{MRT}{v-b} - \frac{a}{v^2}$$

follows

$$-\frac{dp}{dv} = \frac{MRT}{(v-b)^2} - \frac{2a}{v^3}$$

and

$$-MRT \cdot \frac{dp}{dv} = p^2 - \frac{a(a-2b MRT)}{v^4}$$

by approximation.

So we can calculate the quantity Δ_v from $\Delta_v = \frac{(1+at)(\Delta_p)'}{p^2}$.

Variation of pressure in mixing.

Atmosph.	Mixture.	T. 433.0	403.0	373.0	343.0
10	$\frac{3}{4}$	10.06	10.07	10.08	10.08
	$\frac{1}{2}$	10.04	10.05	10.07	10.10
	$\frac{1}{4}$	—	10.06	10.07	10.09
30	$\frac{3}{4}$	30.67	30.85	31.20	—
	$\frac{1}{2}$	30.56	30.81	31.25	—
	$\frac{1}{4}$	—	30.78	31.06	—
50	$\frac{3}{4}$	52.4	54.4	—	—
	$\frac{1}{2}$	52.1	54.3	—	—
	$\frac{1}{4}$	—	53.1	—	—

If we take $T = 403^\circ$ and $x = \frac{3}{4}$, as an example from Mr. KUENEN's table, which I have reproduced here, we find at

$$p = 10 \dots \Delta_v = 0,0010$$

$$p = 30 \qquad \Delta_v = 0,0014$$

$$p = 50 \qquad \Delta_v = 0,0026$$

Also from Mr. KUENEN's values, about which he himself remarks, that they show but little regularity, we get the impression, that the exact determination touches the limit of the errors of observation.

From the observation at 433° we find if $x = \frac{1}{4}$

$$p = 10 \qquad \Delta_v = 0,00095$$

$$p = 30 \qquad \Delta_v = 0,00118$$

$$p = 50 \qquad \Delta_v = 0,00152$$

According to the formula (4) Δ_v must be smaller at a higher temperature, which is also confirmed by the calculated values; for the rest the increase is not so quick at 433° as at 403° . But I repeat, what I said before, that though the approximative formula gives a constant value for Δ_v , we want more accurate formulae, to indicate the real course.

Let us compare, in order to judge about the degree of approximation, with which the thesis of Mr. AMAGAT holds true, the calculated quantity Δ_v with the value of Δ_p . By Δ_p we represent the difference between the pressure of a mixture and that pressure, which we should find if the law of DALTON held good.

If we take in a volume v first $1 - x$ molecules of the first substance and if we call p_1 the pressure, after that x molecules of the second substance, with the pressure p_2 ; and finally a mixture with pressure p , then $\Delta_p = p - (p_1 + p_2)$.

In the first case the molecular volume is $\frac{v}{1-x}$; in the second case $\frac{v}{x}$, and for the mixture v . So we have the three following equations :

$$p = \frac{MRT}{v - b_x} - \frac{a_x}{v^2}$$

$$p_1 = \frac{MRT(1-x)}{v - b_1(1-x)} - \frac{a_1(1-x)^2}{v^2}$$

and

$$p_2 = \frac{MRTx}{v - b_2x} - \frac{a_2x^2}{v^2}$$

and from this by approximation

$$\Delta_p = p - (p_1 + p_2) = \frac{2 MRT b_{12} x (1-x) - 2 a_{12} x (1-x)}{v^2}$$

or

$$\Delta_p = 2 \frac{(1 + \alpha t) b_{12} - a_{12}}{v^2} x (1-x).$$

If we first restrict ourselves, when discussing the value of Δ_p , to this approximative formula, which is sufficiently accurate under a small pressure, we see 1°. that Δ_p varies greatly with the density, that it is even proportional to the square of the density; 2°. that Δ_p depends on the composition of the mixture in the same way as Δ_v , and 3°. that the sign of Δ_p depends on the sign of $(1 + \alpha t) b_{12} - a_{12}$. This expression cannot be considered as small, and does by no means disappear, when the two components are the same.

In this case $b_{12} = b_1$ and $a_{12} = a_1$ and the value of Δ_p is also of the same order as the deviation of the pressure in the investigation of the law of BOYLE, and varies also inversely as the square of the volume. Also for Δ_p there is a temperature, at which it is 0, just as is the case with $p - p'$, if p' is the pressure according to the law of BOYLE and p the observed pressure. Below this temperature Δ_p is negative, above it, on the contrary it is positive. The agreement of the course of Δ_p with that of $p - p'$, when the volume gradually decreases, is nearly perfect.

When the volume is continually decreasing, a maximum value for $p - p'$ is found in those cases, in which this difference is negative for a large volume, and in this case a volume may be

reached, at which $p - p'$ has descended again to 0, reversing its sign when the volume is still more diminished. The same holds true for Δ_p ¹⁾.

In order to show this, the approximative value of equation (5) does not suffice. A more accurate value of Δ_p is:

$$\Delta_p = 2x(1-x)\left\{b_{12}(1+a_x)(1-b_x)\frac{(v-f)(1+\alpha t)}{(v-b_x)[v-b_1(1-x)](v-b_2x)} - \frac{a_{12}}{v^2}\right\},$$

if we represent by f the quantity

$$(b_1 + b_2)x(1-x) + \frac{[b_2x - b_1(1-x)]^2}{2b_{12}}.$$

If we calculate $p - p'$, we find

$$p - p' = \frac{(1+a)(1-b)b(1+\alpha t)}{v(v-b)} - \frac{a}{v^2}.$$

If $a > (1+a)(1-b)b(1+\alpha t)$, $p - p'$ is negative, when the volume is large, but positive when

$$v < \frac{b}{1 - \frac{b}{a}(1+a)(1-b)(1-\alpha t)}$$

Δ_p has, it is true, a more intricate form than $p - p'$. But this is more in appearance than in reality.

If $a_{12} > b_{12}(1+a_x)(1-b_x)(1+\alpha t)$, Δ_p is negative, when the volume is large, but positive when v does not differ much from b_x . It has in reality no significance that the sign would be again reversed for other values of v also, e. g. between b_x and $b_1(1-x)$, because in a volume smaller than b_x the mixture could not take place.

A series of values for Δ_p , which Mr. KUENEN gives from his observations on mixtures of CO₂ and CH₃Cl and which we reproduce here, may be used to test the properties of Δ_p pointed out here.

¹⁾ These results have already been deduced by MARGULES from the observations of ANDREWS. Wien Sitz. Ber. 1889, Band XCVIII, Seite 885. See also B. GALITZINE. Wied. Ann. Band XLI.

Deviation from the law of DALTON.

Vol.	T	433.0		403.3		373.0		343.0	
		p.	Δ	p	Δ	p.	Δ	p.	Δ
0 015	$\frac{3}{4}$	74.43	-5.9	62.8	-6.8	50.0	-8.95	35.7	-11.7
	$\frac{1}{2}$	81.14	-9.1	71.3	-9.9	60.0	-11.9	48.8	-13.2
	$\frac{1}{4}$	—	—	78.5	-7.4	68.3	-8.8	57.7	-10.4
0 030	$\frac{3}{4}$	44.5	-1.6	39.7	-2.0	34.5	-2.6	28.93	-3.35
	$\frac{1}{2}$	46.2	-2.9	41.8	-3.2	37.0	-3.7	32.31	-4.10
	$\frac{1}{4}$	—	—	43.9	-2.1	39.5	-2.5	34.94	-2.96
0 045	$\frac{3}{4}$	31.58	-0.78	28.59	-0.96	25.46	-1.22	22.13	-1.55
	$\frac{1}{2}$	32.30	-1.38	29.50	-1.52	26.55	-1.78	23.62	-1.95
	$\frac{1}{4}$	—	—	30.50	-0.93	27.69	-1.16	24.83	-1.36
0 060	$\frac{3}{4}$	24.46	-0.46	22.30	-0.56	20.06	-0.70	17.72	-0.90
	$\frac{1}{2}$	24.83	-0.81	22.79	-0.89	20.66	-1.04	18.55	-1.14
	$\frac{1}{4}$	—	—	23.35	-0.55	21.30	-0.66	19.23	-0.78

$$\Delta = p - (p_1 + p_2).$$

In the approximative equation (5) the value of Δ_p does not vary, when we interchange x and $1-x$. In the more accurate equation this is no longer perfectly true, but the asymmetry is only perceptible for very small volumes.

Mr. KUENEN finds, at $T=403^{\circ}, 3$

if $v = 0,06$	and $x = \frac{1}{4}$	$\Delta_p = -0,55$ Atm.
	$x = \frac{3}{4}$	$\Delta_p = -0,56$ »
if $v = 0,045$	and $x = \frac{1}{4}$	$\Delta_p = -0,93$ »
	$v = \frac{3}{4}$	$\Delta_p = -0,96$ »
if $v = 0,03$	and $x = \frac{1}{4}$	$\Delta_p = -2,1$ »
	$x = \frac{3}{4}$	$\Delta_p = -2,0$ »
if $v = 0,015$	and $x = \frac{1}{4}$	$\Delta_p = -7,4$ »
	$x = \frac{3}{4}$	$\Delta_p = -6,8$ »

At lower temperatures the agreement is less perfect than is the case for the three first mentioned volumes in the above table; but

in general more perfect than in the case of the last volume. The rule that Δ_p is proportional to $x(1-x)$ involves that for $x = \frac{1}{2}$ a value must be found that is $\frac{4}{3}$ times larger than that for $x = \frac{1}{4}$ or $\frac{3}{4}$. From Mr. KUENEN's value a greater proportion is generally found for it. At $T = 403^\circ$, the proportion found in the case of the four before-mentioned volumes is $\frac{0,89}{0,555}$, $\frac{1,52}{0,954}$, $\frac{3,2}{2,05}$ and $\frac{9,9}{7,1}$; so values varying between 1,6 and 1,4.

At $T = 373^\circ$, the values found were $\frac{11,9}{8,875}$, $\frac{3,7}{2,55}$, $\frac{1,78}{1,19}$ and $\frac{1,04}{0,68}$; so they are varying between 1,5 and 1,35.

But at $T = 343^\circ$, we may consider the proportions found as equal to $\frac{4}{3}$.

According to the remarks, deduced from equation (6), the dependence of Δ_p on the volume must be expected less great than proportional to the squares of the density. This may be considered as being confirmed by the values, given by Mr. KUENEN. So Mr. KUENEN finds, if $v = 0,6$ $\Delta_p = 0,81$ and if $v = 0,03$ $\Delta_p = 2,9$ and if $v = 0,045$ $\Delta_p = 1,38$ etc.

The dependence on the temperature, for which according to equation (5)

$$(-\Delta_p)_{T_2} - (-\Delta_p)_{T_1} = 2 \frac{\alpha b_{12} (T_1 - T_2)}{v^2} x(1-x)$$

would hold, and according to which formula for the same value of x and v the differences of the observed values of Δ_p would be proportional to the differences of temperature, is not confirmed by KUENEN's figures. But in all this we must not overlook the fact that the Δ_p 's are already differences of observed quantities and not the observed quantities themselves, while $\{(-\Delta_p)_{T_2} - (-\Delta_p)_{T_1}\}$ are again differences from these differences. It is therefore to be regretted, that there are so few observations, which may be used for this investigation, and specially that Mr. AMAGAT has not been able to continue his investigation of the mixture CO_2 and N_2 ¹).

If we try to calculate a value for a_{12} from the observed value of Δ_p , e.g. from the observation with $v = 0,06$, $x = \frac{3}{4}$, $T = 373$, which gives $\Delta_p = -0,7$, this can of course, only be done by

¹) C. R. Acad. des Sciences. 11 Juillet 1898.

assuming a value for b_{12} . If we assume 0,0024, a value which lies between $b_1 = 0,0020$ and $b_2 = 0,0029$, while an error in this value will have comparatively little influence on the value of a_{12} , we find

$$a_{12} = 0,7 \times 0,0036 \times \frac{8}{3} + 0,00328 = 0,010 .$$

From Mr. KUENEN's values for the variation of pressure ($-\Delta_p$), we find the following value for a_{12} , which has been calculated by means of the approximative equation.

$T = 343$	$v = 0,06$	$v = 0,045$	$v = 0,03$
$x = \frac{1}{4}$	0,0105	0,0117	0,0101
$x = \frac{1}{2}$	0,0112	0,0109	0,0104
$x = \frac{3}{4}$	0,0116	0,0114	0,0110
$T = 373$			
$x = \frac{1}{4}$	0,0096	0,01086	0,0100
$x = \frac{1}{2}$	0,0096	0,0104	0,0097
$x = \frac{3}{4}$	0,0093	0,0099	0,0095
$T = 403,3$			
$x = \frac{1}{4}$	0,0088	0,0086	0,0086
$x = \frac{1}{2}$	0,0099	0,0097	0,0093
$x = \frac{3}{4}$	0,0089	0,0087	0,0084

For the calculation of a_{12} from the results at $v = 0,015$, given by Mr. KUENEN, the approximative equation is no longer sufficiently accurate.

For these values of a_{12} we see the same not yet explained phenomenon, generally observed for the values of a_1 and a_2 , namely that they increase at lower temperatures. The advantage of the equation which has served to calculate them, is that it is independent of possible changes, which might have occurred in the values of a_1 and a_2 through change of temperature. So the accurate determination of $-\Delta_p$ is as yet the best means of supplying at least one relation between a_{12} and b_{12} . The variability of a_{12} with the temperature, would therefore be no reason to doubt of the values found for a_{12} . There is, however, another circumstance, which makes

me doubt; and that is, that according to the results obtained by me formerly (Verslag Kon. Akad. 27 Nov. 1897) $a_1 + a_2 - 2a_{12}$ would be negative for CO_2 and CH_3Cl or $a_{12} > \frac{a_1 + a_2}{2}$, and this would require a greater value for a_{12} than would be found from Mr. KUENEN's values for $-\Delta_p$. In order that $a_{12} > \frac{a_1 + a_2}{2}$ should hold good, e.g. at $T = 343^\circ$, a_{12} would have to be greater than 0.00126, while from the calculation of $-\Delta_p$, a_{12} is found to be at the utmost 0.0116. That from Mr. KUENEN's observations $a_{12} < \frac{a_1 + a_2}{2}$ follows, is confirmed by the observation, that mixing of CO_2 and CH_3Cl gives increase of volume.

If this is really the case, it would prove that I ought to have expressed myself with still greater reserve than I did in my: "Approximative rule for the course of the plait-curve of a mixture"¹⁾. Though I have drawn the attention to the fact, that the real plait-curve will deviate from the curves drawn, yet I had thought, that the deviations would not be so great, as to make the different types no longer to be distinguished. Yet so great a deviation really occurred in this case.

If we look back on the two rules discussed here: $\Delta_v = 0$ and $\Delta_p = 0$, we are induced to qualify the first rule as an approximative law. Throughout the course of the isotherme, from an infinite volume down to the smallest possible volume of the substance, there may be deviation, but the deviation remains within finite limits. The second rule holds perfectly good for infinite rarefaction, but it would be utterly impossible to apply it also to liquid volumes. Such a law may be qualified as a *loi-limite*. Considered from this point of view, the law of BOYLE too is not an approximative law, but only a *loi-limite*.

Chemistry. — Prof. VAN BEMMEL EN reads a paper of Dr. F. A. H. SCHREINEMAKERS on: „*Equilibriums in systems of three components. Change of the mixing-temperature of binary mixtures by the addition of a third component.*”

Among the different systems composed of the components *A*, *B* and *C* we suppose the case, that in two of the binary systems f.i. *A—B* and *A—C* two liquid phases can appear, but not in the system *B—C*. An example of this we find f.i. in the system formed of:

¹⁾ Verslag Kon. Akad. Nov. 1897.

water, alcohol and succinonitrile, two liquid layers only appearing in the binary systems water-succinonitrile, and alcohol-succinonitrile but not in the system water-alcohol. A second example, which Dr. SCHREINEMAKERS has now subjected to an investigation is the system: water, phenol and aniline; in the system phenol-aniline there do not appear two liquid layers, in the systems water-phenol and water-aniline there do. Yet the isotherms acquired in both systems are entirely different as shown in figures 1 and 2. In fig. 1 the letters *W*, *A*, *N* indicate the components water, alcohol, succinonitrile; in fig. 2 the components are water, aniline and phenol, indicated by the letters *W*, *An* and *Ph*.

In both systems we shall only consider part of the appearing equilibria viz the appearing liquid phases that can be in equilibrium with each other; the equilibria in which solid phases appear are not considered here.

Let us begin with the system: water, alcohol, succinonitrile; at temperatures above $56^{\circ}5$ only homogeneous liquids are possible and no separation into two layers can appear. The ζ -surface in fig. 1 above the triangle *WAN* is consequently in every point convex-convex downwards. Here and in future we only consider the

sheet of the ζ -surface that belongs to the liquid phase¹⁾. When the temperature falls there appears at $56^{\circ}5$ a plaitpoint and at lower temperatures a plait is developed ending on the plane *WN* ζ and continually extending in case of a fall in the temperature. In fig. 1 *d* is the projection of the plaitpoint at $56^{\circ}5$; the different lines drawn, each of them ending in two points of the line *WN* are the connodal lines at the different temperatures; they are all situated in such a way, that those belonging to lower temperatures lie outside those of higher temperatures. In case of a fall in the temperature the plait from the side *WN* extends more and more. At $\pm 32^{\circ}$ there appears however a second plait on the ζ -surface, originating in the plane *AN* ζ , in a point, the projection of which is indicated by *e*; consequently we now get at a lower temperature

¹⁾ Vid. J. D. VAN DER WAALS, Proceedings of the Royal Academy of Amsterdam 1897, 209.

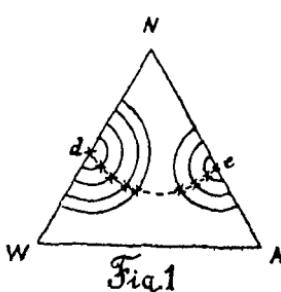


Fig. 1

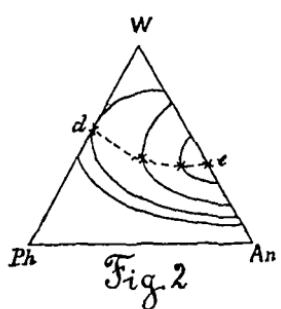


Fig. 2

besides the connodal lines ending on the side WN , also connodal lines ending on the side NA . Both plaits extend more and more when the temperature falls and the two connodal lines approach each other continually; the crosses on the dotted line de indicate the plait-points of the different connodal lines. We are therefore led to suppose that in the end the two plaits merge into each other; yet it was impossible to demonstrate this experimentally, because, in consequence of the appearance of the solid phase succinonitrile, the left connodal line below $\pm 5^{\circ}5$ and the right one below $\pm 4^{\circ}5$ indicate only less stable equilibria.

Quite different are the phenomena in the system water, anilin and phenol in fig. 2. Here there appears namely but *one* plait on the ζ -surface. Above 167° a separation of a liquid in two layers can never occur in this system; the ζ -surface is in every point convex-convex downwards. At 167° there appears a plait-point on the side $WAn\zeta$, its projection being e . As the temperature falls the plait extends and we get connodal lines as in fig. 2, in which the crosses again indicate the plaitpoints. The plait continues to extend and reaches at 68° the plane $WPh\zeta$ in a point, the projection of which is indicated by d ; the connodal line has a form like the one which in fig. 2 touches the side PhW in point d . At still lower temperatures the plaitpoint on the ζ -surface disappears; the plait extends from one side to the other, so that two entirely separate convex-convex parts are formed. The connodal line now has fallen into two parts, separated from each other, both ending on the sides PhW and AnW . At further lowering of the temperature the connodal line keeps its form; the two parts move however continually farther from each other and at last some of the points represent less stable states, as solid phases, viz phenol and a combination of phenol and aniline are formed.

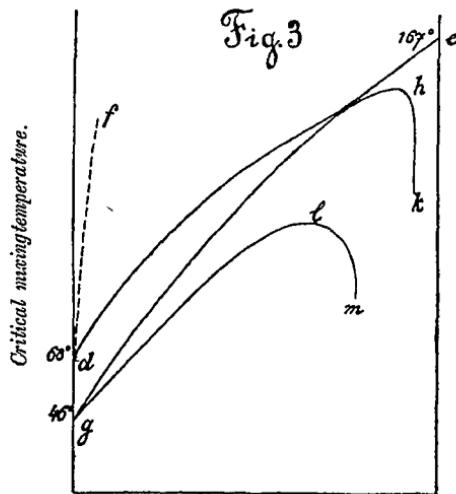
The preceding investigations have led to a theoretical and experimental investigation of different curves; in these pages a survey is given of some of the experimental investigations.

Let us take the connodal lines of fig. 2; below 68° no plaitpoint appears on them; at 68° it appears in point d and the temperature rising it moves along the line de from d to e where it disappears at 167° . It is however very difficult to realize these plaitpointcurves experimentally; neither has this been done in the different systems investigated by Mr. SCHREINEMAKERS. He has however determined another curve, which will not much deviate from it in the two systems mentioned above. It is the line of the critical mixingtemperatures which coincides with the former in at least 2 points,

viz. in the points *d* and *e*: the critical mixing-temperatures of the binary systems.

If we start with a mixture of phenol and aniline, adding water to this binary mixture, two liquid phases are formed, passing into one when heated; this mixing-temperature depends upon the quantity of water added. The highest mixing-temperature we are able to reach by the addition of water, may be called the critical mixing-temperature of water with the said binary mixture. In varying the proportion of phenol and aniline in the binary mixture, another critical mixing-temperature will be obtained by the addition of water.

Fig. 3



Molec. aniln on 100 Molec phenol + aniln.

In fig. 3 is shown on the X-axis the composition of the binary mixture; on the Y-axis the critical mixing-temperature of this binary mixture with water. We then get the line *de* in which *d* indicates the critical mixing-temperature of phenol with water, and *e* that of aniline with water. The line *de* may be drawn according to the table given below.

Composition of the binary mixture

in Molec. aniline on

100 Molec.

Phenol + Aniline

Critical mixingtemperature
after the addition of
water.

0	68°
11.58	95°
25.4	114°—115°
37.3	127°
50	139°—140°
62.8	148°
76.07	155°5
87.66	163°
100	167°

For each of the binary mixtures of phenol and aniline, given in the above table the mixing-temperature is determined by the addition of different quantities of water and the critical temperature deducted from it.

We see therefore that the critical mixing-temperature of the binary mixtures increases in proportion as they contain more aniline —

decreases on the contrary in proportion as they contain more phenol. The extreme limits are 68° and 167° .

Dr. SCHREINEMAKERS has also investigated how the critical mixing-temperature of water with phenol is changed by the addition of sodiumchloride. In the following table the results are given.

Composition of the binary mixture
in Molec. Na Cl on 100 Molec.

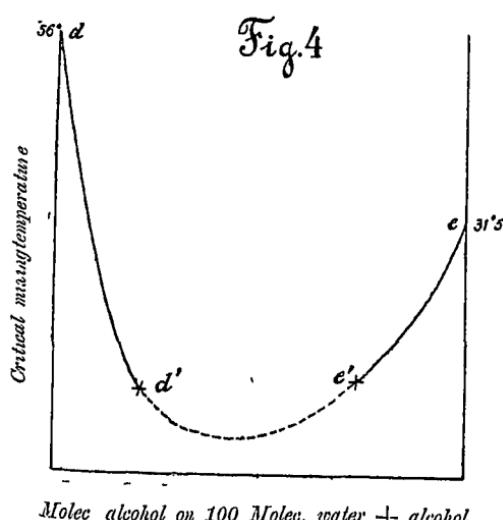
$(H_2O + NaCl)$

		Critical mixingtemperatures after the addition of phenol.
0		68°
0.304		78°
1.579		123°
2.084		144°

If we now indicate in fig. 3 on the X-axis the binary mixture viz. Molec. Na Cl on 100 Molec. ($NaCl + H_2O$), and on the Y-axis the critical mixingtemperature of those mixtures with phenol, we get the dotted line df , rising very quickly. A similar line also exists in the system investigated by SCHREINEMAKERS of water-succinonitrile and sodiumchloride.

By the addition of aniline or Na Cl we therefore get a rise in the critical mixingtemperature of phenol with water; the same occurs also if the pressure is augmented, as lately shown by Dr. N. J. VAN DER LEE¹⁾. A fall in the critical mixingtemperature can also occur, as Dr. SCHREINEMAKERS found with alcohol.

A line quite different from that in fig. 3 is found by SCHREINEMAKERS in the system: water, alcohol and succinonitrile.



If we start from the binary mixture water and alcohol, we can again get two liquid layers by the addition of succinonitrile. In fig. 4 the composition of the binary mixture: water and alcohol is read on the x-axis, while on the y-axis we find the critical mixingtemperature by the addition of succinonitrile. The course of the line $dd'e'e$ is known from the following table:

¹⁾ Dissertation. The influence of pressure on the critical mixingtemperature. Amsterdam 1898.

Composition of the binary mixture in Molec. Alcohol on 100 Molec. Alcohol + water.	Critical mixingtemperature after the addition of succinonitrile.
0 Point <i>d</i>	56°
5.7	41°5
12.1	30°5
25.9	12°
30.1 Point <i>d'</i>	7°5
66.6 Point <i>e'</i>	8°5
70.4	11°
84.6	21°
100 Point <i>e</i>	31°5

Of the dotted line *d'e'* only a small part could be realized; the greater part it is however impossible to determine. Yet we are led to suppose that the line *d'e'* will run as drawn in the figure viz.: with a minimum-temperature at $\pm 3^\circ$ coinciding with the point in which in fig. 1 the two connodal lines merge into each other. The difficulty of determining the course of the curved line lies in the fact of the two liquid-phases presenting there less stable equilibriums; the few cases in which Dr. SCHREINEMAKERS succeeded in obtaining them, were insufficient to deduct the critical mixingtemperatures from.

Besides the question of the critical mixingtemperatures of ternary systems, we can set ourselves many other problems one of which I shall discuss.

Take a binary mixture viz. one of water and phenol; the mixing temperature of such a mixture depends upon its composition; f.i. the mixingtemperature of a mixture containing 10.9 pCt. of phenol is $\pm 46^\circ$. This mixingtemperature is altered not only by pressure, but also by the addition of a third component; it may be raised or depressed. An example of a rise Dr. SCHREINEMAKERS has determined experimentally by taking as a third component aniline; he found that the mixingtemperature began by rising from 46° to about 163° , to fall again after a further addition of aniline. In order to draw this line in fig. 3, only the components phenol and aniline of the ternary mixture are considered. Owing to the proportion of water and phenol being constant in all the mixtures, the entire composition is known. We have:

Molec. aniline on 100 Molec. phenol + aniline.	Mixingtemperature.
0	46°
46.57	127°
67.68	148°
79.99	157°
86.18	160°
89.9	162°
93.6	161°5
95.66	± 157°
97.47	± 142°

If in fig. 3 the number of Molec. aniline on 100 Molec. Phenol + aniline is indicated again on the X-axis, and on the Y-axis the mixingtemperature, we get a line *g h k* as is drawn in fig. 3 which: 1° starts from *g* to a higher temperature, 2° touches the line *d e* 3° reaches its maximum temperature in *h* and 4° further on bends down to lower temperatures.

The preceding line *g h k* is for a mixture containing 10.9 pCt. Phenol and 89.1 pCt. water; for mixtures of other compositions we also get other lines. F. i. if we take a mixture of 63.7 pCt. phenol and 36.3 pCt. water, it has just like the former its mixingtemperature at ± 46°. When adding aniline we then get the following:

Molec. aniline on 100 Molec. phenol + aniline.	Mixingtemperature.
0	46°
21.64	90°
29.58	97°
43.72	107°
51.93	109°
60.48	108°5
71.97	102°
78.35	92°

These values are indicated in fig. 3 by *g l m*, which like *g h k* begins at *g*, reaches its maximum temperature at *l*, and then bends down to lower temperatures.

So we have now got two lines, both starting from point *g* in fig. 3. From each point below *d* two more such lines may be drawn; but from *d* itself but one. To find this we must set out from the critical solution of water and phenol. But this is not exactly known. Dr. SCHREINEMAKERS took a solution, containing 35.77 pCt. phenol which certainly lies near the critical one. He found:

Molec aniline on 100 Mol

phenol + aniline.	Mixingtemperature
0	± 68°
20.46	105°5
23.83	120°
50.68	135°
68.69	144°
76.88	145°
80.18	144°
87.20	± 137°

The line that may be deducted from this table is however not given in fig. 3; as far as it is continued, it is entirely above *glm* but intersects *ghk* in a point situated between *g* and the intersection of the two curves *ghk* and *de*.

Physics. — „On the accurate determination of the molecular weight of gases from their density.” By Prof. J. D. VAN DER WAALS.

From the equation:

$$\left(p + ap_0 \frac{v_0^2}{v^2} \right) (v - b v_0) = \Sigma^{1/3} m N s^2$$

follows

$$\Sigma^{1/3} m N s_0^2 = p_0 v_0 (1 + a) (1 - b)$$

and

$$\Sigma^{1/3} m N s^2 = p_0 v_0 (1 + a) (1 - b) (1 + \alpha t).$$

If the quantity *N* is equal for two different gases, it follows from the thesis that at the same temperature $\Sigma^{1/3} m s^2$ is the same, that for the two gases $p_0 v_0 (1 + a) (1 - b)$ has also the same value, and therefore also $v_0 (1 + a) (1 - b)$.

If the whole quantity of the substance is equal to the molecular weight, $v_0 = \frac{m}{d_0}$, so that the equality of $v_0(1+a)(1-b)$ may be written

$$\frac{m}{d_0}(1+a)(1-b) = \frac{m'}{d'_0}(1+a')(1-b')$$

or

$$m : m' = \frac{d_0}{(1+a)(1-b)} : \frac{d'_0}{(1+a')(1-b')}.$$

The quantity $\frac{d_0}{(1+a)(1-b)}$ is the normal density, which would also have been found at 0° , if the law of BOYLE had been true. Let us represent it by d_n .

If we determine the density of a gas at 0° and under the pressure p_0 (at which the quantities a and b have been determined), we have only to divide this density by $(1+a)(1-b)$, to find the normal density, and to this the molecular weights are proportional.

If the density is determined at another temperature and under another pressure, a volume v_0' is calculated from the pressure, the temperature and the volume read, making use of the formula:

$$\frac{p v}{1 + \alpha t} = p_0 v_0'$$

By means of this volume a density $(d_0)' = \frac{m}{v_0'}$ is found, and we have to investigate in what proportion it stands to d_n .

Let us calculate for this purpose the proportion of v_0 and v_0' . From

$$p = \frac{p_0 v_0 (1+a)(1-b)(1+\alpha t)}{v - b v_0} - \frac{a p_0 v_0^2}{v^2}$$

follows

$$v_0' = \frac{p v}{1 + \alpha t} \frac{1}{p_0} = (1+a)(1-b)v_0 \frac{v}{v - b v_0} - \frac{a}{1 + \alpha t} v_0 \frac{v_0}{v}$$

or

$$\frac{v_0'}{v_0} = (1+a)(1-b) \left\{ 1 + b \frac{v_0}{v} \dots \right\} - \frac{a}{1 + \alpha t} \frac{v_0}{v}.$$

If we restrict ourselves to very large volumes only, so to observations under a small pressure, we may put

$$\frac{v_0'}{(1+a)(1-b)v_0} = 1 - \frac{v_0}{v} \left\{ \frac{a}{(1+a)(1-b)(1+\alpha t)} - b \right\}$$

or

$$\frac{d_n}{d'_0} = 1 - \frac{p v_0}{p_0 v_0 (1+a)(1-b)(1+\alpha t)} \left\{ \frac{a}{(1+a)(1-b)(1+\alpha t)} - b \right\}$$

$$\frac{d_n}{d'_0} = 1 - \frac{p}{p_0 (1+a)(1-b)(1+\alpha t)} \left\{ \frac{a}{(1+a)(1-b)(1+\alpha t)} - b \right\}.$$

If we take for p a certain fraction of the critical pressure e.g. $\frac{1}{76}$ (as LEDUC does „Annales de chimie et de physique, Sept. 1898”), then

$$p = \frac{\pi}{76} = \frac{1}{76} \frac{a}{27} \frac{p_0}{b^2} \quad \text{or} \quad \frac{p}{p_0} = \frac{1}{76} \frac{a}{27 b^2}$$

and

$$\frac{d_n}{d'_0} = 1 - \frac{1}{76} \frac{1}{27} \frac{a}{b(1+a)(1-b)(1+\alpha t)} \left\{ \frac{a}{b(1+a)(1-b)(1+\alpha t)} - 1 \right\}.$$

By means of the critical temperature

$$1 + \alpha t_k = \frac{8}{27} \frac{a}{b(1+a)(1-b)}$$

we find

$$\frac{d_n}{d'_0} = 1 - \frac{1}{76 \times 8} \frac{T_k}{T} \left(\frac{27}{8} \frac{T_k}{T} - 1 \right)$$

or

$$\frac{d_n}{d'_0} = 1 - 0,001645 \frac{T_k}{T} \left(\frac{27}{8} \frac{T_k}{T} - 1 \right).$$

For $\frac{T_k}{T} = \frac{8}{27}$ the correction is 0 and the density found is the normal density. At this temperature a gas follows the law of BOYLE, of course only under a slight pressure.

For $\frac{T_k}{T} < \frac{8}{27}$ or $T > \frac{27}{8} T_k$ the correction is negative, as is the case with hydrogen at 0° . In this case $d_n > (d'_0)'$. Then the value of $\frac{d_n}{(d'_0)'} = \frac{d_n}{d'_0}$ has a maximum for $T = \frac{27}{4} T_k$. For hydrogen at 0° the

value of T has already surpassed this limit. The fact that such a maximum value exists may be understood, if we consider that a gas under a constant pressure is ever more rarefied, when the temperature rises — so that at $T = \infty$, a pressure $= \frac{\pi}{76}$ would require an infinite volume, and correction would be unnecessary.

The condition for the observation under a pressure of $\frac{\pi}{76}$, without the vapour being saturated is, that T must not descend below a certain limit, which we shall put at $\frac{T_k}{1,6}$.

For $\frac{T_k}{T} = 1,6$ we find $\frac{d_n}{(d_0)'} = 1 - 0,0116$, so that the normal density is more than 1 pCt. smaller than that which is furnished by the observation.

If the assumption of $a_t = a \frac{T_k}{T}$ agrees better with the observations, than the supposition that a is constant, we should have to put:

$$\frac{d_n}{(d_0)'} = 1 - 0,001645 \left(\frac{T_k}{T} \right) \quad \left[\frac{27}{8} \left(\frac{T_k}{T} \right)^2 - 1 \right],$$

in which case for $\frac{T_k}{T} = 1,6$ the normal density would be more than 2 pCt. smaller than would follow from the observations.

Astronomy. — „Some remarks upon the 14-monthly motion of the Pole of the Earth and upon the length of its period”. By Dr. E. F. VAN DE SANDE BAKHUYZEN (Communicated by Prof. H. G. VAN DE SANDE BAKHUYZEN).

(Read in the Meeting of October 29th 1898).

1. In the recent N°. 446 of the *Astronomical Journal* another essay is given by Dr. CHANDLER on the motion of the Pole of the Earth, in which he discusses the observations performed in the years 1890—1898 and employs the older series to investigate anew the length of the 14-monthly period. On this last point he contends the opinions formerly emitted by H. G. VAN DE SANDE BAKHUYZEN and recently by me (Proceedings of the Royal Academy, Amsterdam. June 1898). To this latter paper he devotes a note running as follows:

„The memoir last referred to did not arrive until the present

„article was written, but I interpolate this statement with regard to it in order to enable astronomers to decide as to the justness of the views therein set forth. Both of the gentlemen of the Leiden observatory strenuously maintain that the mean period is more than „431 days, and that it is invariable. The formula V" (that is the result given by E. F. v. d. S. B.) „is deduced by a peculiar and „arbitrary treatment of the results of observation, its initial epoch „being based on the Leyden observations alone, on the alleged „ground that its errors are far smaller than those of all other series, „which are rejected. I must however deny the propriety of assigning „a weight of zero, relative to Leyden, to the extensive and precise „series at Pulkowa between 1863 and 1882 with the Vertical Circle „and Prime Vertical Transit."

I shall now take the liberty to add on my part some remarks to these opinions of Dr. CHANDLER. At the same time I shall make use of the opportunity to consider the problem of the length of the 14-monthly period somewhat more closely, which consideration will naturally lead to the discussion of the results on this point arrived at by CHANDLER in his last paper.

2. In the first place in regard to the grievances raised by CHANDLER against my manner of treatment I will grant at once that, by not using the results obtained at Pulkowa in the years 1863 to 1882, I would have committed a gross error, if it had been my purpose to include in my investigations, in an independent way, the observations before 1890. This however was in nowise the case. It was simply my intention to submit to a discussion only those obtained in the period 1890 to 1897; but as from these alone the length of the 14-monthly period could naturally be derived with but slight accuracy I had recourse to the results formerly deduced and compiled by H. G. v. d. SANDE BAKHUYZEN. It seemed undesirable however to use all these results. In the first place, for reasons to be stated hereafter, I thought it necessary to exclude those of an epoch before 1860. Further consideration then led me to restrict myself, in the deduction of a *provisional* result, as far as concerns the observations between 1860—1880, wholly to the Leyden results. I determined on this course because these proved to have much smaller mean errors than all others of the same time, *in as far as they had been treated by H. G. v. d. S. BAKHUYZEN*, whilst moreover these Leyden results proved to lie about midway between the others; so that by including these the final result could not be modified to any considerable amount.

I might perhaps have pointed out still somewhat more clearly the

entirely preliminary character of my result for the length of the period, if I had not thought my meaning sufficiently evident. At all events I assuredly think that, in formulating my result, I have not lost sight of the prudence necessary under these circumstances. Thus I give, beside my result of 431.11 days, also the one which would follow if the mean epoch found by H. G. v. d. S. BAKHUYZEN were combined with mine, viz. 430.36, whilst finally I observe that for the last 35 years the length of the period *cannot have differed considerably from 431 days* and that *such a great variability as CHANDLER assumes*, is now already contradicted by the observations. So I believe I may state that the words of Dr. CHANDLER: „strenuously maintain that the mean period is more than 431 days, „and that it is invariable” show but very inaccurately the standpoint taken up by me¹⁾.

Let this suffice to answer CHANDLER's observations about the treatment followed by me; his remarks concerning the facts themselves will be presently considered.

3. Before discussing the results furnished by my later computations on the length of the period, I will concisely state the results arrived at by CHANDLER in 1894 (Astr. Journ. N°. 322) and those lately deduced by him. His formula of 1894 gave as *Epochs of minimum* in the 14-monthly motion :

$$T = 2402327^d + 428^d.6 E + 55^d \sin \Psi$$

in which, with a sufficient approximation

$$\Psi = (t - 1865.25) \cdot 5^{\circ}.48 = E \times 6^{\circ}.43$$

From this there results for the length of the period, osculating for the epoch E :

$$P = 428^d.6 + 6^d.2 \cos (E \times 6^{\circ}.43). \quad (2)$$

So the length of the period may vary from $434^d.8$ to $422^d.4$ and the cycle of this change embraces 56 periods or 66 years. The maximumlength would have been reached in 1865, the minimumlength would take place in the present year 1898.

In his last paper CHANDLER starts with this formula and tests it by the observations of 1890 to 1897. He does not use the x and y of ALBRECHT, but values derived by himself, which however agree with

¹⁾ Neither are his words accurate, where they concern H. G. v. d. S. BAKHUYZEN. See a. o. Astr. Nachr. N°. 3275, page 163 at the top.

the former in their general course. The length of the period with which he starts thus amounts to about 423 days and from the observations a correction is found for it of + 5 days, which however, as CHANDLER observes, must be quite uncertain, it not being sure that the length of the yearly period is *exactly* a year. Meanwhile, later on, a correction of + 4 days for the length of the period is assumed beside such a one of + 8 days for the mean epoch and, as CHANDLER thinks it proved that the length of the period is variable, he accounts for the correction by a quadratic term added to the formula of the epochs which thus becomes:

$$T = 2412646^d + 427^d.0 E - 0^d.08 E^2 \dots \dots \quad (3)$$

where the initial epoch is placed 24 periods later than that of the preceding formula.

Tested by the older observations this formula proved to satisfy fairly those since 1835, but not at all those of POND, which leave for the epoch a deviation of 166 days. Although formerly CHANDLER set great store by POND's observations, it yet seems that he desires to have the elements of formula (3) regarded as „the revised elements” he wished to determine. It is true that a doubt about this conclusion arises by reading in the „conclusions” which, in another part of the paper (p. 107), are derived from „substantially all the competent „testimony available” (b) „that the mean period since 1825 is 428 days „within a small fraction of a day”, whilst formula (3) gives us for this quantity $431^d.6$, and (d) that the hypothesis of a change in the period uniform with the time is incompatible with the observations before 1860, whilst in conclusion (e) a change per saltum between 1830 and 1860 is called also incompatible with the facts. Leaving this for what it is, I shall in what follows, indicate formula (3) as CHANDLER 1898.

The differences between the epochs computed according to this formula and to that of 1894 are rather small between 1870 and 1894, but increase rapidly beyond these limits. So we find for CH 98—CH 94 in 1830 — 126 d., in 1860 + 38 d., in 1898 + 25 d. and in 1900 + 32 d.

4. In the first place I investigated more closely what the observations from 1890—97, taken by themselves, can teach us about the length of the 14-monthly period. In my former paper I examined the x of the three last years only; now I did the same for the three first years and then I acted in the same way for the y .

I thus obtained the following results for the mean epochs of

maximum, to which I add those for the whole of the period 1890—96.

	Observ.	Obs.—E. B.	Obs.—Ch.94	Obs.—Ch 98
<i>x</i>				
1890—1896	2412439	— 6	+ 13	+ 6
1890—1892	2412006	— 7	+ 3	0
1894—1896	2413300	— 7	+ 28	+ 13
<i>y</i>				
1890—1896	2412438	— 7	+ 12	+ 5
1890—1892	2412007	— 6	+ 4	+ 1
1894—1896	2413298	— 9	+ 26	+ 11

If we derive the length of the period from those couples of partial results, lying three periods apart, we shall find:

$$\text{from } x \quad 431^d.3$$

$$, \quad y \quad 430 .3$$

The surprising agreement with the results obtained from great intervals of time had of course to be regarded as partly accidental. Now in order to investigate more closely what accuracy might be arrived at, I fell back on the original values for the coordinates *x* and *y* as they have been derived by ALBRECHT. In my preceding paper I gave on page (53) 12 a comparison of these values with those computed by my formula. In entirely the same way I now made comparisons with formulae in which 423 and 428 days were successively assumed for the length of the 14-monthly period, but which agreed for the rest, mean epoch included, (which mean epoch coincides approximately with 1893.0) with those employed for the former comparison. These lengths of the period were taken from CHANDLER's two formulae.

I shall not here communicate these comparisons themselves, but shall give only the sums of the squares of the deviations and the mean values of the latter, repeating also the values formerly found with the length of the period 431^d:

Period	$\Sigma \Delta x^2$	M. dev.	$\Sigma \Delta y^2$	M. dev.
431	1207	$\pm 0''.040$	1582	$\pm 0''.046$
428	1322	$\pm 0 .042$	1651	$\pm 0 .047$
423	1699	$\pm 0 .047$	2083	$\pm 0 .052$
"	1708	$\pm 0 .047$	1976	$\pm 0 .051$

We see that a 428 days' period satisfies the original observations almost as well as a 431 days' period.

On the other hand a 423 days' period leaves considerably greater errors, which are but slightly diminished by deducing anew also the yearly motion, as is shown by the numbers given in the last place.

So our result is, that the observations from 1890—97 prove in themselves a 423 days' period to be improbable; but much farther than that we cannot go.

5. In the second place I had recourse again to the older series of observations, but, before discussing this investigation, I will state that, now again, it lays no claim to completeness. I have only again combined the results of observations treated already by others, with each other and with my results for 1890—1897 and from these I have drawn such conclusions as seemed most probable to me. The observations of Pulkowa 1863—1875 only make an exception, as for these I made a computation myself founded on the results arrived at by IVANOFF in two important papers.¹⁾

First came the question how far we may go back in the employment of older observations and this again depends upon that other question, whether we assume in the 14 monthly motion a lasting continuity, or whether we do not exclude the possibility that more or less sudden changes may take place.

As is already remarked, CHANDLER includes amongst the conclusions formulated in his last paper, also this one, „that a change per saltum „between 1830 and 1860 is incompatible with the facts”. To me on the contrary it seems that there is every reason to assume the possibility of such a change between 1840 and 1860.

This statement is based in the first place on the values for the amplitudes as they have been found before 1860 and after that

¹⁾ A. IVANOFF, Variations de la latitude de Poukovo déduites des observations 1863—75. (*Mélanges math. et astr. T. VII.*) St. Pétersbourg 1894.

A. IVANOFF, Recherches définitives sur les variations de la latitude de Poukovo (*Bull. Acad. Pétersb. Serie V. T. II.*) St. Pétersbourg 1895.

time, and which follow below. Here and there I have inserted the results of two different treatments of the same series of observations.

Series of Observations.	Amplitude.	Authority.
Greenwich Mural C .. 1825—1836	0".126	Ch. A. J 315
Greenwich Mural C .. 1836—1850	0 .060	Ch. A. J. 320
Pulkowa Prime Vert. . 1840—1855	0 .035	" " " 296
Pulkowa Vert. C. 1840—1849	0 .056	H. G. B. A. N. 3275
" " " " " "	0 .08	Ivanof Rech déf.
Greenwich Tr. C 1851—1858	0 .069	H. G. B. A. N. 3261
Greenwich Tr. C. 1858—1865	0 .175	H. G. B. A. N. 3261
Washington Prime Vert 1862—1867	0 .126	" "
Leiden Fund. Stars... 1864—1868	0 .156	Result Wilterlink.
" Polaris..... 1864—1874	0 .158 ¹⁾	" "
Greenwich Tr. C. 1865—1872	0 .233	H. G. B. A. N 3261
Pulkowa Vert. C. Pol. 1863—1870	0 .226	" "
" " " " 1871—1875	0 .179	" "
" " All the St. 1863—1875	0 .127	Ivanof Rech déf, E F B
" Prime Vert . 1875—1882	0 .236 ²⁾	Ch. A. J 297
" Vert. C 1882—1891	0 .145 ³⁾	Nyrén Bull. Pétersb. T. 35
" " " " " "	0 .139	H. G. B. A. N. 3261
Greenwich Tr. C 1880—1891	0 .141	Ch. M. N. 53 119
Madison 1883—1890	0 .152	Ch. A. J. 307
Lyon 1885—1893	0 .175	" " 334
Summary Albrecht ... 1890—1896	0 .148	E. F. B. Ac Amst. 1898.
" 1890—1892	0 .167	Result E. F. B.
" 1894—1896	0 .131 ⁴⁾	" "

¹⁾ These results deviate slightly from those communicated by H. G. v. d. S. B. in A. N. 3261. The investigation of WILTERLINK will be shortly published by him in detail.

²⁾ The result 0".33 given by NYRÉN for 1875—1878 in Bull. Pétersb. Vol. 35 is certainly too great, as it is influenced by the yearly motion.

³⁾ Result for the entire motion, upon which however the yearly motion seems to have had but slight influence.

⁴⁾ Results now deduced by me for the partial groups.

This summary shows pretty clearly that the amplitude was found to be considerably smaller in the years 1836—1858 than in the following period. For a number of series the mean errors have been deduced (see A. N. 3261) and the consideration of these strengthens the conclusion which admits as probable the reality of the observed difference. On the other hand no variability of the amplitude is to be found after 1860 and we may conclude at least that the observations make a more or less sudden change between 1850 and 1860 much more probable than a periodic or continually increasing one. Now the dynamical theory of the rotation of a sphere not absolutely solid, either as a whole or in some of its parts, leads to the same result. It teaches us,¹⁾ that with slow secular displacements of mass the axis of the greatest moment of inertia is entirely followed in its motions by the axis of rotation; that with periodic displacements the axis of rotation will get a motion of the same period as that of the axis of inertia, which is added to its own motion, but that in the case of sudden displacements of mass the axis of inertia is the only one to shift its position, so that the opening of the cone, described by the axis of rotation around it, changes, introducing thereby a discontinuity in the motion of the latter. The amplitude changes and in general also the phase, but after that the motion continues in its old period.

May we however be led in this problem by a dynamical theory? CHANDLER denies this strenuously. He thinks it has proved itself a blind guide in this case, and that he who would follow it would betray reprehensible conservatism.

It is a fact that misplaced conservatism has frequently delayed the development of science and, if it were still necessary, the beautiful discovery of CHANDLER himself of the motion of the Pole named after him, would prove once again that an unprejudiced investigation of the observations, without being guided by any theory, can lead a problem in the right paths and render an important service to science.

But on the other hand we are justified I think in not granting the conclusion that the most simple theory is erroneous or incomplete, before such a theory is shown to be decidedly incompatible with the observations.

At the same time a theory, even a somewhat imperfect one, if it be only based in general on correct foundations, is certainly entitled

¹⁾ See a. o. HELMERT. Die math. und phys. Theorieen der höheren geodäsie. Vol. II page 417.

to some consideration in those cases, where the observations cannot as yet furnish the necessary information. Such a case being before us we are justified in not wholly disregarding what it teaches.

6. According both to observation and theory therefore a more or less sudden displacement of the axis of rotation between 1850 and 1860 must be regarded as possible and so I think that for the present only observations after that time may be employed to deduce the length of the period.

In the following table all the epochs of maximum after 1858, that have been determined, are brought together, at least those which were accessible to me and which seemed more or less trustworthy. In the first place all the results of H. G. v. d. S. BAKHUYZEN have been inserted, together with those of WILTERDINK for Leyden; further several ones deduced by CHANDLER, then my result from the observations 1890—1896 and finally an epoch of maximum deduced by me from all the observations with the vertical circle at Pulkowa 1863—1875, as they have been treated by IVANOFF.

Series of observations.	E.	Epoch.	Weights.	O.—E.B.I.	Auth.
Greenwich Tr. C ...1858—65	—18	2400745	1	— 60	H. G. B.
Washington Pr. Vert. 1862—67	—14	2506 ¹⁾	2	— 24	"
Pulkowa V. C. Pol. 1863—70	—13	3035 ¹⁾	2	+ 74	"
Leyden Fund. stars..1864—68	—12	3394	2	+ 2	Wilt.
" Polaris1864—74	—12	3386	2	— 6	"
Greenwich Tr. C ...1865—72	—12	3435	1	+ 43	H. G. B.
Pulkowa Vert. C ...1863—75	—10	4277	4	+ 23	Iv., E. F. B.
" V. C. Pol. 1871—75	— 8	5146 ¹⁾	2	+ 30	H. G. B.
" Prime Vert. 1875—82	— 3	7290	2	+ 18	Ch.
" Vert. C.....1882—91	+ 3	9867	4	+ 8	H. G. B.
Greenwich Tr. C ...1880—91	+ 3	9870	1	+ 11	Ch.
Madison1883—90	+ 5	2410704	1	— 16	"
Lyon1885—93	+ 6	1151 ²⁾	2	0	"
Summary Albrecht..1890—96	+ 9	2439	6	— 6	E. F. B.

¹⁾ CHANDLER also discussed these series of observations; his results deviate resp. only + 4, + 5 and — 5 days.

²⁾ GONNESSIAT, whose observations of 15 polar stars have been employed here, found himself an epoch 3 days later. Bull. Astr. Vol. XI. Afterwards a formula with 4 terms has been deduced by him. C. R. T. 124. page 930.

For that purpose I employed his table on page 269 of the „*Recherches définitives*” and resolved the 14 equations founded thereon without paying regard to the weights assigned. The epoch obtained by me agrees entirely with the epoch deduced from a curve by IVANOFF himself.

The column E contains the rotation-numbers of the maxima; for the initial epoch was taken the mean maximum epoch of my preceding paper. The following column contains the epochs of maximum reduced to Greenwich and against these the weights have been inserted which I assigned to those results. It was difficult to determine these weights accurately on account of the evidently considerable systematic errors. It was not allowed to take as their exclusive measure the mean errors derived from the agreement of the observations of a single observatory *inter se*; so they have been determined according to a rough estimation. I adopted the values assumed by H. G. v. d. S. BAKHUYZEN, and for the remaining series I acted in an analogous manner. The column Obs.—E.B.I. contains the deviations from my formula deduced in the preceding paper and the last contains the authorities from which the several results were borrowed.

I at once omitted the series of Greenwich finally not included by H. G. v. d. S. BAKHUYZEN in his computation, their results being already contained in those of the other series. On the other hand I have inserted, besides the epoch deduced from IVANOFF's results for all the observations with the Vertical Circle at Pulkowa 1863—1875, also those deduced by H. G. v. d. S. BAKHUYZEN, from Polaris only, as observed resp. by GYLDEŃ and NYRÉN. True, the former result is founded on a much greater number of observations, but it is possible that the mixing up of the results of both observers has done more or less harm, a point which IVANOFF himself also discusses in his first paper page 516.

I have now tried to correct my first formula with the aid of the results compiled in this way, and have rigorously resolved for that purpose all the equations they furnished, having due regard to their weights. At first sight the differences Obs—E.B.I seem to betray a non-linear course, but on closer examination this proves to be only apparent, at least for the greater part, and on account of the occasionally considerable differences between close-lying epochs I thought I was not allowed to depart even now from the simple supposition of a constant length of the period. I made two solutions: including the first time the result according to IVANOFF and omitting those of

the Polaris-observations of GYLDÉN and NYRÉN and the second time including the two latter results instead of the former¹⁾.

So I obtained:

$$\begin{array}{lll} \text{1st solution: } & \Delta \text{ epoch} & + 0^{\text{d}.1} \\ & \Delta \text{ length of the period} & + 0.06 \\ \text{2nd solution: } & \Delta \text{ epoch} & + 4^{\text{d}.7} \\ & \Delta \text{ length of the period} & - 0.45 \end{array}$$

We see that it makes rather a considerable difference whether we

	Obs—E.B.IIa	Obs—E.B.IIb	Obs—Ch.94	Obs—Ch.98
Greenwich Mural C.... 1825—1836 ²⁾	+142 d.	+116 d	+ 44 d.	+163 d.
" " 1836—1850 ²⁾	+ 22	0	+ 2	+ 7
Pulkowa Prime Vert.... 1840—1855 ²⁾	— 9	— 30	— 23	— 27
" Vert. C..... 1840—1849 ³⁾	— 50	— 71	— 59	— 70
Greenwich Tr. C..... *1851—1858 ³⁾	— 92	— 108	— 82	— 127
Greenwich Tr. C..... 1858—1865	— 59	— 73	— 61	— 98
Washington Prime Vert. 1862—1867	— 23	— 35	— 38	— 61
Pulkowa Vert. C. Pol.. 1863—1870	+ 75	+ 63	+ 56	+ 37
Leyden Fund. Stars.... 1864—1868	+ 3	— 8	— 19	— 34
" Polaris..... 1864—1874	— 5	— 16	— 27	— 42
Greenwich Tr. C..... 1865—1872	+ 44	+ 33	+ 22	+ 7
Pulk. Vert. C. All the St. 1863—1875	+ 23	+ 14	— 5	— 13
" " Pol. 1871—1875	+ 30	+ 22	— 3	— 4
" Prime Vert... 1875—1882	+ 18	+ 12	— 18	— 7
" Vert. C..... 1882—1891	+ 8	+ 5	— 12	0
Greenwich Tr. C..... 1880—1891	+ 11	+ 8	— 9	+ 3
Madison 1883—1890	— 16	— 18	— 25	— 19
Lyon..... 1885—1893	0	— 2	— 3	0
Summary Albrecht.... 1890—1896	— 7	— 7	+ 13	+ 6

¹⁾ I also made a solution in which the length of the period was assumed as uniformly variable, but I do not mention it here, as the result seemed wholly illusory.

²⁾ According to CHANDLER.

³⁾ According to H. G. VAN DE SANDE BAKHUIZEN.

follow one way or the other in reference to the observations of Pulkowa.

For the initial epoch and the length of the period itself we obtain in the two cases:

IIa	2408565	431.17 days.
IIb	2408570	430.66 ,

Although after all the first solution seems preferable, I have given below the deviations of the observations from both, besides those from CHANDLER's two formulae of 1894 and of 1898. In order to show in what relation the results of the observations before 1858 stand to those of later years I also include the former.

The consideration of the deviations for the observations 1858—1896 shows that the agreement for CHANDLER's formulae, notwithstanding their greater intricacy, is not better than that for mine. If, in order to compare in this respect E.B. IIa with Ch.94 and Ch.98, we omit, as is only just, the two Polaris-series of Pulkowa, we shall find that the sum of the squares of the residuals multiplied by the weights is even smallest for E.B. IIa. The distribution of weights, however is of very great influence on these results.

With regard to this period (1858—1896) therefore I should like to give as the results of my investigation :

1°. For the present there is no sufficient reason to assume in the 14-monthly motion since 1860 a non-uniform velocity.

2°. The length of the period in these years has not deviated much from 431 days.

These results clash entirely with those of CHANDLER's last paper and little change has been brought about in the conclusions, at which I arrived in my previous communication agreeing in the main with the anterior results of H. G. v. d. S. BAKHUYZEN.

The epochs according to both solutions IIa and IIb coinciding about 1893 and no reason existing not to adopt for the length of the period the round number of 431.0 days, lying between both solutions, I assume for the present as final result:

Elements II of the 14-monthly motion since 1860.

Epoch of maximum for Greenwich	2412446
Length of the period	431 ^d .0
Amplitude	0."156 ¹⁾

¹⁾ Mean value deduced from the previous summary.

In the second place with regard to the period before 1858, I think as yet little can be said about it. Whilst the much smaller amplitude found in this period makes it fully justifiable in my opinion, not to connect the results for that period with the later ones, I dare not deduce anything from the observed epochs themselves. The results 1836—1858 are very uncertain on account of the small amplitude and I cannot give an opinion about the certainty of the results of the observations of POND.

Physics. — Communication No. 44 from the Physical Laboratory at Leiden by Dr. H. KAMERLINGH ONNES. „*A standard open manometer of reduced height with transference of pressure by means of compressed gas.*”

(Read in the meeting of October 29th 1898)

§ 1. *The Principle.* In order to make accurate determinations of high pressures to about 100 atmospheres, open mercury-manometers are indispensable. If we deduce the pressure from the compression of any kind of gas in a closed manometer by making use of the equation of condition of this gas, determinations with open manometers form the basis of the measurements and in making accurate measurements it will prove desirable to test if possible the indications of the closed manometer by comparing them with those of the open manometer. But wherever we want to determine the pressure with greater accuracy than is secured by the equation of condition of the gas with which the closed manometer is filled, there is no other way than making the measurements by means of an open manometer, and that with an apparatus which admits of a high degree of accuracy.

The frequent use made of closed manometers¹⁾ for the experiments in the Leiden laboratory and the necessity to measure the pressure with great accuracy in the case of some determinations (especially

¹⁾ If we can measure a range of pressures in a comparatively short time with great accuracy the graduating of closed manometers after we have filled them becomes so simple that we may omit the measurements from which in other cases the value of the scale is deduced. In principle a closed manometer graduated in this way, as a measuring-apparatus is equal to the metal-manometer, but it is preferable to the latter in so far as its indications when the necessary corrections are applied, are perfectly reliable and probably much more sensitive. The graduating of the closed manometer after its construction relieves the observer from those determinations that take up much time and are very uncertain. The accuracy which can be attained in closed manometers to 100 atmospheres is sufficient for the gauging of ordinary metal-manometers, which in order to be reliable must be tested repeatedly and which are specially used as indicators of operations when employed in accurate measurements.

to pressures of about 60 atmospheres), made an absolute manometer a real want for a long time past. It is impossible to place there open large manometers with a single continuous mercury-column following the example of REGNAULT, CAILLETET and AMAGAT. Only in special cases there will be an opportunity for doing this and generally one will have to have recourse to the principle (first applied by RICHARDS in 1845 and afterwards by others) to shorten the open manometer by dividing it into several not very high manometers placed side by side in which the pressure to be measured causes the column of mercury in the first manometer to rise, while a pressure is applied above the mercury which pressure again is measured by transferring it to the second manometer where it causes the mercury to rise under an excess of pressure which is measured by the third manometer, etc. RICHARDS and his followers have the pressure transferred by water. This means is also given by THIESEN¹⁾ and his design has been taken as a basis in constructing the normal-manometer of the Physikalisch-Technische Reichsanstalt, an apparatus which though made after the principle of those formerly mentioned fulfills far higher conditions by its whole construction.

In the (diagrammatic) serpentine syphon of upright and reversed U's when in equilibrio the upper **U** parts contain water, the lower **U** parts contain mercury and if a pressure is applied to one end, all the menisci are set in motion at the same time, so that the pressure might be deduced from the moving of one of the menisci. Adjustments however are made on all the menisci separately, and in this way only, as WIEBE rightly remarks²⁾, a standard instrument is obtained. Yet the moving of all the menisci together has been one of the difficulties which checked the operations with the standard-manometer in the Phys. Techn. Reichsanstalt. For there, although the apparatus was intended also for determinations of higher pressure, they have contented themselves with attaining 20 atmospheres, a limit not even high enough for testing the manometers of the Government Survey of Steam Boilers in this country by direct comparison with the absolute manometer. (Among the manometers which I have tested by means of the open manometer for the Chief Government Surveyor of Steam Boilers, there were some to 30 atmospheres; among those tested by me for the Chief Constructors of H. M. Navy there were some to 60, and others even to 100 atmospheres).

¹⁾ Zeitschrift f. Instrumentenkunde. 1881, p. 114.

²⁾ Compare his interesting paper in the Zeitschrift für Comprimirte Gase 1897, p. 98, 25, 82, 101.

Another defect of the Reichsanstalt-apparatus is that both the weaker and the greater pressures are indiscriminately measured in tubes of such a size of bore as is required to resist the greatest pressure that those pressures whether great or small are deduced from the and equilibrium of one and the same number of columns of liquid of the same length, and adjustments on one and the same number of mercury-menisci. And so in the case of low pressures the uncertainty resulting from the correction for capillary depressions and for the temperature and the reading error are unnecessarily great. For the purpose mentioned in the beginning, this apparatus which for the rest serves to test metal-manometers is of no value.

When I reflected for the first time upon the problem how to measure regularly with an absolute manometer pressures, even greater than those determined by REGNAULT on the tower of the Collège de France, at the Leiden laboratory where I could not dispose of a great height, the shortened manometers mentioned above were unknown to me. I have considered the transference of pressure by means of a liquid which allowed of contact with steel and mercury, yet after all I did not make use of this means. At first sight it seems that the division of the manometer into separate manometers, in which the pressure is transferred from the one to the other by means of compressed gas offers more difficulties than the transference of the pressure by water so simple in its principle. Yet it seemed to me that these difficulties, at least there where the experimentalists are accustomed to work with gases under high pressure, could be easily surmounted. Especially the ordinary steel bottles filled with gas under high-pressure may be utilized to bring the required tension into the space between the two succeeding columns of mercury, by admitting gas into the connecting-tube of the manometers and this with great accuracy with the aid of finely-regulating high-pressure-cocks.

By adhering to this idea I have already succeeded so far that in a room of the laboratory pressures to 60 atmospheres can be determined by one direct measurement with an apparatus constructed according to this principle and consisting of 15 partial manometers connected together. Seven of the partial manometers connected together may be used as a differential manometer of 28 atmospheres under a pressure of 100 atmospheres, so that after having determined at once directly the pressure of 60 atm. one can, with this pressure as a basis by means of a second determination with a column of mercury of 28 atm. attain a pressure of 88 atm. and again proceeding from this pressure attain 100 atm. by means of a third determination. This apparatus (represented in fig. I) is constructed

by the ordinary means offered by the laboratory. It can easily be taken to pieces and cleaned, leakages can easily be detected and repaired and it admits of a high degree of accuracy in reading. Even in use it answers expectations better than I dared to expect. It now seems to me that a shortened manometer consisting of a tube of everywhere equal bore which is alternately filled with water and mercury would offer more difficulties and admit of less accuracy than this apparatus consisting of parts which each can be used either as manometer or as differential manometer, of which each has a size of bore agreeing with the pressure to be determined therein, and to which more partial manometers for higher pressures can be added without any alteration being made in those which exist already.

2. *The separate manometers.* At first I had thought of using tubes of 12 atmospheres length and to fasten these on a wall of a staircase. On nearer consideration of several sources of error in the observations it seemed to me desirable however to arrange the whole apparatus at least to pressures of 60 atmospheres in one and the same room along the wall. Quick reading, so desirable in many respects, was only possible when the single tubes were not longer than 3.14 M. so that the height of the mercury in the tubes could be 3.04 M., corresponding to 4 atmospheres.

Every separate manometer consists of one single piece of glass, made of tubes sealed together, which can be properly cleaned and dried. The mercury once brought into the manometer-tube remains there and has contact only with the carefully cleaned glass and the dry gas which causes the pressure. Where a meniscus is to be adjusted the manometer-tube should be wide enough for sufficiently obviating the uncertainty resulting from the capillary depression, and for the rest so wide as the pressure which the separate manometer shall have to resist will admit of. The manometer is on both ends provided with suitable contrivances by means of which it may be connected with the other parts of the apparatus, and the tubes of which it consists are bent in such a way that these joints together with the others can be brought very near to each other at the bottom of the apparatus near the stopcocks.

I used three different types of glass tubes. The first type is represented by tube *A* in fig. 1. One leg of the U-shaped wider tube (12 m.m. diameter) is half the length of the other, to both legs capillary tubes have been sealed on which are bent downwards and reach below the lower part of the U. Tubes of this kind *A* may generally be used where we want to read a whole range of positive pressures from 0 to 4 atm., (from 0 to $m_o' - m_o$) on the same tube.

One might construct a manometer entirely with tubes of that type, each for r atmospheres, in which the additional pressure above $n r$ atmospheres might be read on a tube proof against $n r$ atmospheres, while each of the n preceding tubes were adjusted on a difference of pressure of r atmospheres but this manometer when adjusted to a high pressure, would contain more compressed gas than is necessary.

This is not desirable, for with unaltered section of the tubes at the place of the mercury-menisci, and with unaltered distance of the latter, change of temperature in the gas confined between the two columns of mercury, will bring about a displacement of the menisci which will be the stronger as the volume of the confined gas is larger. So we must try to limit as much as possible the volume of the space filled with gas. This is also desirable in order not to waste any gas during the operations; which gas, with a view to the preservation of the apparatus and in order to enable us to apply the corrections with exactitude must be perfectly dry and pure; finally because the effect of a possible explosion of a tube filled with gas at a high pressure, is in proportion of the quantity of the gas compressed therein.

In order to economize gas we use the second type represented by the tubes B_1, B_2 in figure 1, which is also easier to be handled than A . These tubes consist of a wider upper cylindrical vessel, b and a lower cylindrical vessel, a , in which the mercury-menisci are adjusted, of a capillary g , (comp. fig. 2) through which the mercury rises and of the connecting capillaries f , and h , bent downwards. The diameter of both vessels is such as would be chosen for a manometertube of class A , intended for the same pressure and smaller in proportion to the augmentation of the pressure for which it is destined. The following are the diameters of manometers for:

Atmospheres	Inner diameter	Outer diameter
12	9 mM.	12.8 mM.
20	8	13.5
32	6,5	13.5
60	6,5	15.5

In order to be able to take the lower cylinder as short as possible we choose a narrow capillary for the ascension of the mercury, in doing this however we must take into account that this has a great influence on the motion of the mercury and makes that the adjusting requires more time. As the co-efficient of friction of mer-

cury is much greater than that of gas, one might take the capillary through which only gas flows, narrower than that through which mercury flows, without causing thereby considerable delay in adjustment. As the dimensions of those capillary tubes however do not offer any difficulties, and it is not desirable to take them narrower than is strictly necessary (comp. § 7) I used capillaries of nearly the same section both for gas and mercury.

The section of the capillary tubes is from $0,75 \text{ m.m}^2$. to $1,37 \text{ m.m}^2$. The length of the upper cylindrical vessel is 8 cm. This length leaves more than sufficient room in adjusting the meniscus, without the accurate determination of the height of the mercury being damaged. The cylinder may not be shorter because when the mercury rises, danger might arise for the mercury to overflow which would cause all the other manometers to overflow likewise. The whole apparatus would temporarily become unfit for use and even the joints on the board to which the stopcocks are fixed might be endangered.

The length of the lower cylinders is calculated after the available capillaries and the upper cylinders. It has been taken into account that when the lower cylinder contains so much mercury as necessary to fill the whole capillary and the whole upper cylinder, a space of 4—5 cm. long remains above the mercury. If this precaution had not been taken, gas bubbles that might rise (in blowing off the manometers) would cause the mercury to overflow, which as mentioned above must be prevented carefully.

An open manometer cannot consist of tubes of the type *B* only as in these we can only adjust at a fixed difference of pressure (position of the mercury nearly in the middle of the upper cylinder). With n of these tubes, each measuring a difference of pressure of r atmospheres one can adjust accurately at nr atmospheres only. For adjustment at the additional pressure above nr atmospheres we want another tube, viz. the tube *A*, the first in the plate.

In constructing shortened manometers we can very successfully avail ourselves of tubes of the third type, *C* (comp. fig. 1). These are distinguished from the type *A* by their having the two branches of the U equal in length so that we may adjust both positive and negative differences of pressure, according as we adjust either at r_1 on the left and r_2 on the right or conversely at r_1 on the right and at r_2 on the left. Suppose that we have adjusted the tubes *A* and *B₁* to *B₈* together at 35 atmospheres and that we want to measure the pressure of 34.8 atmospheres, this can be obtained, (the adjustment in all preceding tubes remaining unaltered), by adjusting the mercury in tube *C* at — 0.2 atmospheres difference of pressure.

The tube shown in fig. 1, serves to adjust at positive and negative differences to within $\frac{1}{2}$ atmosphere.

§ 3. *The connections and the stop-cock board.* In the apparatus represented here, tube *A* at one end has contact with the atmosphere by means of a drying tube *Q*, coated with P_2O_5 by which it is protected from moisture. And so in this case an india-rubber tube is sufficient connection. At the other end however the ordinary contrivance which is found at both ends of all other separate manometers has been fastened. To the thick-walled glass capillary (comp. fig. 4) *p*, a steel capillary *q* has been cemented which carries an overpipe *W* with a nut *U*. This nut is screwed (comp. fig. 1) on the corresponding tap (of T_1) of the stop-cock board; the steel capillary gives elasticity to this joint. The communication between the other manometers is also obtained by screwing the connection-nut on to the **T**-pieces, *T*, on the stop-cock board. The horizontal branches of the **T**-pieces form the connection between the successive manometers, the steel stop-cocks in the vertical branches of the **T**-pieces form the communication of the space between the lower meniscus in one manometer and the upper meniscus in the next manometer with the main feeding tube *H*, from which when brought to a sufficient pressure, gas can be admitted in that space in order to apply the desired pressure on the columns of mercury.

This main feeding tube (of copper with brazed on branches) is filled by the bronze stopcock *X* through the steel cross-piece *N* from a cylinder with gas compressed at a sufficient pressure — dry air or oxygen or hydrogen, for lower pressures sometimes dry carbonic acid — which gas passes over P_2O_5 (in the drying tube *I*) before entering the apparatus. The cross piece *N* enables us at the same time to make communication between the manometer and the apparatus in which we want to measure the pressure; such an apparatus in the drawing being a metal-manometer as represented by *M*. This metal-manometer and the stop-cock *Z* are also used to measure the pressure in an apparatus which is joined to *Z*, before causing communication between this apparatus and the open manometer by means of the stop-cock *Y*.

4. *The adjustment at a given pressure.* The admission of the compressed gas in the different spaces in the manometers must be performed with the greatest caution. In the first place we have to prevent the overflowing of mercury from one manometer into the other; we have alluded already to the disagreeable consequences of this. Then we must remark that when we cause mercury to rise in one of the manometer-tubes *B*, f. i. and admit therefore (keeping the

stop-cock K_2 shut) pressure into the lower cylinder, the gas in the closed space between the upper meniscus in B_2 and the lower meniscus in the preceding manometer will be compressed and consequently the lower meniscus in B_1 will be further depressed. A similar displacement is repeated in all preceding manometer tubes at the adjustment of each following manometer tube. If finally we want to get the desired adjustment in all cylinders we must begin because the spaces between two manometers are not provided with separate stop-cocks for drawing off so as not to add unnecessarily to the number of stop-cocks and connecting pieces, by causing the mercury to rise in the manometer to less than the required height i.e. only so far that it will be brought to the required height by the following adjustments in the other tubes. In order to facilitate the filling these heights have been indicated by distinct marks (comp. x_1, x_2 in fig. 1).

In order to bring the apparatus to a given pressure, (at $46\frac{1}{4}$ atmospheres f.i. as in the drawing) we must find out which multiple of the whole height is nearest to that pressure, how many tubes of the type B we shall have to use therefore and which additional pressure has to be adjusted in A .

Then all stopcocks except K_0 , KA and X , are opened wide and gas is carefully admitted through X until the difference of level in A has the required amount (in this case $2\frac{1}{4}$ atmospheres) within about 6 c.M., after which X is shut. In all the other manometer tubes the pressure rises to the same height, but the position of the mercury remains unaltered during this process. Then shut K_1 and open X carefully until the mercury in B_1 has reached x_1 ; shut K_2 (x is shut) and admit gas through X until the mercury in B_2 has reached x_2 , etc. until the mercury in B_{10} has reached x_{10} , shut X and K_{11} and admit pressure through X till the topsurface of the mercury in B_{11} in this special case is just in the middle of the upper cylinder. Then the menisci in all the preceding manometer tubes are in a proper position.

A given pressure can always be adjusted accurately within a small fraction of an atmosphere.

If differences should remain or arise between the pressure adjusted and the pressure we want to determine, we can take away these differences by means of the differential tube C which in the case represented here was not used. K_{12} to K_{15} are left open, K_{16} is shut and more gas is admitted through X , but at the same time we draw off through KA just as much as to leave the menisci in B_{11} in the same position, and then the desired difference of pressure in C is adjusted. After this X and KA are shut.

In order to stick to the principles of the construction of our apparatus we should have at hand C tubes of different sections and we should always have to join on such a tube as agrees most with the pressure to be measured.

From what precedes it appears that when we have adjusted at a certain pressure and we want to pass to another this can be done easiest by 4 atm. at a time. In order to rise 4 atmospheres we must let the last tube B_{11} go back to the mark x_{11} by drawing off through KA , shut KA and K_{12} and admit gas through X until the required adjustment in B_{12} is attained. In order to let the mercury fall 4 atm. we must act conversely, viz. draw off through KA till the mercury in B_{12} has fallen; shut KA and open K_{12} and again admit gas through X until the mercury in B_{11} has reached the middle of the upper cylinder.

If we want to pass over to pressures higher by a fraction of 4 atm. say 2 atm. then we must proceed more cautiously; as a rule one should work with all stop-cocks shut, because all menisci will vary at a higher adjustment in A , but it will often be sufficient to bring A almost to the desired new pressure through the stopcock K_1 and to bring the other menisci to a proper position by admitting a higher pressure through X , if only we have left unimpaired the reserve space in the upper cylinders. In none of these cases the adjustment will offer any difficulties.

It is however more difficult to let the pressure fall by differences of less than 4 atm. We then must first adjust at 4 atmospheres lower and must raise the pressure to the desired pressure by adding the remaining difference, if the position of the mercury in tube A permits. If not, we shall have to empty the main tube, to blow off the manometers, and to raise the pressure anew from 0 to the desired pressure. For in the tube A we can only adjust at a rising pressure, because at a falling pressure mercury would overflow from the upper receptacle B_1 .

If we should want an apparatus in which we can adjust with the same rapidity at rising as at falling pressures, this should consist entirely of A tubes and we should have to read the additional pressure above the multiple of 4 atmospheres on the tube containing the highest pressure. Or we should have to add to a manometer consisting of B -tubes a differential tube C of 2 atmospheres, or two tubes C of 1 atmosphere each to which as in the case of the separate manometers a special supply-stopcock has been joined. The remarks made previously concerning the diameter apply to these tubes unless one should be satisfied for all pressures with

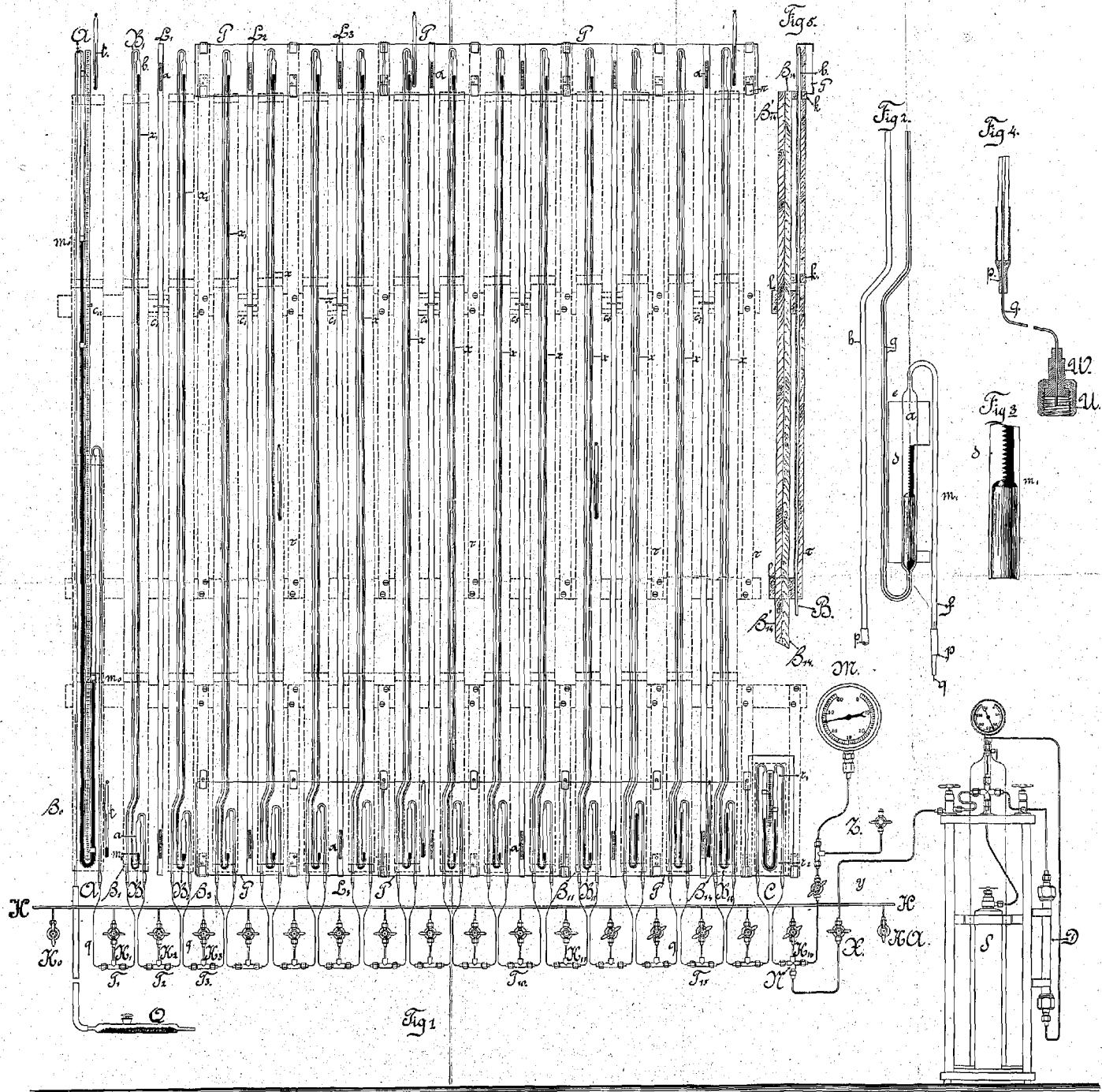
reading 2 (or 4) menisci in tubes calculated for the highest pressure.

The standard-manometer as a matter of course is used to determine pressures, that are carefully kept constant for a long time, and by preference by repeated readings from one and the same adjustment. And so there was no reason for using the above-mentioned auxiliary means.

5. *The reading.* The position of the menisci is read on tubular measuring rules, suspended between the manometer tubes in rings of CARDANUS $c_0, c_1 \dots$, which rules besides promoting the accuracy of the measurements, have the advantage of being easily applied to the apparatus and removed from it. The measuring rule used with the manometer of type *A* in which adjustments are made at different heights is a brass tube graduated in m.m. over its whole length. The measuring rules used with tubes of the type *B* are ordinary brass tubes to which only at both ends short graduated scales α are fixed. We adjust at the menisci in the tubes *B*, all of which are at an elevation of nearly 1 M. or nearly 4 M. above the ground with the aid of two theodolites. One of these is placed on an isolated observation pillar in the room, 4 M. in front of the wall against which the apparatus is fixed. Vertically above this one a second theodolite is placed on a table (3.5 M. high) resting on the same pillar. Next to this isolated elevation or observation-tower steps provided with a platform, such as are generally used for observing purposes in the laboratory, are placed.

In this our case it was very difficult to make the menisci clearly visible in the way in which it is generally done viz. by sliding along the tubes a piece of black paper tubing until the rim of this piece of tubing almost touches the meniscus and the meniscus stands out as a dark reflection against the bright background (white paper), especially because in order to protect the observer when he opens or shuts stopcocks, or looks for leaks and his eye is very close to the apparatus, sheets of plate glass *P* (fig. 1) have been placed in front of the manometer receptacles. The sheets of plate glass are supported by clamps fixed to beams *r* which are screwed on to the wooden strips *l* fixed everywhere along the walls of the laboratory for fastening purposes.

Mr. J. C. SCHALKWIJK, assistant at the laboratory to whom I am much indebted for the care bestowed upon the apparatus, has hit upon the idea to place behind the meniscus a drawing of black teeth against a white surface (comp. fig. 2 and 3) which is reflected on the meniscus. The coming together of the reflection of the teeth on the meniscus with the drawing of the teeth enables us to determine accurately the place of the top of the meniscus.



The method of adjustment of Mr. SCHALKWIJK will prove very useful in many other cases.

The illumination is obtained by means of incandescents lamps which are moved along at a suitable distance of the apparatus.

In order to read the position of the mercury in the tube *A* or in the small tube *C*, we can, if perfect precision is not necessary and we therefore need not take the readings by means of a levelling telescope, utilize indicators fixed to pieces of tubing, which can be moved to and fro round the glass tube.

§ 6. Remarks on the constructing of the manometer. The apparatus is constructed by Mr. G. J. FLIM chief of one of the instrumentmakers workshops of the laboratory, whose able help was of great value. Although the single pieces of manometer-tubes are very large the constructing and handling especially of the type *B* did not offer great difficulties. But it need hardly be mentioned that all the operations have to be done very carefully and cautiously, if so complicated an apparatus as the manometer is not to have any leak or other defect. To persons who want to construct such an apparatus the following remarks may be useful. In order to make a tube of the type *B* the separately blown cylindrical vessels with short capillary tubes (about 15 c.M.) bent in the proper form (comp. fig. 2) are put together with the other parts either suspended from pulleys or placed on stands, provided with proper india-rubber tubes in which we can blow, and sealed together by means of the hand blow-pipe (fed by a water blow pump). In this way tubes of any length can be sealed together. As the manometertube of the type *B*, consists principally of capillary tubes it forms a very elastic whole. It is mounted on a loose board between cork by means of clamps, cleaned with boiling, and dried with the same precautions as the glass used for the thermometer described in the Proceedings of Mai 30th '96, p. 40¹⁾. During the boiling the tube with the auxiliary board are placed in a sloping position, entirely filled and with both branches plunged into small basins, and heated by means of a BUNSEN-flame regularly over its whole length. After the drying the ends are provided with the steel caps, *r*, (prepared beforehand) to which the steel capillaries (treated as the thermometer capillaries comp. Proceedings May 30th '96, p. 41)²⁾ with the overpipe *W* have been fastened (fig. 4). The steelcapillary (comp. Proceedings Dec. '94, p. 168)³⁾ (long 35 cM.) has been

¹⁾ Comm. of the Phys. Lab. at Leyden 1896, n°. 27 p. 8.

²⁾ Ibid. p. 9.

³⁾ Comm. of the Phys. Lab. at Leyden 1894, n°. 14 p. 8.

screwed as l. c. p. with marine-glue into the cap and into the overpipe, and after this the joint has been soldered with tinmans solder in order to shut up the soft marine-glue firmly while the seam is protected from the action of the mercury.

The fastening of the steel caps to the glass capillaries which are widened towards the ends requires special care (comp. the cementing of thermometer-capillary, Proceedings May 30th 1896, p. 41)¹⁾. It is done as then by means of red sealing-wax, that answered the purpose better than several other mixtures that have been examined since that for this purpose. In addition to the precautions in the case mentioned l. c. we had to prevent the sealing-wax from having contact with the steel capillary until all the air has escaped, as else the passage for the air would be blocked up, which would cause a defective closure.

We continued the pushing on of the cap under moderate heating as when cementing the volumenometer cap (l. c. p.) until the sealing wax became visible between the steel capillary and the glasswall. In this way we succeeded in getting perfectly tight connections, only the joint between the glasscapillary and the steelcapillary would sometimes give rise to defects and as the reliability of this joint is of great importance to the usefulness of the manometer we are still trying to get a better connection. After the joints have been made the manometertube is ready to be tested by pressure in order that we may detect leakages. In order to fill the manometertube with mercury we cement a small thin-walled glasscapillary the point of which has been sealed, on the end of the steelcapillary which projects beyond through the overpipe, then we exhaust the manometer by means of the mercury pump and the point is broken off under mercury. By admitting air we then can expel as much mercury as is desirable.

Each manometertube is fastened on a separate board β_1 by means of 4 clamps k (comp. fig. 5) between cork and this board is screwed on to a board β_2 which is fastened to the wall by means of fixing it to a strip l .

The T-pieces on the board on which the stopcocks are fastened, are forged of steel and have been pierced afterwards. The high-pressure stopcocks (the ordinary Leiden pattern) are also entirely made of steel in order not to be attacked by the mercury which might accidentally get into the horizontal connection-canals of the T-pieces; they have

¹⁾ Comm. of the Phys. Lab. at Leiden 1896. №. 27, p. 8.

been placed higher than these connection-canals so as to make it more difficult for the mercury to get into them. The footplates of the stopcocks are fastened by means of wood-screws on the stopcock-board. The connection of the stopcocks with the steel T-pieces is obtained again by marine-glue fastened by soldering-work; all these parts and the coupling by nuts and taps proved perfectly reliable. After the manometertubes have been connected, the whole apparatus is exhausted through the stopcock K_0 (at the same time coupling the left side of A to the airpump) and is filled with gas from the main feeding tube.

When there are still leakages in the apparatus after this has been performed, these are shown by the motion of the mercury columns when the manometer is brought to pressure. They are then detected by passing a pencil with soap-water or oil along the suspected seams, and generally they can easily be repaired without removing any of the separate manometers from the apparatus (which however would not offer the least difficulty owing to the elasticity of the glass capillaries, the flexibility of the steel capillaries, the mobility of the boards and the suitable position of the joints).

7. *Corrections.* In computing the pressure from the read height of the mercury the ordinary corrections for open manometers must be applied.

I need not dwell upon the well-known small corrections which the theodolite-readings (comp. § 5) might require, from the telescope being not perfectly level in the way of reading which is followed here; I will only suggest that the correction for the capillarity may be applied by determining the height of the meniscus by means of the reading telescope. We have however not yet much experience about that correction and till now we tried to equalize the two menisci in one and the same tube as much as possible by knocking. There are however two corrections resulting from the peculiar construction of the apparatus on which I shall have to make a few remarks.

The first correction is that for the difference of pressure in the compressed gas near the lower meniscus in the one and near the upper meniscus of the next manometertube. If the corrections must be very small we might use for the supply of pressure compressed hydrogen, this may be of advantage for very high pressure, at 60 atmospheres or 45600 m.m. the correction for all hydrogen columns together is not more than about 9 m.m., at 100 atm. or 76000 m.m. not more than 21 m.m. so that these corrections have only to be applied to measurements, which are very accurate. But as the slightest

contamination of the hydrogen has a great influence on the correction, this gas can only be applied when one disposes of a compression-pump as constructed at the Leiden laboratory (Proceedings, Dec. '94, p. 168)¹⁾ by means of which the hydrogen can be re-compressed without its becoming impure. Compressed air at 60 atm. (about 120 m.m.) gives a correction which can be applied with perfect certainty.

In the case of lower pressures we can use commercial carbonic acid from cylinders, if only one takes good care to dry it. At a comparison at about 48,6 atm. we read on the manometer, after having applied all the corrections except those for the connecting tubes between the separate manometers, a total column of mercury of 36943,3 m.m. The correction for the carbonic acid columns was but to — 144,8 mm.

After the correction for the columns of compressed gas in the manometer has been applied we must still take into account the column of the gas between the level of the last meniscus of the manometer and the level of the place at which we want to measure the pressure in the apparatus. The determination mentioned above referred to the pressure in apparatus which were placed in a distant room; these were connected with the manometer by a conduit (40 M. long) filled with carbonic acid; the difference of level amounted to 0,65 M. and the correction was + 6,2 m.m.

A second correction peculiar to this apparatus, must be applied when the mercury in the manometertubes stands not perfectly still, as even the slowest motion of it through the narrow tubes requires a measurable pressure. If a velocity of 0.1 m.m. in a minute is observed at the menisci in one of the separate manometers, this agrees with an excess of pressure and with a correction of about 0.1 m.m. For the streaming of the gas this correction would be uncertain as gas may overflow without this being perceived at the movement of the menisci. Therefore the tubes along which the gas flows from the one manometer to the other may not be too narrow. The diameter of the steel capillaries is 0.6 m.m. They don't add to the friction more than the glass capillaries and these are taken comparatively wide (comp. § 2) so that when the apparatus is in a good condition we need not pay attention to the motion of the gas.

Usually the filling of the tubes is done very slowly, the filling of each tube takes about 4—5 minutes. Each time at the closing of the stop-cocks the equilibrium is fairly well attained. If however a small

¹⁾ Commun. fr. the Physical Laboratory at Leiden. 1894. N°. 14 pg. 7.

excess of pressure had to be applied it appeared that even in the remotest tube the mercury soon stands still. After the adjustment is made we usually wait 10 minutes to make sure that no further change occurs. During that time several readings can be prepared or taken, amongst others those of the 8 thermometers, indicating the temperature.

As the temperature during the observations is kept as equable as possible, and as the cylindrical vessels have been taken as short as possible (comp. § 2) the movement of the menisci of the mercury-columns through the alteration of temperature of the gas, after the adjustment has been made, is very slight.



KONINKLIJKE AKADEMIE VAN WETENSCHAPPEN
TE AMSTERDAM.

PROCEEDINGS OF THE MEETING
of Saturday December 24th 1898.

(Translated from: Verslag van de gewone vergadering d'er Wis- en Natuurkundige Afdeeling van Zaterdag 24 December 1898 Dl. VII).

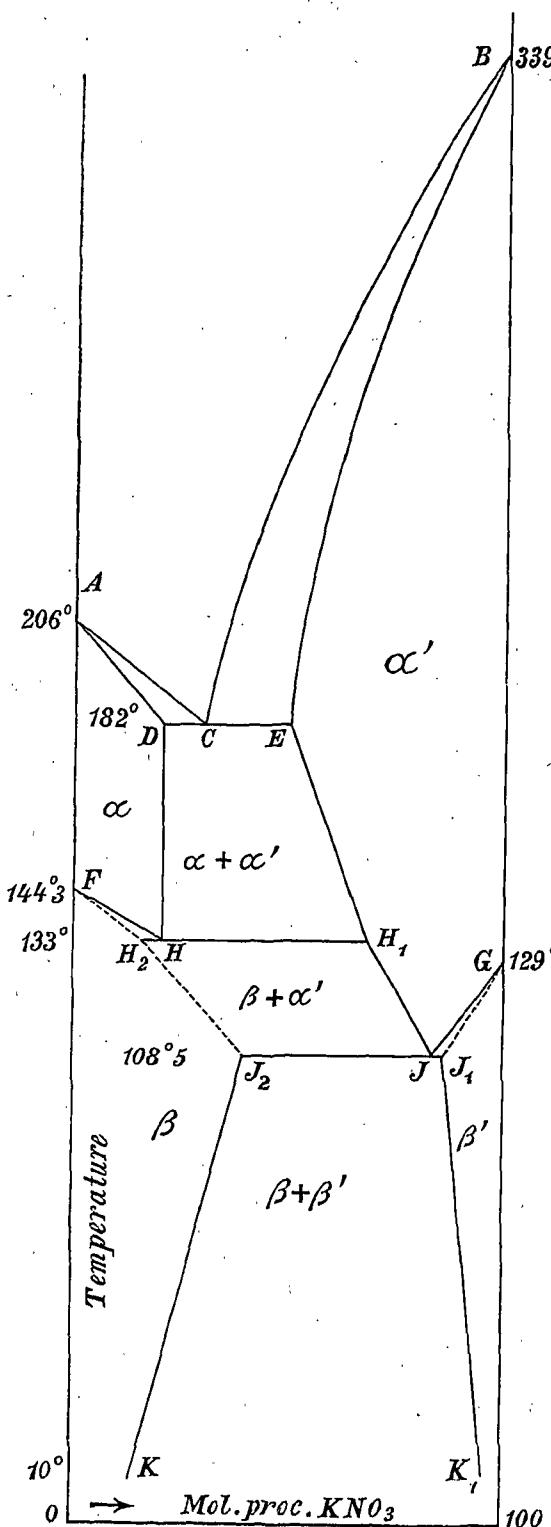
CONTENTS: „On mixed crystals of nitrate of kalium and nitrate of thallium.” By Prof. H. W. BAKHUIS ROOZEBOOM, p. 229. — „Variation of volume and of pressure in mixing (II).” By Prof. J. D. VAN DER WAALS, p. 232. — „Measurements on the course of the isotherms in the proximity of the plaitpoint and particularly on the course of the retrograde condensation in a mixture of carbonic acid and hydrogen.” By Dr. J. VERSCHARFELT. (Communicated by Prof. H. KAMERLINGH ONNES), p. 243. — „Measurements on the magnetic rotatory dispersion of gases.” By Dr. L. H. SIERTSEMA. (Communicated by Prof. H. KAMERLINGH ONNES), p. 243.

The following papers were read:

Chemistry. — „*On mixed crystals of nitrate of kalium and nitrate of thallium*”. Dissertation written by Dr. C. VAN EIJK and presented by Prof. H. W. BAKHUIS ROOZEBOOM who reads on this subject as follows:

Some time ago (vide Proceedings of the meeting of the Academy of 24 Sept. last, page 101) speaker has given a theoretical development of the course of congealing of a molten mixture of two substances, from which mixed crystals are deposited when cooling and of the course of transformation of such crystals, in case one of the two components or both of them pass into another solid modification.

Nitrate of kalium and nitrate of thallium now are the first examples of a system of two substances, of which we have acquired a general survey of the course of these two phenomena over the entire field of concentration, and in which the observations agree in all respects with the course deducted theoretically.



Let us first consider the congealing (see figure).

The congealing temperature falls in mixtures of 0—31.3 mol.pCt. K NO₃ from 206° (melting point of Tl NO₃) to 182° (AC). From these liquids mixed crystals are deposited, containing more Tl NO₃ than the liquid, namely 0—20 pCt. K NO₃ (AD). These are heavier than the liquid.

Starting from KNO₃ the congealing temperature falls equally from 339°—182° (BC) while from the molten mass of 100—31.3 pCt. K NO₃ mixed crystals are deposited containing 100—50 pCt. K NO₃ (BE) being therefore richer in K NO₃. These crystals are specifically lighter than the molten mass.

The congealing both in points of AC and of BC takes place in such a temperature-interval, as comprised between the points of AC and AD or BC and BE showing the same

concentration. At the minimum congealing-point C the liquid of 31.3 pCt. K NO₃ solidifies to a conglomerate of two kinds of mixed crystals, one with 20 pCt. and the other with 50 pCt. K NO₃. Notwithstanding both kinds belong to the rhomboedric system, the mixing series is not continuous.

Below 182° three different conditions are therefore possible in the solid state: homogeneous mixed crystals (α) of 0—20 pCt. K NO₃, homogeneous of 50—100 pCt. K NO₃ (α') and conglomerates of the limit-crystals of 20 and 50 pCt. in variable relations, in accordance to the composition of the fused mass, taken as a starting point.

The mixed crystals, either homogeneous or conglomerate undergo in case of further cooling, transformations from the rhomboedric to the rhombic form.

For K NO₃ that transition was known at 129°.5 (G).

For Tl NO₃ Dr. van Eijk found for the transition temperature 144° (F).

The determination of these temperatures for these two substances, and for their mixed crystals, was effected either by observing the delay in the rise or fall of the temperature, effected by the heat of transformation, or by determining the temperature at which the transformation becomes noticeable by the refractive power or the exterior of the crystals. These latter observations are made both microscopically and with the naked eye.

Between 182° and 144° no transformation takes place; we have only found, from the course of the transformation at a lower temperature, that the limits of the homogeneous crystals α and α' diverge somewhat more at a falling temperature. At 133° these limits have become: 20 pCt. and 69 pCt. K NO₃.

The transformation-temperature of the homogeneous α -crystals falls with increasing K NO₃ proportion from 144°—133° (line FH); the correct position of line FH_2 , indicating the rhombic β -crystals formed, is however not known. The transformation temperature for all conglomerates of α and α' mixed crystals is at about 133° and from the fact that this temperature remains constant between 20 pCt. and 69 pCt. K NO₃ the situation of the points H and H_1 is deducted and the course of the lines DH and EH_1 .

At 133° all the rhomboedric α -crystals of the conglomerate are transformed in rhombic β -crystals.

Below 133° the conglomerate consists therefore of β - and α' -crystals. The limits of these two kinds are proved equally to change in case of a further fall in the temperature (lines H_2J_2 and H_1J)

namely both to a higher KNO_3 proportion so that these limits have become at $108^\circ.5$: 40 pCt and 84 pCt.

The homogeneous rhomboedric α' -crystals only begin below $129^\circ.5$ to be transformed in rhombic mixed crystals β' .

For crystals of 100—84 pCt. KNO_3 the transformation temperature falls from $129^\circ.5$ — $108^\circ.5$ (*GJ*). At $108^\circ.5$ the composition is attained, also possessed by the α' mixed crystals, which were already present in the conglomerate with β -crystals from 133° . Below $108^\circ.5$ all the α' -crystals in the conglomerates are now transformed in rhombic ones. The transformation is completed at this temperature.

In consequence of the discontinuity of the mixing-series there remain however below $108^\circ.5$ still 3 mixing-types of rhombic crystals viz. homogeneous β and β' crystals and conglomerates of both limit-crystals.

The limits of mixing and consequently the composition of the two component parts in the conglomerates, being at $108^\circ.5 \pm 40$ pCt. and 84 pCt. KNO_3 , diverge still more with falling temperature, so that they have become at 10° : 15.5 pCt. and 96.5 pCt. ($J_2 K$ and $J_1 K_1$).

At the lower temperatures these limits are easily found, by determining the limits of the mixtures in the crystals that are deposited from aqueous solution.

Physics. — „Variation of volume and of pressure in mixing (II):”

By Prof. J. D. VAN DER WAALS.

For a simple substance we may calculate how much its volume under a given pressure and at a given temperature differs from that which it would have occupied, if it had perfectly followed the law of BOYLE.

Let us call the real volume v , and that which it would occupy according to the law of BOYLE v' , then the quantity to be examined is $v' - v$. Let the quantity of matter be equal to the molecular weight.

From

$$p = \frac{MRT}{v'} = \frac{MRT}{v-b} - \frac{a}{v^2}$$

follows

$$v' = \frac{v}{1 + \frac{b}{v-b} - \frac{a \cdot 1}{(1+a)(1-b)(1+at) \frac{1}{v}}} .$$

If we put $\frac{a}{(1+a)(1-b)(1+\alpha t)} = a'$, we find

$$v' - v = \frac{a' - b - \frac{v}{v-b}}{\frac{v}{v-b} - \frac{a'}{v}} = \frac{a'v(v-b) - bv^2}{v^2 - a'(v-b)}$$

or

$$v' - v = \frac{a' - b - \frac{a'b}{v}}{1 - \frac{a'}{v} + \frac{a'b}{v^2}}.$$

If $v = \infty$ we find for the increase of volume in consequence of the causes which make a substance deviate from the law of BOYLE, the value $a' - b$.

If we had not solved the problem in general, but had restricted ourselves to a first approximation, we should have found this special value $(a' - b)$ as apparently the general one. So in my paper on „variation of volume and of pressure in mixing” I have found for Δ_v

$$x(1-x) \left\{ \frac{a_1 + a_2 - 2a_{12}}{(1+a_x)(1-b_x)(1+\alpha t)} - (b_1 + b_2 - 2b_{12}) \right\},$$

but only because the value of Δ_v had been determined by approximation — and the question is not without interest, can the real course of Δ_v be determined? For the quantity $v' - v$ for which the accurate value has been found above, this may of course be done — and it will be possible to show that a similar course is to be expected for Δ_v .

If we proceed to the discussion of

$$v' - v = \frac{a' - b - \frac{a'b}{v}}{1 - \frac{a'}{v} + \frac{a'b}{v^2}}$$

We first observe that for $v = \frac{a'b}{a' - b}$ the quantity $v' - v = 0$. This is the same value for which the quantity $p - p'$, treated in my former communication, is equal to 0.

Two cases, however, are possible, if beginning with $v = \infty$ we diminish the volume gradually.

When diminishing v , the value of $v' - v$ may either decrease continually down to 0, or it may first increase to a certain maximum, and then decrease. Which of the two cases occurs will depend on the value $\frac{a'}{b}$. If we put by first approximation

$$v' - v = \left(a' - b - \frac{a' b}{v} \right) \left(1 + \frac{a'}{v} \right)$$

or

$$v' - v = a' - b + \frac{a'(a' - 2b)}{v}$$

we see, that when $a' > 2b$, $v' - v$ will begin to increase and vice versa. The condition $a' > 2b$ leads to $\frac{T_k}{T} > \frac{16}{27}$. When $T < \frac{16}{27} T_k$, $v' - v$ will begin to increase and vice versa. The condition for the limiting value of $v' - v$ being positive, is $a' > b$ or $T < \frac{27}{8} T_k$.

Our result is therefore that for half of the temperatures, at which $v' - v$ is positive, this quantity will begin to increase still more.

This result will be modified to some extent, if we take also the variability of b with the volume into account, — for this particular problem it is sufficient to know the first correction term — but as long as the accurate value of b is not known, it may be better to consider b as invariable —, also because the laws found in this supposition are generally much simpler. Not before a sufficient number of reliable observations are at our disposal, at which the result of this and similar calculations may be tested, it will be possible to investigate, whether this and the following calculated correction-terms of b can account for the deviations.

If we consider b as independent of the volume, we find by differentiating $v' - v$, that a maximum is found at

$$\frac{v}{b} = \frac{a'}{a' - 2b}.$$

In order to find v positive, a' must be greater than $2b$, which agrees with the preceding result.

If we substitute this value of $\frac{v}{b}$ in $v' - v$, we find for the maximum value of $v' - v$

$$(v' - v)_{\max.} = \frac{b}{\frac{4b}{a'} - 1} = \frac{b}{\frac{32T}{27T_k} - 1} .$$

As a matter of course it is better to restrict ourselves to temperatures above the critical. Below this temperature $v' - v$ is not single-valued and the inquiry after the value of $v' - v$ for such points on the theoretical isothermal as for which the pressure would be negative, has no sense. If we take as utmost case $T = T_l$, the maximum value $v' - v$ has risen to $\frac{27}{5} b$.

For the value of the proportion of the maximum amount of $v' - v$ and the initial value, we find

$$\frac{1}{\left(\frac{32T}{27T_k} - 1\right)\left(\frac{27T_k}{8T} - 1\right)} ,$$

T to be taken between $\frac{27}{16} T_k$ and T_k . This proportion which at $T = \frac{27}{16} T_k$ is equal to unity, reaches the value of 2.27 at $T = T_l$.

Now we can imagine the course of $v' - v$ at all temperatures above the critical.

1°. for $T > \frac{27}{8} T_k$ the limiting value of $v' - v$ is negative and equal to $b\left(\frac{27T_k}{8T} - 1\right)$, so contained between $-b$ and 0. For decreasing value of v , the quantity descends to the value $-b$ at $v = b$. There is neither maximum nor minimum.

2°. for T between $\frac{27}{8} T_k$ and $\frac{27}{16} T_k$ the limiting value is positive and keeps between 0 and b . For decreasing value of v , $v' - v$ descends down to $-b$. So there must be a value of v , at which $v' - v$ changes its sign. This value of v has been calculated above, and has been found to amount to $\frac{a'b}{a' - b}$, for which we may write

$$v = \frac{b}{1 - \frac{8}{27} \frac{T}{T_k}}.$$

This value for v lies between ∞ and $2b$.

As in the preceding case there is no question of a maximum value.

3°. for T between $\frac{27}{16} T_k$ and T_k , the limiting value of $v' - v$ is contained between b and $2\frac{3}{8}b$. When the volume decreases, $v' - v$ begins to increase, till a certain maximum value is reached, which amounts, however, at the utmost to 2.27 times the limiting value. It decreases afterwards, reaches the value 0 and ends with $-b$.

4°. for values of $T < T_k$ only those values of v have to be considered, which lie beyond the limits of coexisting gas- and liquid-volumes, and $v' - v$ loses even its theoretical signification for such volumes for which the pressure on the theoretical isothermal is negative. For $T = \frac{27}{32} T_k$ the isothermal touches the p -axis at $v = 2b$.

There the quantity $v' - v$ is infinite, because if $p=0$, v' is also infinite. But this does not mean at all that for the volumes which may be realized at that temperature, $v' - v$ will increase to a high amount.

Let us take e. g. $T = \frac{27}{32} T_k$. In order to find the highest value for $v' - v$, we have to determine the value of v for saturated vapour and to substitute that value of v in the formula

$$v' - v = \frac{a' - b - \frac{a' b}{v}}{1 - \frac{a'}{v} + \frac{a' b}{v^2}}.$$

Let $\frac{b}{v}$ be taken equal to $\frac{1}{25}$, — and as at this temperature $a' = 4b$ we find

$$v' - v = b \frac{\frac{8}{25} - \frac{4}{25}}{\left(1 - \frac{2}{25}\right)^2},$$

so a value which is not much higher than the limiting value. If

we follow the border-curve from $T = T_k$ down to very low temperatures, at which the vapour volumes are very large, a large proportion may be found in the beginning, but very soon that proportion must become nearly equal to unity.

By means of the knowledge of $v' - v$, we may conclude to the course of Δ_v , the increase of volume when two substances mix under constant pressure. If we call the molecular volume of the first component under a given pressure v_1 , of the second component v_2 and of the mixture v , then v'_1 and v'_2 and v' are equal to $\frac{MRT}{p}$ and so equal to each other.

For $\Delta_v = v - (1 - x)v_1 - x v_2$ we may therefore write

$$\Delta_v = (1 - x)(v'_1 - v_1) + x(v'_2 - v_2) - (v' - v),$$

and so we find Δ_v from the values of $v' - v$ for each of the substances.

For $v_1 = v_2 = v = \infty$ we find

$$\Delta_v = (1 - x)(a'_1 - b_1) + x(a'_2 - b_2) - (a'_x - b_x).$$

If we put $(1 + a_1)(1 - b_1)$, $(1 + a_2)(1 - b_2)$ en $(1 + a_x)(1 - b_x)$ equal to 1, we obtain equation (4) of my former communication (Proceedings Nov. 1898, p. 181). There also the same approximation is applied in equating $p v_1$, $p v_2$ and $p v$. But from the result now obtained, it appears clearly that

$$x(1 - x) \left\{ \frac{a_1 + a_2 - 2a_{12}}{1 + \alpha t} - (b_1 + b_2 - 2b_{12}) \right\}$$

is only the limiting value of Δ_v for infinite rarefaction and we are led to the question, whether something further may be deduced with regard to the course of Δ_v under increasing pressure, and whether it may be possible to account for the fact that the values for Δ_v which may be deduced by means of KUENEN's observations, do not show that symmetry which follows from the factor $x(1 - x)$ and which appears so clearly from this observations of Δ_p .

If we substitute the values found for $v'_1 - v_1$, $v'_2 - v_2$ and $v' - v$ we find

$$\Delta_v = (1 - x) \frac{a'_1 - b_1 - \frac{a'_1 b_1}{v_1}}{1 - \frac{a'_1}{v_1} + \frac{a'_1 b_1}{v_1^2}} + x \frac{a'_2 - b_2 - \frac{a'_2 b_2}{v_2}}{1 - \frac{a'_2}{v_2} + \frac{a'_2 b_2}{v_2^2}} - \frac{a'_x - b_x - \frac{a'_x b_x}{v}}{1 - \frac{a'_x}{v} + \frac{a'_x b_x}{v^2}}$$

in which v_1 , v_2 and v represent the volumes, occupied by the components and by the mixture under the same pressure. As long as we restrict ourselves to large volumes, we may take v_1 , v_2 and v as being equal to each other, and then we find for the second term of Δ_v (which must be divided by v) the value:

$$(1 - x) a'_1 (a'_1 - 2 b_1) + x a'_2 (a'_2 - 2 b_2) - a'_x (a'_x - 2 b_x).$$

The sign of this quantity for the different values of x will decide whether Δ_v , when the value of v is decreasing, will decrease or increase and the value which this quantity possesses, will be decisive as to the degree of this variation. This quantity disappears for $x = 0$ and $x = 1$, and must therefore have $x(1 - x)$ as a factor. If the remaining factor were independent of x , there would again be symmetry in the values of Δ_v , also when the volume decreases. If, however, this factor depends on x , there remains symmetry in the limiting value of Δ_v , but this symmetry must disappear sooner or later, when the pressure increases. The remaining factor has the following intricate form, which I shall give in the five parts, into which it may be divided:

$$(3 - 3x + x^2) a'_1 (a'_1 - 2 b_1) \dots \dots \dots \quad (\alpha)$$

$$+ (1 + x + x^2) a'_2 (a'_2 - 2 b_2) \dots \dots \dots \quad (\beta)$$

$$- 2(1 - x)^2 \left\{ a'_{12} (a'_1 - 2 b_1) + a'_1 (a'_{12} - 2 b_{12}) \right\} \dots \quad (\gamma)$$

$$- 2x^2 \left\{ a'_{12} (a'_2 - 2 b_2) + a'_2 (a'_{12} - 2 b_{12}) \right\} \dots \quad (\delta)$$

$$- x(1 - x) \left\{ 4a'_{12} (a'_{12} - 2 b_{12}) + a'_1 (a'_2 - 2 b_2) + a'_2 (a'_1 - 2 b_1) \right\} \quad (\epsilon)$$

If we put $a'_1 + a'_2 - 2a'_{12} = \Delta_a$ and $b_1 + b_2 - 2b_{12} = \Delta_b$, the sum of these five terms may be brought under the following form:

$$(a'_2 - a'_1) \{ (a'_2 - b_2) - (a'_1 - b_1) \} + 2 \Delta_{a'} \{ (1-x)(a'_1 - b_1) + x(a'_2 - b_2) \} - \\ - 2 \Delta_b \{ (1-x)a'_1 + x a'_2 \} - 2x(1-x)\Delta_{a'}(\Delta_{a'} - 2\Delta_b).$$

So this quantity taken $\frac{x(1-x)}{v}$ times represents the first correction of Δ_v , whereas $x(1-x)(\Delta_{a'} - \Delta_b)$ represents the limiting value of this quantity — so that for v near zero the value of Δ_v is equal to

$$x(1-x) \left\{ (\Delta_{a'} - \Delta_b) + \frac{(a'_2 - a'_1) \{ (a'_2 - b_2) - (a'_1 - b_1) \} + 2 \Delta_{a'} (a'_1 - b_1) - 2 \Delta_b a'_1}{v_1} \right\}$$

and for x near 1 equal to

$$x(1-x) \left\{ (\Delta_{a'} - \Delta_b) + \frac{(a'_2 - a'_1) \{ (a'_2 - b_2) - (a'_1 - b_1) \} + 2 \Delta_{a'} (a'_2 - b_2) - 2 \Delta_b a'_2}{v_2} \right\}$$

So there is a distinct asymmetry, as soon as $a'_2 - b_2$ is sensibly greater than $a'_1 - b_1$, but the different cases that may occur are so numerous, that it is better for the present not to enter into a further discussion, as the experimental data are wanting.

Yet it should be observed that the asymmetry is not so great as to account for the circumstance that Mr. KUENEN's values for $(\Delta_p)^{(1)}$ at $x = \frac{3}{4}$, $x = \frac{1}{2}$ and $x = \frac{1}{4}$ differ so little, while with scarcely an exception the highest value is given at $x = \frac{3}{4}$.

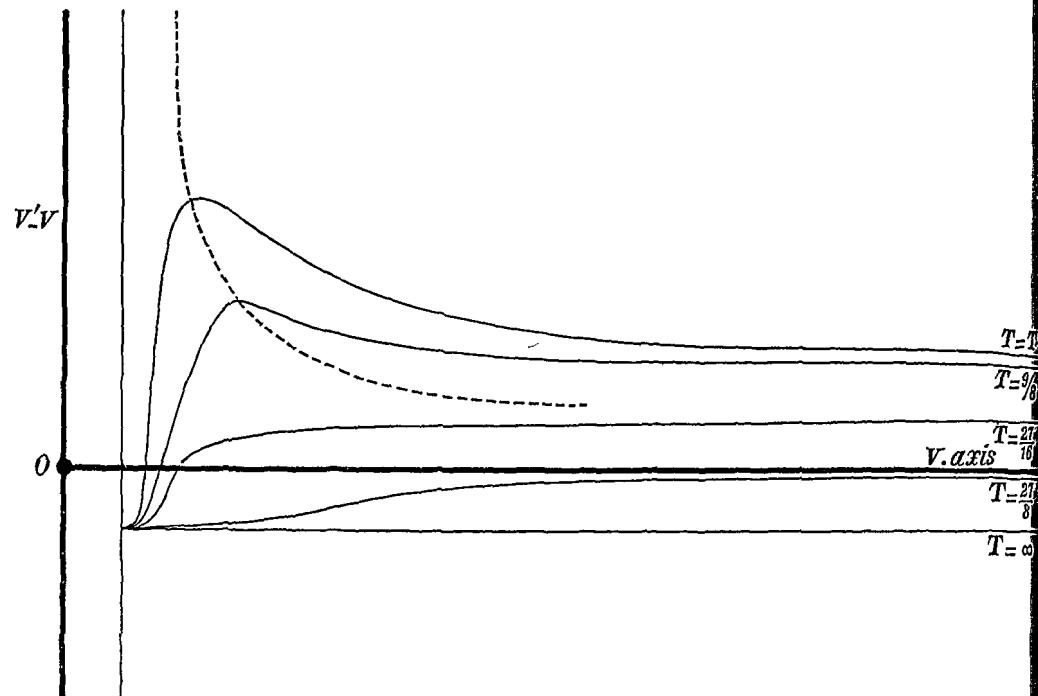
As, however, the accurate value of Δ_v requires too intricate calculations to draw general conclusions from them, we shall have recourse to the graphical representation, in order to give at least an idea of the course of this quantity.

Let us consider for this the formula :

$$\Delta_v = (1-x)(v'_1 - v_1) + x(v'_2 - v_2) - (v'_x - v),$$

from which appears, that Δ_v may be considered as resulting from three separate quantities, viz. $v'_1 - v_1$, $v'_2 - v_2$ and $v' - v$. Each of these quantities, which have been discussed before, may be represented by curves as have been drawn in the following figure.

¹⁾ Proc. Nov. 26th 1893, p. 184.



In this figure v and $v' - v$ have been taken as axes. The origin is in point 0. At $T = \infty$ $v' - v$ is equal to $-b$ for all values of v . In the figure this has been represented by a line parallel to the v -axis. If the variability of b with the volume could also be taken into account, this straight line would naturally have to be changed into a curve, which approaches asymptotically the position given here when the volume is large and which has come sensibly closer to the v -axis at the smallest volume possible. But then every curve had to be modified, specially in the region of the small volumes.

Under an infinite pressure the value of $v' - v$ is equal to $-b$ at all temperatures. Hence all curves pass through the same point. According as the temperature falls the curves begin higher at $v = \infty$. The curves drawn are those for $T = \frac{27}{8} T_k$ (the temperature at which a substance obeys the law of BOYLE, if the volume is infinite); for $T = \frac{27}{16} T_k$ (the limiting temperature at which $v' - v$ begins to show a maximum value), for $T = \frac{9}{8} T_k$ and $T = T_k$.

The maxima lie on an equilateral hyperbola, which has $v = 2b$ and $v' - v = b$ as asymptotes.

When the volume is rather large, the different curves do not differ sensibly from straight lines parallel to the v -axis.

As soon as T descends below T_k , the quantity $v' - v$ can become very great; this happens, however, between such values of v , as lie within the border curve, and which therefore cannot be realized.

If for the two substances, of which the mixture is to be composed, such curves are construed, whose dimension and form will depend on b and $\frac{a'}{b}$, a point of the mapping of $(1-x)(v_1' - v_1) + x(v_2' - v_2)$ may be found by connecting two points of those curves by a straight line, and by dividing that straight line into parts which are in inverse ratio to x and $1-x$. The two points chosen must always belong to values of v_1 and v_2 , which occur under the same pressure. The curve obtained in this way will approach $v_1' - v_1$ the nearer, the smaller x is, and may to some extent be considered as a mean curve. When $v' - v$ has been construed, Δ_v may be found by taking the difference of the ordinate of the resulting curve of the two first mentioned and of the last mentioned, always for such values of v , as occur at given p .

Though it is true that the course of Δ_v does not follow in particulars from this way of construing, yet some general rules may be derived from it. As has been observed before, we find for $v = \infty$ again the former value, viz. $x(1-x)\{\Delta a' - \Delta b\}$. For the limiting value on the other side we find:

$$-\Delta_v = b_1(1-x) + b_2x - b_x = x(1-x)\Delta_b,$$

so a contraction. In all these cases, in which $\Delta a' - \Delta b$ is positive, a pressure must exist, under which mixture without variation of volume takes place. If the volumes are very great so that $v_1' - v_1$, $v_2' - v_2$ and $v' - v$ remain invariable, Δ_v too will not vary much in absolute value. The variation may however be considerable relative to the limiting value. But it appears at any rate, that if none of the three substances, viz. the components and the mixture, remain considerably below the critical temperature¹⁾, the absolute value of Δ_v remains within finite limits throughout the course of the isothermal.

If we try to calculate $(\Delta_p)'$ directly, viz. the increase of pressure on the mixture, in order to keep the volume equal to the sum of

¹⁾ By the critical temperature of a mixture is understood that temperature which is calculated from a_x and b_x in the same way as for a simple substance.

the separate volumes, we find for it the following expression:

$$\begin{aligned} \frac{(\Delta_p')\{v_1(1-x) + v_2x\}}{x(1-x)} &= - \\ &= -\frac{\frac{v}{v-b_x}(b_1+b_2-2b_{12}) + b_1\left(\frac{v_2-b_2}{v_1-b_1}-1\right) + b_2\left(\frac{v_1-b_1}{v_2-b_2}-1\right)}{(v_1-b_1)(1-x) + (v_2-b_2)x}(1+\alpha t) + \\ &\quad + \frac{a_1\frac{v_2}{v_1} + a_2\frac{v_1}{v_2} - 2a_{12}}{v_1(1-x) + v_2x}. \end{aligned}$$

For volumes which are not too small, which do e.g. not descend below 0,03, we might put by approximation

$$(\Delta_p') = \frac{x(1-x)}{[v_1(1-x) + v_2x]^2} \left\{ \left(a_1\frac{v_2}{v_1} + a_2\frac{v_1}{v_2} - 2a_{12} \right) - \right. \\ \left. - (b_1 + b_2 - 2b_{12})(1 + \alpha t) \right\}.$$

If v_1 and v_2 are left the same and only the value of x is changed, $(\Delta_p)'$ is a maximum if $\frac{x}{1-x} = \frac{v_1}{v_2}$. This leads to the rule, that the maximum value of $(\Delta_p)'$, which for small values of p is found in mixtures for which $x = 1/2$, passes, when the pressure increases, towards mixtures, in which that component is in excess which is most compressible. As Δ_v reverses its sign, when the volume is very small, this must also be the case for $(\Delta_p)'$.

If $a_1 - b_1(1 + \alpha t) = 0$, the first component follows the law of BOYLE. In the same way the second component, if $a_2 - b_2(1 + \alpha t) = 0$. If $a_{12} - b_{12}(1 + \alpha t) = 0$ the law of DALTON holds good. And finally if $(a_1 + a_2 - 2a_{12}) - (b_1 + b_2 - 2b_{12})(1 + \alpha t) = 0$, there is no variation of volume, and the law of AMAGAT holds true. All this only in the supposition of large gasvolumes. Let us call the temperatures, at which these four relations are fulfilled: t_a , t_b , t_c and t_d . If $t_a > t_b > t_c$, then $t_d > t_c$. The supposition $t_a > t_b > t_c$ is fulfilled, if the two components can form mixtures, the critical temperature of which lies below that of the components¹⁾. Then t_c is the lowest of these four special temperatures.

¹⁾ Moleculartheorie. Phys. Chem. V. 2, p. 149.

On the other hand t_c is the highest of these four temperatures, if there should exist mixtures, the critical temperature of which is higher than that of the components. In general the following relation exists between these four temperatures:

$$b_1 t_a + b_2 t_b = 2 b_{12} t_c + (b_1 + b_2 - 2 b_{12}) t_d .$$

If we call t_m the temperature that fulfils the condition that

$$(b_1 + b_2) t_m = b_1 t_a + b_2 t_b ,$$

t_m lies always between t_c and t_d , while the distance between t_m and t_c is smaller than that between t_m and t_d .

Physics. — „Measurements on the course of the isotherms in the proximity of the plaitpoint and particularly on the course of the retrograde condensation in a mixture of carbonic acid and hydrogen.” By Dr. J. VERSCHAFFELT (Communication №. 45 from the Physical Laboratory at Leiden, by Prof. H. KAMERLINGH ONNES.)

(Will be published in the Proceedings of the next meeting.)

Physics. — „Measurements on the magnetic rotatory dispersion of gases.” By Dr. L. H. SIERTSEMA (Communication №. 46 from the Physical Laboratory at Leiden, by Prof. KAMERLINGH ONNES.)

(Will be published in the Proceedings of the next meeting).

KONINKLIJKE AKADEMIE VAN WETENSCHAPPEN
TE AMSTERDAM.

PROCEEDINGS OF THE MEETING
of Saturday January 28th 1899.

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The following papers were read:

Geology. — „On Brackish-water deposits of the Melawi in the interior of Borneo”. By Prof. K. MARTIN.

It is more than forty years since the mining engineer R. EVERWIJN's geognostical expeditions in the „Wester-Afdeeling” of Borneo. Since that time several explorers have been occupied with the geology of that country and nevertheless all our present knowledge of the geological formation of West-Borneo is still based on incoherent data only of which we do not even possess a rough geognostical sketch map.

During the last years the mining engineer N. WING EASTON has been intrusted to make a closer investigation of this country and

from the examination of the fossils forwarded by him to Leyden it was possible to point out inter alia the existence in Borneo of a jurassic system¹⁾.

His last consignments from the basin of the Kapoeas contained sedimentary rocks with hundreds of fossils upon which I wish to give some preliminary information. Of the objects to which I wish particularly to draw attention only a few have been collected at the main river somewhat below the mouth of the Melawi whereas the greater portion were found in the region of the last named important left branch of the Kapoeas. The localities are situated either on the said Melawi or on its tributaries the S.²⁾ Kajan and the S. Tebidah which flow into the S. Kajan. Taken altogether the fossils have been collected in ten different places and an examination of important material containing numerous organic remains from most of these localities was possible. This examination however proved to be rather difficult, the objects being as a rule exceedingly brittle thus requiring to be prepared with the greatest care which could only be done with the aid of a needle.

The fossils are embedded in clay some of which contains calcium carbonate and turns into marl; in some cases they also form the organic centres of marl concretions. In other cases the clay becomes sandy and often it serves only as the cementing material which keeps the thick mass of fossils together. Shell-breccia are numerously represented.

Among many hundred specimens of shells and periwinkles obtained from this material, only seven species can be named viz. *Arca melaviensis spec. nov.*, *Cyrena subtrigonalis* P. G. KRAUSE, *Cyrena subrotundata* P. G. KRAUSE, *Corbula dajacensis* P. G. KRAUSE, *Melania melaviensis spec. nov.*, *Paludomus gracilis* P. G. KRAUSE spec., and *Paludomus crassa* P. G. KRAUSE spec., but the greater part of these species vary so much as to render the determination very difficult. The material collected by G. A. F. MOLENGRAAFF at the upper Kapoeas especially at the S. Pinoh and the S. Lekawai (also branches of the Melawi) on being examined by P. G. KRAUSE was found to contain also the above mentioned species; but the material at his disposal for his researches being defective in several respects, he has not been successful in giving a complete determination of some species (*Arca* and *Melania*) while to others he has given different generic names (*Paludomus*).

Having compared the petrifications of the different localities in the

¹⁾ See: Sammlg. d. Geol. R. Museums in Leiden. Bd. V. (KRAUSE, MARTIN, VOGEL)

²⁾ S = Soengai, Malayan name for river.

collections of MOLENGRAAFF and WING EASTON it becomes evident that in the basin of the McLawi a deposit occurs which taken as a whole is of the same age. This deposit evidently corresponds with the formation stated by EVERWIJN to be tertiary, his statement however was not sufficiently proved. The fauna of these beds exhibits different facies which is due in all probability to the quantity of salt having varied during the process of formation and also having been present in greater or lesser quantities in different localities. A number of species (*Melania* and *Paludomus*) living in fresh water have been found amalgamated with the species of *Cyrena* and *Corbula* which are prevalent amongst these fossils and were inhabitants of brackish water as also the species of *Arca* which may have lived in close proximity to the sea. This amalgamation would chiefly take place during the rainy season.

It is exceedingly difficult to settle the age of the strata in regard to the character of the fauna as sketched above. A direct determination based solely on the occurrence of the species is impossible as none of the above mentioned species corresponds with any described species either from India or the Malay Archipelago. Neither is it of any use to compare these fossil species with the terrestrial shells of Nias described by WOODWARD and which up to the present time have been considered by others to be tertiary. During a recent visit to London I have had an opportunity of studying these Nias forms and feel assured that they decidedly are not tertiary but are, as WOODWARD supposed, of a more recent date.

A comparison with the „intertrappean beds” of India, which constitute a connecting link between cretaceous and tertiary, seems to be of the greatest importance. Their fauna still allows relations to be traced to the Laramie-group of North America and similar relations also exist between the Laramie-group and the upper cretaceous of Ajka in Hungary; consequently this fauna was not from a geographical point of view so well differentiated in those days as at the present time. The fauna of the McLawi beds does not show any features to correspond with that of the „intertrappean beds” the Laramie-group etc., on the contrary containing only still living Indian genera amongst which is the genus *Paludomus* it resembles very closely in character that of the present day.

Separating from *Paludomus* the genera *Pyrgulifera* Meek and *Cosinia* Stache which certainly do not belong to it, there remains only a single group which must be considered one of the most characteristic of the Indian fauna. The occurrence of the genus *Paludomus* is restricted to this area, being found in India, Further

India, Sumatra and especially in Ceylon and Borneo. Amongst the still living Bornean forms there are found some species closely allied to the fossil ones; *P. gracilis* in particular closely resembles *P. Everettii* Smith from Sarawak.

Setting on one side its dissimilarity to the fauna of the „inter-trappean beds” and on the other its resemblance in one highly characteristic point to the fauna of the present day we may conclude that the Mélawi deposit must be of more recent date than the said Indian strata, which constitute a transitional formation between cretaceous and tertiary. For the above reasons the Melawi deposit cannot be older than tertiary but may be of eocene age as, not one of the seven aforesaid species is known to belong to the present fauna and may be considered as extinct, the habitus of two of these species (*Melania melaviensis* and *Cyrena subtrigonalis*) is moreover somewhat aberrant when compared with still living forms. Meanwhile a more exact determination of its age than „tertiary” cannot be given with any certainty and it is for this reason and to avoid all misunderstandings that I propose to call this deposit by the generalizing name of „Mélawi group” in reference to the region where it has originally been found.

From the material which has been examined, the Mélawi-group can be traced from the S. Tempoenak below Sintang eastwards along the Mélawi as far as the S. Lekawai and also on the S. Kajan to a certain point somewhat above Maboeck¹⁾. This tertiary (eocene?) brackish-water sediment can only have been formed with the assistance of rivers and brooks which perhaps belonged to the same river system that is now represented by the Mélawi and its tributaries and accordingly the beds of these streams may have been cut out of the sediments which they themselves have contributed to form.

1) At present it is impossible to trace a further extention of the Mélawi-group. I can only state that I have also received through the agency of WING EASTON a consignment of petrifactions from the river Silat (flowing into the Kapoeas near Silat above Sintang) which contains a fauna differing from that of the Mélawi-group.

Particularly numerous amongst which are the remains of *Vivipara* which is represented by at least two different species and there also occurs a species of *Corbula* (s. str.).



The latter seems to be quite distinct from *C. dajucensis* while amongst the species of *Vivipara*, a highly characteristic form is found which I propose to name *V. Eastoni*. The figures of two specimens of this new species are inserted here.

A very remarkable characteristic of this species is the presence of sharp and prominent keels, one of which situated at the angle of the whorls projects very strongly.

It is possible that this deposit of the river Silat is older than the Mélawi-group, but certainly it is not older than cretaceous.

Chemistry. — „On Hydrogel of oxide of iron (ferric-oxide)“.
By Prof. J. M. VAN BEMMELLEN.

In the Recueil des Travaux Chimiques des Pays-bas¹⁾ Mr. SPRING stated some time ago that he had prepared a hydrate of oxyd of iron: d'une composition définie. He had got it by allowing to dry — by exposure to the air — the gelatinous precipitate produced by ammonia in a diluted ferri-solution and that after having washed it out. After 72 days the equilibrium became stable, and after 5 months it was found to be unchanged. It answered to the composition $\text{Fe}_2\text{O}_3 \cdot 4\text{H}_2\text{O}$. Above sulphuric acid it lost water so that after 3 days the composition was $\text{Fe}_2\text{O}_3 \cdot 1.78\text{H}_2\text{O}$ (calculated by me).

Formerly, in 1888²⁾ and 1892, I have communicated, that the gelatinous oxide of iron had no stable composition, but was an absorption-alloy of Fe_2O_3 with water. I had also found the number $\pm 4.0\text{H}_2\text{O}$ in one of my preparations, but I considered it as an alloy, that only answered to a whole number by chance. The opinion that a fixed hydrate had been obtained, is contrary to the nature of hydrogels. For the composition is continuously dependent on: 1st. the structure of the colloid, which in its turn depends upon the circumstances of the gel-formation; 2nd. the modifications that the structure has undergone by the further treatment; 3rd. the concentration of the gaseousphase, with which the gel co-exists; 4th. the temperature.

Owing to my having determined the composition only with a few vapour-tensions, SPRING's communication led me to determine the entire isotherm at 15° viz. of dehydration of rehydration and of redehydration and to examine carefully how the Hydrogel behaves when exposed to the air, the vapour-tension of which is continually changing.

Accordingly I prepared again a Hydrogel from a highly diluted ferric-solution³⁾ and worked with it at once, while it was still quite fresh. Besides I determined the same curves for the preparation already investigated by me in 1882, which had now grown 16 years older (III); and in the third place for a preparation made in 1891, which had been under water for seven years (II). These determinations were made in the same manner as formerly for the hydrogel of SiO_2 and of CuO ⁴⁾. On the graphic sketch I have

¹⁾ 18.222 (1898).

²⁾ Recueil VII 106—114. J. f. prakt. Ch. 46—529.

³⁾ 1 Part Fe_2O_3 on 66 p. water = 1 Mol. Fe_2O_3 on 585 p. water.

⁴⁾ Vide Verslag Kon. Akad. of 28 Janr. 1893 and 29 June 1895. At present the equilibria are determined at vapour-pressures of 12°—11°—10°—10—9°—8°—7°—6°—5—4°—2—0° m.m. both for A and for Z↓ and Z↑.

made use of the same signs (see fig.) viz. $A \downarrow$ for the curve of dehydration of water, $Z \uparrow$ for that of the rehydration of water and $Z \downarrow$ for that of the redehydration of water. The equilibrium at a certain vapour-tension was only gradually approximated on $A \downarrow$, the more slowly in proportion to the latter's differing less from the tension of saturated vapour (12.7 m.m.). At 12² m.m. a loss in 1 month's time was still noticeable after 5 months; at 9 m.m. the loss in 1 month's time was no longer noticeable after 3 months; and at 0° m.m. after one week. On the contrary the equilibrium in $Z \uparrow$ and $Z \downarrow$ was obtained within 1—2 days, except on part of $Z \downarrow$ (from point O to O₁) where the phenomenon of Hysteresis showed itself again.

From the figure we see that the curves agree with those for the Cu O- and the Si O₂-hydrogel.

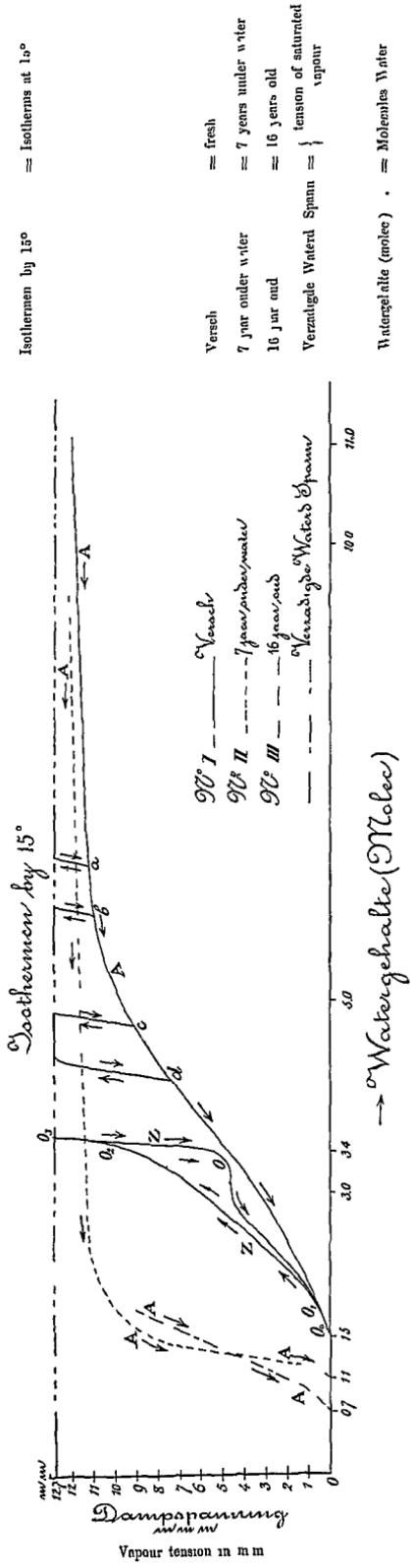
The proportion of water at 15° is greater than that of these two (I only give here that at 12² and at 0 m.m. vapour-tension).

	12 ² m.m.	0° m.m.
Fe ₂ O ₃	± 9.5	1.5 ¹
Cu O	± 7	1.1
Si O ₂	± 6	0.2 ¹

but is equally continuously dependent on the vapour tension.

Point O, which shows a break in the continuity in silicic acid (in consequence of a transformation in the gel and the diminution of the power of absorption resulting therefrom) and at the same time the turnpoint of the process of de-hydration, this point is not so clearly seen in the hydrogel of Fe₂ O₃. But it betrays itself in the following manner. If the gel, arriving in any point of the line $A \downarrow$, is provided again with water from this point (by the exposition to a higher vapour-tension up to 12⁷ m.m.) it only absorbs ± 0,2 H₂O; comp. for this on fig. 1 the lines $A \uparrow$, starting from the points a, b, c, d. This rule however only holds good if the proportion of ± 3,5 H₂O is not yet reached; below this the gel absorbs in case of rehydration so much water, that the proportion in tension of saturated vapour amounts to ± 3,6⁵. It is therefore about here that the possibility of reversing the further process of the dehydrations begins, and there is a point which answers in that respect to the turningpoint O in silicic acid. At the end of the dehydration in point O₀ (vapour-tension = 0° m.m.) the appearance of air-bubbles

J. M. VAN BENMELLEN, On Hydrogel of Oxide of Iron (ferric oxide).



on the immersion in water is only observed in a slight degree¹⁾. It must therefore be inferred that but few cavities are formed during the dehydrations on the branch O—O₀. This phenomenon is however much better noticed in the gel II (see below). The final curve of the rehydration Z↑ again has the points O₁ and O₂, which appear at the redehydration of water as in the case of silicic acid. For the rehydration curve Z↓ does not converge, neither with Z↑ nor with A↓. It is true that the difference for the branches O₃O₂ and O₁O₀ is slight, but just as with silicic acid, the branch O₂O is vastly divergent from O₁O₂. The same hysteresis-phenomenon appears. Line Z↑, especially the branch O₁O₂ can only be realized in the direction ↑ (rehydration), the line O O₁ in the direction ↓ (dehydration). An intensified power of absorption is created on Z↑, that leaves its influence on Z↓. At the redehydration (Z↓) a point O appears, showing the same phenomena as in silicic acid. Starting from this point O₁ the equilibrium is obtained very slowly on the branch O O₁; a weakening of the power of absorption takes place, that leaves its influence behind, so that at the rehydration it is not the branch O O₁ that is run back, but the branch O₁O₂. As a result, here as with silicic acid return-curves may be realized within the figure O O₁ O₂ O, if rehydration or redehydration takes place, starting from a point on the line O O₁ or on the line O₁O₂. Two of these intermediate curves, one of the dehydration ↓ and one of the rehydration ↑, are indicated in the figure by dotted lines.

Modifications. The modification caused by time, by which the power of absorption is weakened, is also the same as with silicic acid. The figure shows this for a Gel, that has grown 16 years older (curve A↓ of III). The proportions are lower again than were found before, when the same substance was seven years older than at the first determination. The fact of the Gel having been under water for seven years has also weakened the power of absorption (curve A↓ of II). For the sake of clearness the curves Z↑ and Z↓ have been omitted. They differ very little from the A↓ line, and only diverge from it at higher vapour-tensions (at about 10—11 m.m.), as is shown in the following table:

¹⁾ In silicic acid many cavities are formed, which condensate a considerable volume of air. Vide Verslag Kon. Akad. of April 6, 1898, p. 498—506.

m.m.	III (16 years old).		II 7 years under water.	
	Z ↑	Z ↓	Z ↑	Z ↓
	Mol H ₂ O		Mol H ₂ O	
9	1.5 ⁶	1.5 ⁶	1.5	1.5
10	1.6 ⁴	1.6 ⁶	1.6 ⁶	1.7 ¹
10 ⁷	1.7 ⁴	1.7 ⁷	1.8 ⁶	1.9 ⁶
11 ⁶	2.2 ¹	2.4 ⁷	3.0 ²	4.5
12 ²	3.3 ⁶	3.5 ¹	6.7	7.9 ⁶
12 ⁷ saturated	$\pm 8.5^1)$		8.6 ²⁾	

¹⁾ Obtained in the course of some months.

²⁾ After 8 days.

The same phenomenon presents itself as with silicic acid. The more it is modified, the more water it can absorb at the higher vapour-tensions. The hysteresis is removed to that region. The Gels II and III at rehydration can absorb up to more than 8 Mol. H₂O, while I can come no further than to $\pm 3.6^5$.

It is highly remarkable that III requires months for it, II only days. This too points to a difference in structure. Now it is very remarkable that Gel III does not show any air-bubbles escaping at the point O₀, when immersed in water, while Gel II does. In Gel II there appear immediately on the surface and at the rims of the transparent (so exceedingly thin) films at numerous places smaller and larger air-bubbles. Consequently the gel contains cavities, but their joint volume is not by far so great as in silicic acid. In connection herewith seems to be that II absorbs pretty quickly a considerable amount of aqueous vapour at a vapour-tension of 12 m.m., while III, which contains no cavities, does this so extremely slowly.

It results from the above that no chemical hydrate can be obtained by drying the Gel by exposure to the air, and that moreover the composition must change with the change in the vapour-tension of the air. This was also proved by daily weighing a quantity of I exposed to the air (freshly prepared and pressed between two porous plates) for half a year. In the beginning it contained ≈ 18 Molec. H₂O on 1 Molec. Fe₂O₃ and contained therefore much water inclosed. With decreasing velocity the proportion had fallen after 9 days to 5.4 H₂O; from that period it came sensibly under the influence of the changes in the vapour-tension of the air, so that it decreased irregularly in weight and sometimes increased. After 1 $\frac{1}{3}$ month the

proportion had fallen to 4,64 H₂O, and for the next three months this proportion varied according to the vapour-tension of the air, between 4,64 and 4,55 H₂O. Unaltered weight, rising or falling were found to agree with the indications of the hygrometer (of KLINKERFUES). Meanwhile a little decline was observable. If the vapour-tension of the air, which in that time varied between 10 and 8 m.m. had fallen to 7 m.m. and had remained stationary for some time, the composition \pm 4 H₂O would have been attained. For this amount has been obtained in the Gel I after some months at *that* vapour-tension. That composition would have risen but little if the vapour-tension of the air had become again greater.

That the composition depends on the temperature, that the power of absorption is weakened by heating to higher temperatures, and is destroyed at incandescence, all this I have already communicated before ¹⁾.

Dr. KLOBBIE and I have only succeeded in observing the transition of colloidal ferric oxide to a crystalloidal chemical hydrate in case of monohydrate²⁾, by the influence of water at 15° on the hexagonal crystals of the chemical compound Fe₂O₃. Na₂O, in which Na₂O is gradually replaced by H₂O without the crystalline structure (form, transparency, power of polarisation) being modified. Nature provides us with the crystalline Monohydrate: Göthit, that can bear heating to \pm 280° without losing water and without passing into the amorphous state ³⁾.

Chemistry. — MR. VAN BEMMEL presents in the name of MR. B. DE BRUIJN a paper on an investigation held in the Inorganic Chemical Laboratory of the University of Leyden, concerning : „*The equilibrium of systems of three substances, in which two liquids occur*”.

In systems of three components, in which two liquid phases occur, the following cases may be distinguished :

1st. the components A and B form together two liquid phases; equally so the components A and C and B and C.

2nd. the components A and B and A and C form two liquid phases, but the components B and C do not, at least in stable conditions.

¹⁾ Recueil 7. 111—113. Zeitschr. anorg. Ch. 18. 24.

²⁾ J. f. pr. Ch. 46. 523—529.

³⁾ J. f. pr. Ch. 46. 521.

3rd. only the phases A and B form two liquid phases ; A and B or B and C do not do so, at least in stable conditions.

4th. the components in pairs never give two liquid phases at least in a stable condition.

Examples of the first three cases have been investigated by SCHREINEMAKERS ¹⁾; purpose of this investigation was to examine also an example of the last mentioned case. Not until the investigation was nearly finished, did there appear an essay by SNELL on potassiumchloride, acetone and water, by which an example of the fourth case is given ²⁾.

Mr. DE BRUIJN examined the equilibria in the following systems:

Ammoniumsulphate, ethylalcohol and water.

Potassiumcarbonate, methylalcohol and water.

Potassiumcarbonate, ethylalcohol and water.

Sodiumsulphate, ethylalcohol and water.

Guided by the theory of SCHREINEMAKERS on equilibria in systems of three components, in which two liquid phases occur, he succeeded in making a sketch of the equilibria in the systems examined. The composition of the phases is represented in the usual manner by means of a triangle; if the temperature-axis is placed vertically on the plane of the triangle, we get a representation in space.

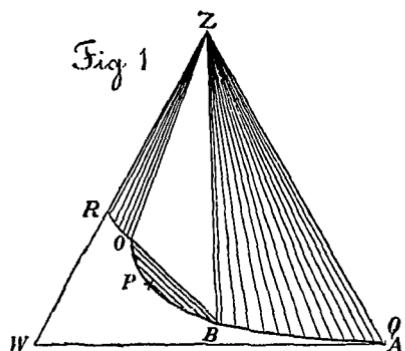
I. The system: ammoniumsulphate, ethylalcohol and water.

Above $\pm 8^\circ$ the general form of the isotherms is as given in fig. 1. The isothermal consists namely of three parts : RO, OPB and BQ, meeting in O and B in an angle. RO and BQ indicate all possible solutions, that can be in equilibrium with the solid $(\text{NH}_4)_2 \text{SO}_4$. OPB is the connodal line with the plait-point P; it indicates therefore the solutions that coexist; the points O and B are the two liquid phases which are not only in equilibrium with each other, but also with solid $(\text{NH}_4)_2 \text{SO}_4$. In case of a change in the temperature the different parts of the isotherm are displaced; at a higher temperature $\angle OZB$ increases; at a fall in the temperature $\angle OZB$ diminishes till at $\pm 8^\circ$ the lines OZ and ZB coincide. The points O, B and P coinciding at this temperature, the isothermal consists at this and lower temperatures of only one curve RO and BQ coming in each other's prolongation; there remain therefore only the equilibria of solid salt with solution. Mr. DE BRUIJN has determined such an isothermal at $6,5^\circ$. Two liquid

¹⁾ Zeitschr. f. phys. Chem. 25 543 26 237 27 95 (1898) 23 417 (1897).

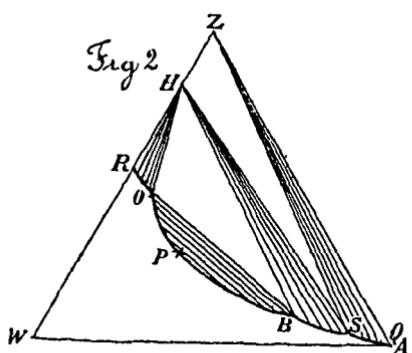
²⁾ Journ. of phys. Chem. 2 457 (1898).

phases are no longer possible in a stable equilibrium. It is true that Mr. DE BRUIJN has noticed them several times, but they disappeared at the appearance of the solid phase.



Mr. DE BRUIJN has also determined several isotherms at higher temperatures, where the isotherms have forms as in fig. 1. As to the connodal lines for the different temperatures, it was proved, that they intersect each other in different manners or cover each other within the limits of the faults of analysis. In order to further investigate this point several sections were determined, in which the proportion of water and alcohol was the same, but the quantity of $(\text{NH}_4)_2\text{SO}_4$ varied. In accordance with the situation of the connodal lines with respect to each other Mr. DE BRUIJN found the following facts. If the proportion of $(\text{NH}_4)_2\text{SO}_4$ is taken for the ordinate and for the abscis the temperature at which two liquid phases become homogeneous or a homogeneous liquid phase is divided into two others, we get, with a greater proportion of alcohol, lines rising with the temperature; with a smaller proportion of alcohol, lines falling with the temperature, between the two, lines showing a maximum, which moves toward a higher temperature with a greater proportion of alcohol. From these different sections follows in accordance with the isotherms given, that it is possible to have solutions of $(\text{NH}_4)_2\text{SO}_4$ in water and alcohol that are homogeneous at a certain temperature, but divide into two layers both in case of a rise and of a fall in the temperature. Mr. DE BRUIJN has made several of these solutions.

II. The system: Potassiumcarbonate, methylalkohol and water.



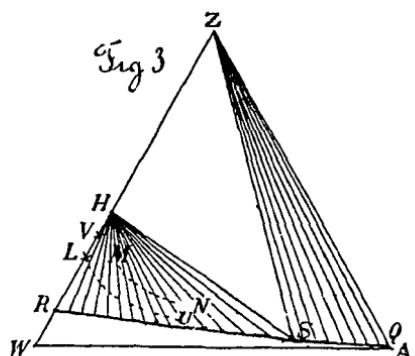
Next to the two liquid phases there appears in this system as a solid phase the hydrate $(\text{K}_2\text{CO}_3)_2(\text{H}_2\text{O})_3$ (at lower temperatures perhaps a still higher hydrate). The isotherm has here the form of fig. 2. RO and BS indicate the solutions that can be in equilibrium with $(\text{K}_2\text{CO}_3)_2(\text{H}_2\text{O})_3$; SQ the solutions in equilibrium with

K_2CO_3 ; OPB is the connodal line with the plait-point P; with each liquid phase of part OP one of PB can be in equilibrium. Mr. DE BRUIJN has determined the situation of the connodal-line OPB at different temperatures. Just as in the preceding system the points O and B approach each other when the temperature is lowered; in the preceding system they met at $\pm 8^\circ$; in this system however no earlier than at $\pm -33^\circ$.

III. *The system : Potassiumcarbonate, ethylalkokol and water.*

The isotherm has here the same form as in the preceding system. The composition of the two liquid phases in equilibrium with the hydrate was determined at temperatures of -18° to 75° . The proportion of salt rises with the temperature, that of alcohol changes less than the amount of the faults of analysis. That a change in the composition of the two liquid phases does however decidedly take place when the temperature is changed may be deducted from the observation, that such homogeneous solutions divide into two liquid phases at a change in the temperature. Different pairs of conjugated points were determined on the connodal line at 17° and at 35° . The connodal lines for different temperatures intersect here too. Sections, as in the case of ammoniumsulphate are not determined here; from a single observation made by SNELL it may be deduced that these sections will not show a maximum here but a minimum.

A great difference in the conduct of methyl- and ethylalcohol is shown by what follows. At 17° the upper layer contains in equilibrium with the solid phase 91,5 pCt. ethylalcohol and 0,06 pCt. K_2CO_3 ; the lower-layer 0,2 pCt. ethylalcohol and 55,2 pCt. K_2CO_3 . The ethylalcohol and the potassiumcarbonate form therefore each of them, with part of the water, a liquid in which the other component hardly occurs. In the system K_2CO_3 , CH_3OH and H_2O the upper layer contains however at the same temperature 69,6 pCt. methylalcohol and 6,25 pCt. K_2CO_3 , the lower layer 5,7 pCt. methylalcohol and 48,4 pCt. K_2CO_3 . Another great difference in the conduct of methyl- and ethylalcohol in these systems is the following. In the system with ethylalcohol the temperature has hardly any influence on the position of the points O and B; the proportion of water and alcohol remaining in two liquid phases, that are in equilibrium with the solid hydrate of K_2CO_3 , nearly unaltered from -18° to $+75^\circ$. In the system with methylalcohol the temperature has on the other hand a great influence on the proportion of water and methylalcohol, the latter being more miscible in presence of the solid salt than water and ethylalcohol.

IV. *The system sodiumsulfate, ethylalcohol and water.*

In this system Mr. DE BRUIJN investigated besides the stable equilibria also some less stable ones. Fig. 3 indicates the general form of the isotherms below $32^{\circ}5$; the drawn curves RS and SQ show the stable equilibria, the dotted curves LU and MN the less stable ones. RS indicates the solutions that can be in equilibrium

with solid $\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$; SQ the solutions in equilibrium with $\text{Na}_2\text{SO}_4 \cdot \text{S}$. S is the solution in equilibrium with $\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$ and Na_2SO_4 . At a rise in the temperature the point S moves to the left and upwards and at $32^{\circ}5$ joins R in V, this point indicating the composition of the aqueous solution that can be in equilibrium with solid $\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$ and Na_2SO_4 . When the temperature rises, the line RS therefore dwindles, while SQ is prolonged; at $32^{\circ}5$ RS disappears and only SQ is left, while S falls on the line WZ. Mr. DE BRUIJN has determined these isotherms for different temperatures experimentally.

Besides the stable equilibria mentioned above viz. $\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O} +$ solution and $\text{Na}_2\text{SO}_4 +$ solution Mr. DE BRUIJN also determined the less stable system $\text{Na}_2\text{SO}_4 \cdot 7\text{H}_2\text{O} +$ solution. The solutions in equilibrium with the hydrate $\text{Na}_2\text{SO}_4 \cdot 7\text{H}_2\text{O}$ are represented by line LU (fig. 3). The latter being situated entirely above the curve RS, the solutions pass into a solution of RS, if the less stable condition ceases to exist.

If it was possible for two liquid phases in stable condition to appear in the systems of equilibrium mentioned above, this is no longer the case here. But on the other hand it is easy to get these systems, if care is only taken that neither Na_2SO_4 nor $\text{Na}_2\text{SO}_4 \cdot 7\text{H}_2\text{O}$, nor $\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$ can come in contact with the two liquid phases. Consequently Mr. DE BRUIJN has succeeded in determining the connodal lines for different temperatures; care being however taken to prevent the liquid phases to come in contact with the air. If the latter happened or if only a particle of $\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$ was introduced, the crystallisation of $\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$ followed at once, and the two layers passed into one. That this must necessarily be the case is shown by fig. 3, the connodal line MN, determined by Mr. DE BRUIJN being situated not only above RS, but even above LU.

Mathematics. — “*The representation of the Screws of BALL passing through a point or lying in a plane, according to the method of CAPORALI.*” By Prof. J. CARDINAAL.

1. This communication must be regarded as a continuation and enlargement of a lecture delivered at the 70th Congress of the “Gesellschaft deutscher Naturforscher und Aerzte” at Dusseldorf (Sept. 1898) and published in the last “Jahresbericht der Deutschen Mathematiker-Vereinigung”. There the method applied in the following pages has been considered in its relation to the “Theory of Screws” by Sir R. St. BALL, so I think I can suffice by beginning with a few brief indications indispensable for the understanding of the purpose of the communication.

2a. The motion of a body considered here is the motion with freedom of the 4th degree; the screws about which motion is possible form a quadratic complex, consisting of all screws reciprocal to a given cylindroid C^3 .

b. We construct the screws passing through a point P and belonging to the complex by drawing perpendiculars through P to the generatrices of C^3 ; each of these perpendiculars moreover intersects two generatrices of C^3 , equidistant from the middle plane (conjugate lines). The locus of these screws is the cone P^2 .

c. In a similar manner we construct the screws situated in a plane π . They envelop a parabola π^2 .

3. The representation of the rays of a quadratic complex has been treated among others by R. STURM and CAPORALI. We find it inserted at large in Mr. STURM’s “Liniengeometrie”, III, pages 272 – 282. The special complex formed by the screws alluded to belongs to the type treated on pages 438—444. Although the results laid down in the following correspond with those obtained there, as could be expected, there is a great difference in the investigation; this difference can be circumscribed as follows:

1st. The proofs are here deduced immediately from the theory of BALL, whereas with STURM they follow as special cases out of the complex.

2nd. The constructions, more particularly a principal construction, are really executed.

4. Fig. 1 represents the axonometrical projection of a cylindroid, whose construction is understood to be known. The nodal line d coincides with the axis OZ ; we suppose further that the rotation

of the generatrices has begun in the plane $Z O Y$; on the line $O-O$ the maximum pitch g has been measured out; on the following lines $I-1$, $II-2$, etc. the succeeding pitches are now continually measured out from the nodal line, the measuring ceasing in the position where the rotation amounts to 180° , thus the whole height of the cylindroid having been described on both sides of the centre O . To explain this we make use in the first place of the circle, which has served to find the length of the pitch of any generatrix, drawn on half the scale of the principal figure, where the length of any generatrix (e. g. $II-2$) is indicated with its corresponding angle of deviation.

In the second place we see axonometrically constructed the pitch curve projected on $X O Y$ with the projections $O-0$, $O-1$ etc. of the generatrices. Further is drawn the perpendicular $A B$ to $II-4$ passing through A on $a \equiv 1-7$. According to $2b$ this is a screw, so it meets the line $a' \equiv I-5$ conjugate to $1-7$.

Remarks. *a.* The projection of the above mentioned pitch curve lies entirely on one side of the axis $O Y$. Evidently this is half the figure we obtain in constructing the curve with the equation: $\varphi = a + 2r \cos^2 \theta$. It is merely a consequence of our peculiar manner of measuring that by the followed construction only half the figure is obtained.

b. In the constructed figure all the pitch values have the same sign: if this were not the case, the figure of the projection would be changed, the curve half drawn, half dotted would show a double point, so the entire projection a fourfold point. This last has now become isolated.

5. Fig. 2 represents the parallel projection of the principal curve of the representation of CAPORALI and must be considered in connection with fig. 1. On the generatrix a of C^3 a point A has been fixed. Through A the screw $A A' B$ has been drawn, being one of the rays of the pencil through the centre A in the plane $A a' \equiv \alpha$. A , as pole of α , determines with α a linear system of the 3^d order of linear complexes.

With these figures we suppose that C^3 lies in the space Σ , the principal curve in the conjugate space Σ_1 . To any linear complex in Σ a plane in Σ_1 corresponds, to any screw a point. The principal curve is the locus of the points in Σ_1 to which corresponds in Σ not only a single screw but a pencil of screws. According to these conventions and to the indication sub N°. 1

we can pass to the analysis of the principal curve. It consists of the following parts:

a. The conic K^2_d in plane δ_1 , locus of the points corresponding to pencils of screws having a ray in common with the pencil (A, α) and whose planes are parallel to the nodal line d .

b. The conic K^2_u in plane v_1 , locus of the points corresponding to pencils of screws having in common a ray with (A, α) and consisting of parallel rays.

c. The line $l_1 \equiv D_d D_u$, intersecting K^2_d and K^2_u , locus of the points corresponding to pencils of screws whose vertices lie on a' and whose planes pass through a .

Both conics have two imaginary points in common; the planes δ_1 and v_1 are the loci of the points corresponding to screws parallel to d and to those in the plane at infinity. They have not been indicated here.

In the figure have further been constructed the vertices T_g and T_k of the two quadratic cones, determined by K^2_d and K^2_u . The line connecting T_g , T_k meets δ_1 and v_1 in M_d and M_u .

6. Now the forms of Σ_1 corresponding to the cones and parabolas of Σ can be found. It is evident that we shall get curves in Σ_1 . Let us take a vertex P and construct a cone of screws P^2 . The construction gives rise to the following remarks:

a. Let us imagine through (A, α) a zero system, formed by two reciprocal systems of points and planes with the property that any point lies in its corresponding polar plane ("Nullsystem" of MOEBIUS), with a linear complex σ situated in it; the polar plane of P intersecting cone P^2 in two generatrices, the corresponding plane σ_1 intersects in two points the curve P_1^2 corresponding to P^2 ; so P_1^2 is a conic.

b. One screw of P^2 intersects a and a' ; so P_1^2 meets l_1 in one point.

c. One screw of P^2 is parallel to d ; so P_1^2 intersects δ_1 in one point not situated on K^2_d .

d. It is very important to determine how many screws of P^2 belong to the pencils of parallel screws having one screw in common with (A, α) . Let us bring a plane parallel to α through P ; two screws of P^2 , m and n , are parallel to the screws m' and n' of pencil (A, α) ; from this we conclude that m and m' are perpendicular to the same generatrix of C^3 ; so they belong to the same pencil of parallel screws, viz. to a pencil having with (A, α) one screw in common. The same can be said of n and n' ; so P_1^2 intersects the conic K_u^2 in two points.

e. Further we must determine how many screws of P^2 belong to pencils of screws, whose planes are parallel to d , having with (A, α) a screw in common. Therefore it is necessary to determine the points common to screws of P^2 and the screws of (A, α) lying at the same time on a generatrix of C^3 . Let us first consider the section of plane α with C^3 ; it consists of the line a' and a conic A^2 . In the second place we must notice that the feet of the perpendiculars through P on the generatrices of C^3 form a conic B^2 , lying in a plane having moreover with C^3 a right line b in common. Both degenerated cubic curves $A^2 + a'$, $B^2 + b$, intersect in three points. As the lines a' and b cannot intersect, those points lie in such a manner that a' meets once B^2 , b once A^2 and A^2 once B^2 . Now the last point L is the only point of intersection satisfying the above condition; so P_1^2 intersects K^2_d in one point.

7. The curves corresponding in Σ_1 with the screws, enveloping the parabolas in the planes π , are determined in the same way. We shall show this briefly.

a. Out of the pole of π with regard to the zero system through (A, α) two tangents can be drawn to the parabola π^2 . So the corresponding curve π_1^2 is a conic.

b. One screw in π_1^2 meets a and a' ; so π_1^2 intersects l_1 in one point.

c. One screw in π lies at infinity; so π_1^2 meets the plane v_1 in one point, not situated on K^2_u .

d. One screw of pencil (A, α) is parallel to the plane π ; both belong to the same pencil of parallel screws; so π_1^2 has one point in common with K^2_u .

e. The line $\underline{\alpha\pi}$ common to α and π meets C^3 in its point of intersection with a' and moreover in two points M and N . Through M pass two tangents of π^2 . One of these tangents contains a point lying on the generatrix m' of C^3 , conjugate to that on which M is situated; the other one is perpendicular to m itself. The latter tangent determines with the screw AM a plane perpendicular to m ; consequently in that plane there is a pencil having a screw in common with (A, α) ; this pencil belongs to those, whose plane is parallel to d . The same reasoning can be applied to N . Consequently the corresponding conic π_1^2 intersects the conic K^2_d in two points.

8. So the conics P_1^2 and π_1^2 , having four points in common with the curve $K^2_u + K^2_d + l_1$, their number in Σ_1 would amount to ∞^4 ; there being however in Σ only ∞^3 cones and curves of the

complex, there must be a closer definition of the conics. This is also evident from the following. All cones P^2 of the complex belong to zero systems of a special kind. The line connecting P and A defines such a special linear complex of rays; it consists of all rays intersecting PA . All these systems have the point A in common; the net of rays through A contains the rays common to these systems and finally the screws common to all consist of the pencil (A, α) and the pencil through A perpendicular to α . Hence it follows:

All zero systems in which the screws of the cones of the complex are situated have in common the pencil through A perpendicular to α , to which in Σ_1 a point C_d on K^2_d corresponds (compare STURM, III p. 276).

In a similar manner is proved that the pencil of screws common to all zero systems of the curves of the complex is the pencil of parallel screws lying in α , to which pencil a point C_u in Σ_1 on K^2_u corresponds.

9. Now a construction will be deduced to find the points C_d and C_u . It is evidently sufficient if we consider the point of intersection of a plane, in which one of the conics P_1^2 or π_1^2 is lying, with K^2_d and K^2_u ; specially that point of intersection which is not at the same time a point of P_1^2 or π_1^2 , for which we can choose in Σ a special cone or conic.

Let us take P on the nodal line d in the point where the generatrices with maximum and minimum pitch g and k meet. Cone P^2 breaks up into two planes through d ; one plane is perpendicular to g , in which a pencil of screws lies with P as vertex, all screws having a pitch equal to that of k ; the second is perpendicular to k , which also contains a pencil of screws having a pitch equal to that of g . To each of these pencils a right line corresponds in Σ_1 ; the first belongs to the cone T^2_k , the second to the cone T^2_g ; further a screw of the degenerated cone P^2 coinciding with d , the corresponding plane of Σ_1 passes through $D_d T_g T_k$; so it intersects K^2_d still in a point, the point C_d that was to be found.

10. From the preceding we easily deduce the construction of the point C_u . Let us bring plane π through g and k . The parabola π^2 breaks up into two pencils of parallel screws consisting of rays perpendicular to g and of rays perpendicular to k , the first having the pitch k , the second the pitch g . So the corresponding plane in Σ_1 passes through two generatrices of the cones T^2_k and

CARDINAAL, The representation of the screws of BALL passing through a point or lying in a plane, according to the method of CAPORALI.

Cylindroid.

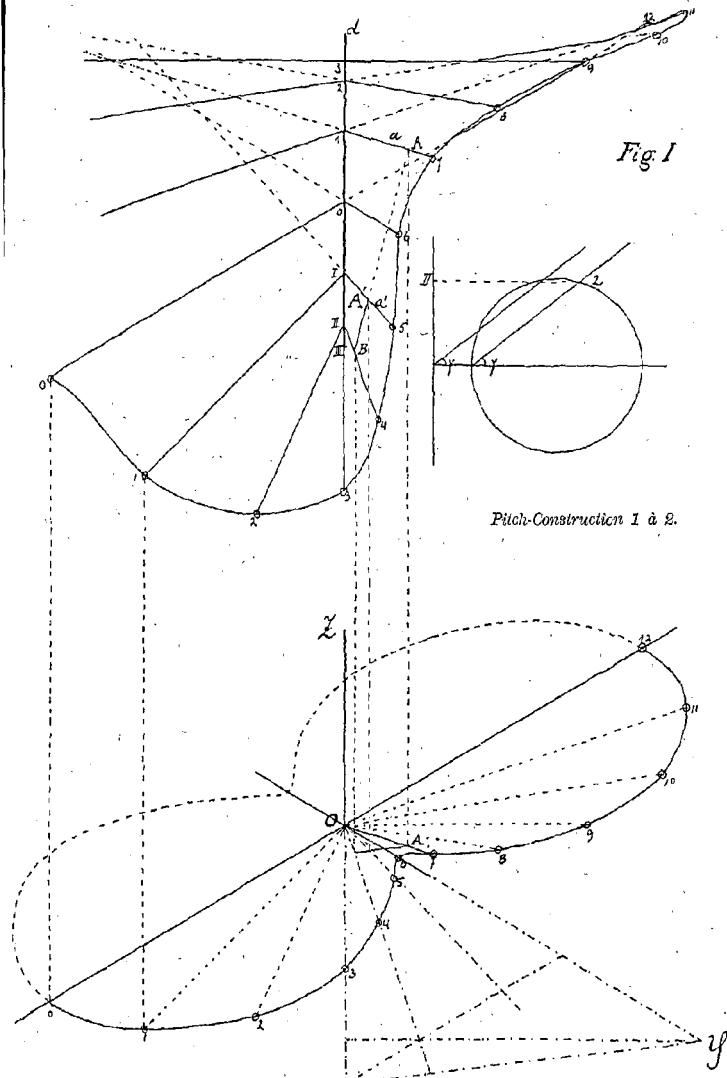
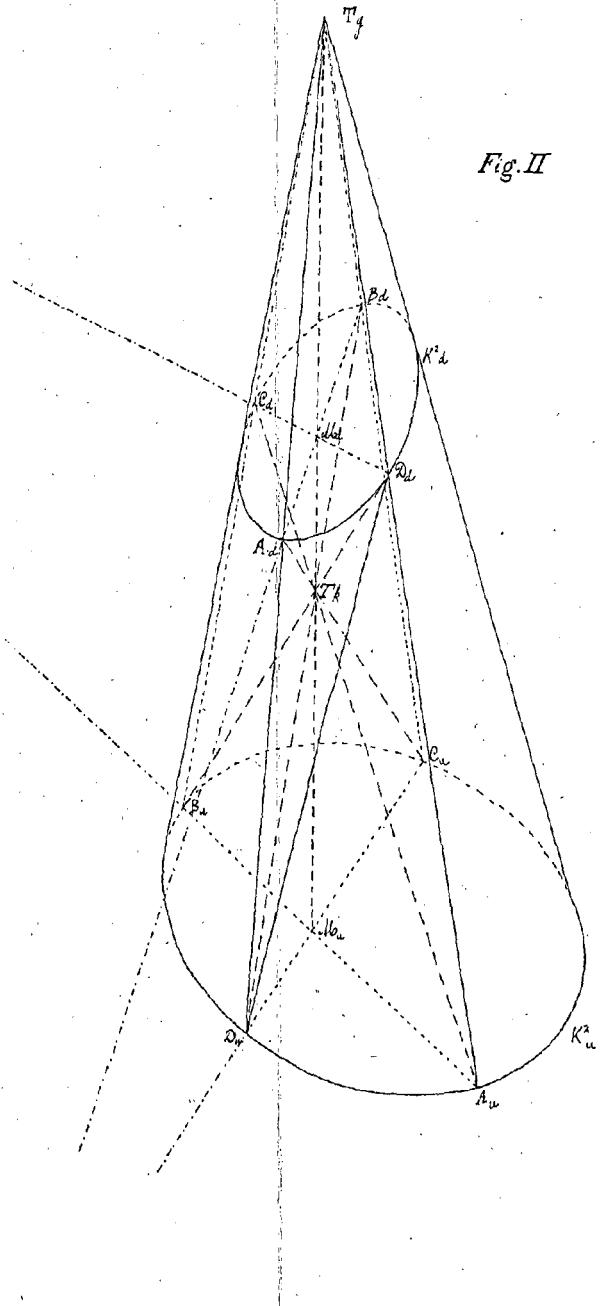


Fig. II



T^2_g and both pencils contain at the same time the ray at infinity of the plane \overline{gk} . The latter is a line through which the bitangential planes of C^3 pass; the point in Σ_1 corresponding to it is D_u . So the point C_u is found by constructing the plane $T^2_g T^2_k D_u$ and determining its point of intersection with K^3_u .

Physics. — „On the vibrations of electrified systems, placed in a magnetic field. A contribution to the theory of the ZEEMAN-effect”. By Prof. H. A. LORENTZ.

(Will be published in the Proceedings of the next meeting.)

Mathematics. — „On Trinodal Quartics”. By Prof. JAN DE VRIES.

1. If we consider the nodes D_1 , D_2 , D_3 of a trinodal plane quartic as the vertices of a triangle of reference, that curve has an equation of the form :

$$\begin{aligned} \Gamma_4 \equiv & a_{11} x_2^2 x_3^2 + a_{22} x_3^2 x_1^2 + a_{33} x_1^2 x_2^2 + \\ & + 2 x_1 x_2 x_3 (a_{12} x_3 + a_{23} x_1 + a_{31} x_2) = 0 . . . (1) \end{aligned}$$

The equations

$$\Phi_2 \equiv b_1 x_2 x_3 + b_2 x_3 x_1 + b_3 x_1 x_2 = 0 , (2)$$

$$\Psi_2 \equiv c_1 x_2 x_3 + c_2 x_3 x_1 + c_3 x_1 x_2 = 0 (3)$$

then represent two conics passing through the nodes.

If the coefficients of these equations satisfy the conditions

$$b_1 c_1 = a_{11}, \quad b_2 c_2 = a_{22}, \quad b_3 c_3 = a_{33}, \quad (4)$$

it is evident from the identity

$$\Phi_2 \Psi_2 - \Gamma_4 \equiv x_1 x_2 x_3 \Sigma (b_1 c_2 + b_2 c_1 - 2 a_{12}) x_3 (5)$$

that the two new couples of points common to Γ_4 and each of the two associated conics Φ_2 and Ψ_2 are situated on the right line corresponding to the equation

$$\Sigma (b_1 c_2 + b_2 c_1 - 2 a_{12}) x_3 = 0 (6)$$

Eliminating with the aid of (4) the coefficients c_1, c_2, c_3 we shall find for the chord common to Φ_2 and Γ_4 the equation

$$\Sigma (a_{11} b_2^2 - 2 a_{12} b_1 b_2 + a_{22} b_1^2) b_3 x_3 = 0 \dots \quad (7)$$

2. If the coefficients b and c are submitted to the two conditions

$$b_2 c_2 = a_{22}, \quad b_3 c_3 = a_{33} \dots \dots \dots \quad (8)$$

only, it results from

$$\Phi_2 \Psi_2 - \Gamma_4 \equiv x_2 x_3 \Omega_2, \dots \dots \dots \quad (9)$$

that it is possible to bring a conic through the nodes D_2, D_3 and the couples of points B_1, B_2 and C_1, C_2 common to Γ_4 and each of Φ_2 and Ψ_2 .

The equation of this conic is

$$\Omega_2 \equiv (b_1 c_1 - a_{11}) x_2 x_3 + x_1 \Sigma (b_1 c_2 + b_2 c_1 - 2 a_{12}) x_3 = 0. \quad (10)$$

We shall call Φ_2 and Ψ_2 *complementary* with regard to D_2, D_3 .

If C_1, C_2 are fixed and the conic Ω_2 varies, the variable pair B_1, B_2 describes an involution I_b^2 determined on it by the pencil $\Phi_2 = 0$.

Evidently the variable chord $B_1 B_2$ is represented by (7), where in connection with (8) only b_1 is variable. Substituting $b_2 c_2$ and $b_3 c_3$ for a_{22} and a_{33} into (7) we shall find for $B_1 B_2$ the new equation

$$(c_3 x_2 + c_2 x_3) b_1^2 + [(b_3 c_2 + b_2 c_3 - 2 a_{23}) x_1 - 2 a_{13} x_2 - 2 a_{12} x_3] b_1 + \\ + a_{11} (b_3 x_2 + b_2 x_3) = 0, \dots \quad (11)$$

which is quadratic with regard to b_1 . So the curve of involution is a conic Ω_2 with the equation

$$4 a_{11} (b_2 x_3 + b_3 x_2) (c_2 x_3 + c_3 x_2) = [(b_2 c_3 + b_3 c_2) x_1 - 2 \Sigma a_{12} x_3]^2. \dots \quad (12)$$

From the symmetry of (12) w. r. to the quantities b and c it is now evident that Ω_2 is at the same time the envelope of the joining lines of the pairs $C_1 C_2$ of the involution I_c^2 which is generated when B_1, B_2 are fixed and Ω_2 is variable. These two *complementary* involutions are characterised moreover by the property that each pair of I_b^2 can be joined to each pair of I_c^2 by a conic containing

also D_2, D_3 ¹). In particular every chord $B_1 B_2$ determines one pair of the I_c^2 and reversely.

3. In connection with (8) the curve of involution \mathcal{H}_2 is fully determined by the value of $(b_2 c_3 + b_3 c_2)$. Substituting $2k$ for this parameter we can represent \mathcal{H}_2 by the equation

$$a_{11} (a_{33} x_2^2 + 2k x_2 x_3 + a_{22} x_3^2) = (\sum a_{23} x_1 - k x_1)^2, \quad \dots \quad (13)$$

which is quadratic w.r. to the parameter k ; therefore the conics \mathcal{H}_2 form a system with index *two*.

Generalizing for shortness the equation (13) to

$$P k^2 + 2Qk + R = 0, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (14)$$

where P, Q, R denote known quadratic forms, the envelope of the system is given by

$$PR - Q^2 = 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (15)$$

By mere reckoning this equation proves to be identical with (1); so the envelope of the system (14) is the given trinodal Γ_4 .

Out of the identity

$$(Pk^2 + 2Qk + R)R - k^2(PR - Q^2) \equiv (Qk + R)^2 \quad \dots \quad (16)$$

follows, that each conic \mathcal{H}_2 touches the curve Γ_4 in the points common to \mathcal{H}_2 and $Qk + R = 0$, or, what comes to the same, in the points common to the conics $Qk + R = 0$ and $Pk + Q = 0$. So the four points of contact of Γ_4 with the conic \mathcal{H}_2 indicated by k_1 are also situated on

$$k_2(Pk_1 + Q) + (Qk_1 + R) = 0 \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (17)$$

The equation (17) being symmetrical in the parameters k_1 and k_2 the conic represented by it will also contain the points of contact of the conic \mathcal{H}_2 indicated by k_2 .

Hence we may conclude that each conic of the net

¹⁾ In a paper published in Vol. XIV of the "Nieuw Archief voor Wiskunde" (pages 198—200) I have pointed out that suchlike *complementary* systems of pairs of points present themselves on the binodal quartics. Likewise the special involutions of § 5 have their analogs on these curves.

$$\lambda P + \mu Q + \nu R = 0 \quad \dots \quad \dots \quad \dots \quad (18)$$

contains two quadruples of points of contact of four times touching conics \mathcal{H}_2 .

The corresponding parameter-values follow out of:

$$\nu k_1 k_2 = \lambda \quad \text{and} \quad \nu (k_1 + k_2) = \mu.$$

4. To determine the pairs of line belonging to the system we substitute $x: \lambda$ for k into the condition that the discriminant of (13) disappears. After reducing we find

$$(a_{23} \lambda - x)^2 \lambda^2 (a_{22} a_{33} \lambda^2 - x^2) = 0 \quad \dots \quad (19)$$

Evidently for $\lambda = 0$ we have the right lines $D_2 D_3$ to be counted twice; for $x = \lambda \sqrt{a_{22} a_{33}}$ we have the four times touching conic

$$(a_{23} - \sqrt{a_{22} a_{33}}) x_1 + a_{13} x_2 + a_{12} x_3 = \pm (x_2 \sqrt{a_{11} a_{33}} + x_3 \sqrt{a_{11} a_{22}}). \quad (20)$$

breaking up into two double tangents.

Likewise $x = -\lambda \sqrt{a_{22} a_{33}}$ gives two double tangents.

Finally $x = a_{23} \lambda$ produces the conic

$$(a_{11} a_{22} - a_{12}^2) x_3^2 + 2(a_{11} a_{23} - a_{12} a_{13}) x_2 x_3 + (a_{11} a_{33} - a_{13}^2) x_2^2 = 0, \quad (21)$$

composed of the two tangents drawn through D_1 to the curve Γ_4 .

On any right line l the conics \mathcal{H}_2 determine a (2,2) – correspondence; the point of section of l with $D_2 D_3$ belonging to the coincidences of this system, there are three curves \mathcal{H}_2 touching a given right line. These agree with the three manners in which the points common to l and Γ_4 can be divided into two pairs. Each of these pairs determines an involution, i. e., a curve \mathcal{H}_2 . This contradicts in appearance only the fact that an I^2 is determined by 2 pairs, for the two points lying in D_1 form a pair common to all I_b^2 .

Evidently a system of four times touching conics is conjugated to each pair of nodes of Γ_4 .

5. If the conics Φ_2 and Ψ_2 alluded to in § 2 are identical, it follows out of the equation

$$\Phi_2^2 - \Gamma_4 \equiv x_2 x_3 \Omega_2. \quad \dots \quad \dots \quad \dots \quad (22)$$

that the conic Ω_2 drawn through D_2 and D_3 touches the curve Γ_4

in both points common to Ω_2 and the conic Φ_2 passing through the three nodes.

It is therefore rational to put

$$b_2 = \pm \sqrt{a_{22}} \quad \text{en} \quad b_3 = \sqrt{a_{33}} \dots \dots \dots \quad (23)$$

Of the two systems pointed out hereby we shall consider only the system determined by the upper sign. All conics Φ_2 belonging to this are defined by

$$\Phi_2 \equiv x_1 x_2 \sqrt{a_{33}} + x_1 x_3 \sqrt{a_{22}} + \lambda x_2 x_3 \sqrt{a_{11}} = 0 \dots \dots \quad (24)$$

So we get

$$\begin{aligned} \Omega_2 \equiv & (1 - \lambda^2) a_{11} x_2 x_3 + 2(a_{12} - \lambda \sqrt{a_{11} a_{22}}) x_1 x_3 + \\ & + 2(a_{13} - \lambda \sqrt{a_{11} a_{33}}) x_1 x_2 + 2(a_{23} - \sqrt{a_{22} a_{33}}) x_1^2 = 0, \end{aligned} \quad (25)$$

which proves that these conics drawn through D_2 and D_3 , touching two times elsewhere, form a system with index two.

For the connecting line of the points of contact B_1 , B_2 we find with the aid of (6)

$$(1 + \lambda^2)(x_2 \sqrt{a_{11} a_{33}} + x_3 \sqrt{a_{11} a_{22}}) = 2\lambda(\sum a_{12} x_3 - x_1 \sqrt{a_{22} a_{33}}). \quad (26)$$

So these chords of contact form a pencil whose vertex is the point common to the lines

$$x_2 \sqrt{a_{11} a_{33}} + x_3 \sqrt{a_{11} a_{22}} = 0, \quad \sum a_{12} x_3 = x_1 \sqrt{a_{22} a_{33}}. \quad (27)$$

Each ray bears two pairs of the involution formed by the pairs B_1 , B_2 . The values of λ corresponding to the pairs borne by the ray

$$\mu(x_2 \sqrt{a_{11} a_{33}} + x_3 \sqrt{a_{11} a_{22}}) + (\sum a_{12} x_3 - x_1 \sqrt{a_{22} a_{33}}) = 0. \quad (28)$$

are found out of

$$\lambda^2 + 2\mu\lambda + 1 = 0 \dots \dots \dots \dots \quad (29)$$

For $\mu = \pm 1$ we get two rays d_0 and d_1 , on which the two pairs coincide; so these are double tangents.

Likewise is proved that the negative sign in (23) produces a

system of conics of double contact drawn through D_2 , D_3 , whose chords of contact pass through the point of intersection of the remaining double tangents d_2 and d_3 .

6. The conics through D_2 , D_3 touching Γ_4 in B_1 determine an I^2 . Of its two double points one, B_2 , is joined with B_1 by means of a ray passing through the point $(d_0 d_1) \equiv D_{01}$; the second B_{II} lies with B_1 on the ray through $D_{23} \equiv (d_2 d_3)$. Now B_1 is a double point of the complementary I^2 ; let us indicate the second double point by B_I . As any pair of the first involution can be united to any pair of the second by a conic containing D_2 , D_3 , there are two twice touching conics Ω_2 , touching Γ_4 in B_I , B_2 and B_I , B_{II} . Hence $B_I B_2$ passes through D_{23} and $B_I B_{II}$ through D_{01} .

So in Γ_4 an infinite number of quadrangles may be inscribed, one pair of opposite sides passing through D_{01} , the other through D_{23} .

Their vertices form the groups of a biquadratic involution.

Each couple of pairs of the involution (B_1, B_2) collinear with D_{01} lies in a conic containing the nodes D_2 , D_3 . This follows readily from the equation

$$\begin{aligned} a_{11} \Gamma_4 - (x_1 x_2 \sqrt{a_{33}} + x_1 x_3 \sqrt{a_{22}} + \lambda_1 x_2 x_3 \sqrt{a_{11}})(x_1 x_2 \sqrt{a_{33}} + x_1 x_3 \sqrt{a_{22}} + \\ + \lambda_2 x_2 x_3 \sqrt{a_{11}}) \equiv a_{11} x_2 x_3 \{ 2(a_{23} - \sqrt{a_{22} a_{33}}) x_1^2 + \\ + [2 a_{13} - (\lambda_1 + \lambda_2) \sqrt{a_{11} a_{33}}] x_1 x_2 + \\ + [2 a_{12} - (\lambda_1 + \lambda_2) \sqrt{a_{11} a_{22}}] x_1 x_3 + (1 - \lambda_1 \lambda_2) x_2 x_3 \sqrt{a_{22} a_{33}} \} . . . \quad (30) \end{aligned}$$

To find the equation of the conic joining D_2 and D_3 with the vertices of one of the above mentioned quadrangles $B_1 B_2 B_I B_{II}$ let us consider the following identity

$$\begin{aligned} \varrho(x_1 x_2 \sqrt{a_{33}} + x_1 x_3 \sqrt{a_{22}} + \lambda_1 x_2 x_3 \sqrt{a_{11}})(x_1 x_2 \sqrt{a_{33}} + x_1 x_3 \sqrt{a_{22}} + \lambda_2 x_2 x_3 \sqrt{a_{11}}) + \\ + \sigma(x_1 x_2 \sqrt{a_{33}} - x_1 x_3 \sqrt{a_{22}} + \mu_1 x_2 x_3 \sqrt{a_{11}})(x_1 x_2 \sqrt{a_{33}} - x_1 x_3 \sqrt{a_{22}} + \\ + \mu_2 x_2 x_3 \sqrt{a_{11}}) \equiv a_{11}(\Sigma a_{11} x_2^2 x_3^2 + 2 \Sigma a_{12} x_1 x_2 x_3^2) . . . \quad (31) \end{aligned}$$

We easily find that this is satisfied if we put

$$\varrho = (\sqrt{a_{22} a_{33}} + a_{23}) : 2 \sqrt{a_{22} a_{33}} \text{ and } \sigma = (\sqrt{a_{22} a_{33}} - a_{23}) : 2 \sqrt{a_{22} a_{33}}$$

By substituting the value of $\lambda_1 + \lambda_2$ following out of (31) into the equation of the conic indicated in the part on the right side of

(30), we find that the conic $(D_2 D_3 B_1 B_2 B_I B_{II})$ is represented by

$$A_{11} x_1^2 - A_{12} x_1 x_2 - A_{13} x_1 x_3 + \nu x_2 x_3 = 0 (32)$$

Here ν is a new parameter dependent on $\lambda_1 \lambda_2$, whilst A_{ik} denotes the minor corresponding to a_{ik} in the determinant formed by the quantities a_{ik} .

Of the biquadratic involution formed by the points B the quadruple $\nu = 0$ is the only one, the points of which are collinear. So the right line $D_{01} D_{23}$ has the equation

$$A_{11} x_1 - A_{12} x_2 - A_{13} x_3 = 0 (33)$$

By remarking that the third diagonal point Δ of quadrangle $B_1 B_2 B_I B_{II}$ is the pole of $D_{01} D_{23}$ with respect to the conic (32), we easily find that the coordinates of Δ are determined by

$$\frac{y_1}{2 A_{12} A_{13} - A_{11} \nu} = \frac{y_2}{A_{11} A_{13}} = \frac{y_3}{A_{11} A_{12}} (34)$$

Thus the locus of Δ is the right line

$$A_{12} x_2 = A_{13} x_3 (35)$$

7. The relation

$$a_{11} (\sum a_{11} x_2^2 x_3^2 + 2 x_1 x_2 x_3 \sum a_{23} x_1) \equiv (a_{11} x_2 x_3 + a_{12} x_1 x_3 + a_{13} x_1 x_2)^2 + \\ + x_1^2 [(a_{11} a_{33} - a_{13}^2) x_2^2 + 2 (a_{11} a_{23} - a_{12} a_{13}) x_2 x_3 + (a_{11} a_{22} - a_{12}^2) x_3^2] . (36)$$

proves that the tangents of Γ_4 passing through D_1 are represented by

$$A_{22} x_2^2 - 2 A_{23} x_2 x_3 + A_{33} x_3^2 = 0 , (37)$$

whilst their points of contact R_1, R_I are determined by the conic

$$Q_1 \equiv a_{11} x_2 x_3 + a_{12} x_1 x_3 + a_{13} x_1 x_2 = 0 (38)$$

Out of the equation

$$Q_2 \equiv a_{12} x_2 x_3 + a_{22} x_1 x_3 + a_{23} x_1 x_2 = 0 (39)$$

of the conic ($D_1 D_2 D_3 R_2 R_{II}$) is found (according to § 2) the equation

$$\lambda x_2 x_3 + \alpha_{23} x_1 x_3 + \alpha_{33} x_1 x_2 = 0 \quad \dots \quad (40)$$

for the pencil of conics complementary to ϱ_2 with respect to D_2, D_3 . Consequently to this pencil belongs

$$\varrho_3 \equiv \alpha_{13} x_2 x_3 + \alpha_{23} x_1 x_3 + \alpha_{33} x_1 x_2 = 0 \quad \dots \quad (41)$$

The points $D_2, D_3, R_2, R_{II}, R_3, R_{III}$ can be joined by a conic. By applying (10) we find that this conic is indicated by

$$\varrho_{23} \equiv A_{11} x_1^2 - A_{12} x_1 x_2 - A_{13} x_1 x_3 + A_{23} x_2 x_3 = 0 \quad \dots \quad (42)$$

So it belongs to the pencil represented by (32). This was easy to foresee, the pairs of tangents through D_2, D_3 furnishing one of the inscribed quadrangles alluded to in § 6.

In a similar manner by considering the pencil complementary to ϱ_1 we deduce that D_2, D_3, R_1 and R_I lie in a conic with the pair of points C', C'' determined by

$$\alpha_{11} \alpha_{23} x_2 x_3 + \alpha_{22} \alpha_{31} x_3 x_1 + \alpha_{33} \alpha_{12} x_1 x_2 = 0 \quad \dots \quad (43)$$

Hence out of the symmetry of this equation follows that C', C'' are also joined by conics with the quadruples D_1, D_2, R_3, R_{III} and D_1, D_3, R_2, R_{II} .

For the conic ($D_2 D_3 R_1 R_I C' C''$) the equation

$$\begin{aligned} \gamma_1 \equiv & (a_{12} A_{12} + a_{13} A_{13}) x_1^2 + (a_{13} A_{23} - a_{12} A_{22}) x_1 x_2 + \\ & + (a_{12} A_{33} - a_{13} A_{33}) x_1 x_3 + a_{11} A_{23} x_2 x_3 = 0 \quad \dots \quad (44) \end{aligned}$$

is furnished by (10).

According to a wellknown property the six antitangential points R lie in a conic. This is confirmed by the identity

$$A_{23}^2 \Gamma_4 - \varrho_{23} \gamma_1 \equiv x_1^2 \varrho_{123}, \quad \dots \quad (45)$$

where ϱ_{123} represents a quadratic form.

But moreover (45) proves that this conic has still the points C' and C'' in common with Γ_4 .

8. According to the identity

$$\begin{aligned}\Gamma_4 \equiv & (a_{33} x_2^2 + 2 a_{23} x_2 x_3 + a_{22} x_3^2) x_1 + (a_{11} x_2 x_3 + \\ & + 2 a_{12} x_1 x_3 + 2 a_{13} x_1 x_2) x_2 x_3 \quad (46)\end{aligned}$$

the tangential points T_1 and T_{I} of D_1 lie on the conic

$$\tau_1 \equiv a_{11} x_2 x_3 + 2 a_{12} x_1 x_3 + 2 a_{13} x_1 x_2 = 0 \quad \dots \quad (47)$$

It is complementary to the conic (43); so there is a conic ε_1 passing through D_2 , D_3 , T_1 , T_{I} , C' , C'' .

It is similarly proved that D_2 , D_3 , T_2 , T_{II} , T_3 , T_{III} lie in a conic τ_{23} .

And now again we can form an identity

$$\lambda \Gamma^4 + \tau_{23} \varepsilon_1 \equiv x_1^2 \tau_{123} \quad \dots \quad (48)$$

out of which follows that C' and C'' are also connected with the six points T by a conic τ_{123} .

Thus the conic τ_{123} of the six tangential points and the conic ϱ_{123} of the six antitangential points intersect in two points lying on Γ_4 .

Mr. BRILL has pointed out that also the six points of inflection are connected by a conic ω (Math. Ann. XII, 106), intersecting Γ_4 still in two points belonging to the conic τ_{123} (Math. Ann. XIII, 182).

Out of the preceding follows that the complementary points are lying on three remarkable conics ϱ_{123} , τ_{123} and ω .

9. Evidently there are four conics Φ_2 , coinciding with their associates, each determining the points of contact of a double tangent. They are represented by

$$\left. \begin{aligned}\delta_0 &\equiv x_2 x_3 \sqrt{a_{11}} + x_3 x_1 \sqrt{a_{22}} + x_1 x_2 \sqrt{a_{33}} = 0, \\ \delta_1 &\equiv -x_2 x_3 \sqrt{a_{11}} + x_3 x_1 \sqrt{a_{22}} + x_1 x_2 \sqrt{a_{33}} = 0, \\ \delta_2 &\equiv x_2 x_3 \sqrt{a_{11}} - x_3 x_1 \sqrt{a_{22}} + x_1 x_2 \sqrt{a_{33}} = 0, \\ \delta_3 &\equiv x_2 x_3 \sqrt{a_{11}} + x_3 x_1 \sqrt{a_{22}} - x_1 x_2 \sqrt{a_{33}} = 0.\end{aligned}\right\} \quad (49)$$

By applying (6) the equations of the double tangents d_0, d_1, d_2, d_3 can be easily found. Of course they can also be deduced from (26).

There are two identities of the form

$$\left. \begin{aligned} I_4 - \delta_0 \delta_3 &\equiv 2 x_1 x_2 \xi_{03}, \\ I_4 - \delta_1 \delta_2 &\equiv 2 x_1 x_2 \xi_{12}, \end{aligned} \right\} \dots \dots \quad (50)$$

δ_0 and δ_3 , δ_1 and δ_2 being complementary w. r. to D_1 and D_2 ; hence it follows that D_1 and D_2 are connected with the points of contact of d_0 and d_3 by a conic $\xi_{03} = 0$ and with the points of contact of d_1 and d_2 by a conic $\xi_{12} = 0$.

Furthermore the identical equation

$$a_{33} I_4 - \xi_{03} \xi_{12} \equiv x_3^2 \vartheta \dots \dots \quad (51)$$

furnishes the proof of the wellknown property that the eight points of contact of the four double tangents lie in a conic

$$\vartheta \equiv \sum A_{11} x_1^2 - 2 \sum a_{13} a_{23} x_1 x_2 = 0 \dots \dots \quad (52)$$

We easily see that the points of contact of d_1 and d_2 determine with the node D_3 a conic with the equation

$$\begin{aligned} \eta_{12} \equiv d_1 d_2 - (a_{12} + \sqrt{a_{11} a_{22}}) \xi_{12} &\equiv (a_{12} + \sqrt{a_{11} a_{22}}) \varphi_3 - \\ &- (A_{11} x_1^2 - 2 A_{12} x_1 x_2 + A_{22} x_2^2) = 0. \end{aligned} \quad (53)$$

Out of the second form of η_{12} now follows that it also contains the antitangential points of D_3 .

In the same way we can show that D_3 is joined by a conic to its antitangential points and the points of contact of d_0 and d_3 .

The six points R lying in a conic φ_{123} , whilst $D_{03} \equiv (R_1 R_{II}, R_2 R_I)$, $D_{02} \equiv (R_3 R_I, R_1 R_{III})$ and $D_{01} \equiv (R_2 R_{III}, R_3 R_{I})$, the inscribed hexagon $R_1 R_{II} R_3 R_I R_2 R_{III}$ has the double tangent d_0 for its line of PASCAL. Similarly is proved that the remaining three double tangents are lines of PASCAL for three other hexagons formed out of points R .

Physics. — „Calculation of the second correction to the quantity b of the equation of condition of VAN DER WAALS.” By Mr. J. J. VAN LAAR. (Communicated by Prof. J. D. VAN DER WAALS.)

In a paper, published in the Proceedings of the Meeting of the Section for Mathematics and Physics of the Royal Academy of Sciences 29th of Oct. 1898 (appeared Nov. 9th 1898), Prof. VAN DER WAALS has pointed out among others how a second correction to the quantity b of his equation of condition might be found. The integrations necessary to it, proving to be extremely tedious and lengthy, have not been calculated at full length.

I then tried to work out these integrations; I shall communicate in short the results found, referring, with respect to the various mathematical developments which led to my results, to a more extensive treatment that will shortly be published elsewhere (in the “Archives du Musée Teyler”).

The form to be integrated¹⁾ ran as follows (see pages 142—143 of the cited Proceedings):

$$\iint \frac{N}{V} \cdot 2\pi (h + a \cos \theta)^2 dh d\theta \times \text{part of segment},$$

in which that part of segment is found to be:

$$\begin{aligned} & \frac{1}{3} a^3 \sin \varphi \cos \varphi \sqrt{R^2 - a^2} + \frac{2}{3} R^3 \tan^{-1} \left(\tan \varphi \frac{\sqrt{R^2 - a^2}}{R} \right) - \\ & - a \sin \varphi (R^2 - \frac{1}{3} a^2 \sin^2 \varphi) \tan^{-1} \frac{\sqrt{R^2 - a^2}}{a \cos \varphi}. \end{aligned}$$

If we now first perform the integration with respect to θ between the limits 0 and θ_1 , where θ_1 is given by the circumstance that the centre C of the third sphere cannot lie within the two spheres A and B (see fig. on page 142), we have to integrate:

$$2 \int_0^{\theta_1} (h + a \cos \theta)^2 d\theta,$$

¹⁾ The angle AMG indicated as C by Prof. v. d. WAALS has here been called φ , whilst the angle indicated as φ has here been called θ .

giving after some reduction:

$$2 \left[(h^2 + \frac{1}{2} a^2) \theta_1 + (2 a h \sin \theta_1 + \frac{1}{2} a^2 \sin \theta_1 \cos \theta_1) \right].$$

If we now call the angle CMA , point C lying on the sphere A , 2ψ , we have evidently, θ_1 being $= 180 - \varphi - 2\psi$:

$$\begin{aligned} \sin \theta_1 &= \sin(\varphi + 2\psi) = \sin \varphi \cos 2\psi + \cos \varphi \sin 2\psi \\ \cos \theta_1 &= -\cos(\varphi + 2\psi) = -\cos \varphi \cos 2\psi + \sin \varphi \sin 2\psi \end{aligned} \quad \left. \right\},$$

or as

$$\sin \varphi = \frac{\frac{1}{2} r}{a} \quad , \quad \cos \varphi = \frac{h}{a} \quad , \quad \tan \varphi = \frac{\frac{1}{2} r}{h}$$

$$\sin \psi = \frac{\frac{1}{2} R}{a} \quad \sin 2\psi = \frac{R}{a^2} \sqrt{a^2 - \frac{1}{4} R^2} \quad \cos 2\psi = \frac{1}{a^2} (a^2 - \frac{1}{2} R^2),$$

$$\text{also } \sin \theta_1 = \frac{1}{a^2} \left[\frac{1}{2} r (a^2 - \frac{1}{2} R^2) + h R \sqrt{a^2 - \frac{1}{4} R^2} \right]$$

$$\cos \theta_1 = \frac{1}{a^3} \left[-h (a^2 - \frac{1}{2} R^2) + \frac{1}{2} r R \sqrt{a^2 - \frac{1}{4} R^2} \right];$$

so, taking into consideration the relation

$$h^2 = a^2 - \frac{1}{4} r^2,$$

after reduction, we find:

$$\begin{aligned} 2ah \sin \theta_1 + \frac{1}{2} a^2 \sin \theta_1 \cos \theta_1 &= \frac{1}{2 a^4} \left[\frac{1}{2} r \sqrt{a^2 - \frac{1}{4} r^2} (3 a^4 - \frac{1}{2} R^4) + \right. \\ &\quad \left. + R \sqrt{a^2 - \frac{1}{4} R^2} (3 a^4 + \frac{1}{2} a^2 (R^2 - r^2) - \frac{1}{4} R^2 r^2) \right]. \quad . \quad (1) \end{aligned}$$

$$(h^2 + \frac{1}{2} a^2) \theta_1 = (\frac{1}{2} a^2 - \frac{1}{4} r^2) \left[\tan^{-1} \frac{-\frac{1}{2} r}{\sqrt{a^2 - \frac{1}{4} r^2}} - 2 \tan^{-1} \frac{\frac{1}{2} R}{\sqrt{a^2 - \frac{1}{4} R^2}} \right], \quad (2)$$

θ_1 being equal to $(180 - \varphi) - 2\psi$.

Now the form to be integrated is

$$4\pi \frac{N}{V} \int [(1) + (2)] \times \text{part of segment} \times dh.$$

To simplify still further we put:

$$\frac{1}{2} r = n R \quad a = y R,$$

so that

$$h = R \sqrt{y^2 - n^2}$$

and

$$dh = \frac{1}{2} R \frac{dy^2}{\sqrt{y^2 - n^2}}.$$

The above mentioned expression for the part of segment passes into:

$$\begin{aligned} \frac{1}{3} R^3 & \left[n \sqrt{(1-y^2)(y^2-n^2)} + 2 \tan^{-1} \left(n \sqrt{\frac{1-y^2}{y^2-n^2}} \right) - \right. \\ & \left. - n(3-n^2) \tan^{-1} \sqrt{\frac{1-y^2}{y^2-n^2}} \right]. \end{aligned}$$

(1) becomes

$$\frac{1}{2} R^2 \frac{1}{y^4} \left[n(3y^4 - \frac{1}{2}) \sqrt{y^2 - n^2} + \left\{ 3y^4 + \frac{1}{2}y^2(1-4n^2) - n^2 \right\} \sqrt{y^2 - \frac{1}{4}} \right].$$

(2) becomes

$$\frac{1}{2} R^2 (3y^2 - 2n^2) \left[\tan^{-1} \frac{-n}{\sqrt{y^2 - n^2}} - 2 \tan^{-1} \frac{\frac{1}{2}}{\sqrt{y^2 - \frac{1}{4}}} \right],$$

so that our integral is now, writing everywhere x for y^2 , transformed into:

$$I = \frac{1}{3} \pi R^6 \frac{N}{V} \int (A + B + C + D) dx,$$

where (paying attention to dh being equal to $\frac{1}{2} R \frac{dx}{\sqrt{x-n^2}}$):

$$\left. \begin{aligned} A &= n \sqrt{1-x} \left[\frac{n(3x^2 - \frac{1}{2})}{x^2} \sqrt{x-n^2} + \frac{3x^2 + \frac{1}{2}x(1-4n^2) - n^2}{x^2} \sqrt{x-\frac{1}{4}} \right] \\ B &= n(3x - 2n^2) \sqrt{1-x} \left[\tan^{-1} \frac{-n}{\sqrt{x-n^2}} - 2 \tan^{-1} \frac{\frac{1}{2}}{\sqrt{x-\frac{1}{4}}} \right] \\ C &= \left[\frac{n(3x^2 - \frac{1}{2})}{x^2} + \frac{3x^2 + \frac{1}{2}x(1-4n^2) - n^2}{x^2} \sqrt{\frac{x-\frac{1}{4}}{x-n^2}} \right] \times \\ &\quad \times \left[2 \tan^{-1} \left(n \sqrt{\frac{1-x}{x-n^2}} \right) - n(3-n^2) \tan^{-1} \sqrt{\frac{1-x}{x-n^2}} \right] \end{aligned} \right\}$$

$$D = \frac{3x - 2n^2}{\sqrt{x-n^2}} \left[\tan^{-1} \frac{-n}{\sqrt{x-n^2}} - 2 \tan^{-1} \frac{1/2}{\sqrt{x-1/4}} \right] \times \\ \times \left[2 \tan^{-1} \left(n \sqrt{\frac{1-x}{x-n^2}} \right) - n(3-n^2) \tan^{-1} \sqrt{\frac{1-x}{x-n^2}} \right]$$

In the integrations following now we shall for the present not pay attention to the limits for h (or x). To simplify the notation I still propose the following abridgments:

$$\sqrt{\frac{1-x}{x-n^2}} = z \quad \sqrt{\frac{1-x}{x-1/4}} = z' \\ \sqrt{(1-x)(x-n^2)} = p \quad \sqrt{(1-x)(x-1/4)} = p'$$

We then easily find for the five parts of the integral $\int A dx$:

$$A_1 = 3n^2 \int p dx = \frac{3}{4}n^2 \left[(2x - (1+n^2))p - (1-n^2)^2 \tan^{-1} z \right] \quad (3)$$

$$A_2 = -\frac{1}{2}n^2 \int \frac{p}{x^2} dx = \frac{1}{2}n^2 \left[\frac{p}{x} - 2 \tan^{-1} z + \frac{1+n^2}{n} \tan^{-1} nz \right]$$

$$A_3 = 3n \int p' dx = \frac{3}{4}n \left[(2x - \frac{5}{4})p' - \frac{9}{16} \tan^{-1} z' \right] \quad (4)$$

$$A_4 = \frac{1}{2}n(1-4n^2) \int \frac{p'}{x} dx = \frac{1}{2}n(1-4n^2) \left[p' - \frac{5}{4} \tan^{-1} z' + \tan^{-1} \frac{1}{2}z' \right]$$

$$A_5 = -n^3 \int \frac{p'}{x^3} dx = n^3 \left[\frac{p'}{x} - 2 \tan^{-1} z' + \frac{5}{2} \tan^{-1} \frac{1}{2}z' \right]$$

as is to be verified by means of the relations.

$$dp = -\frac{1}{2} \frac{2x - (1+n^2)}{p} dx \quad dp' = -\frac{1}{2} \frac{2x - \frac{5}{4}}{p'} dx$$

$$d \tan^{-1} z = -\frac{1}{2} \frac{dx}{p} \quad d \tan^{-1} z' = -\frac{1}{2} \frac{dx}{p'}$$

$$d \tan^{-1} nz = -\frac{n}{2} \frac{dx}{xp} \quad d \tan^{-1} \frac{1}{2}z' = -\frac{1}{4} \frac{dx}{xp'}$$

Furthermore, as

$$\int (3x - 2n^2) \sqrt{1-x} dx = -2 \left[(1 - \frac{2}{3}n^2)(1-x)^{3/2} - \frac{3}{5}(1-x)^{5/2} \right]$$

$$d \tan^{-1} \frac{-n}{\sqrt{x-n^2}} = \frac{n}{2} \frac{dx}{x \sqrt{x-n^2}} \quad d \tan^{-1} \frac{1/2}{\sqrt{x-1/4}} = -\frac{1}{4} \frac{dx}{x \sqrt{x-1/4}},$$

we find for both parts of $\int B dx$:

$$\left. \begin{aligned} B_1 &= -2n \left[(1 - \frac{2}{3}n^2)(1-x)^{3/2} - \frac{3}{5}(1-x)^{5/2} \right] \tan^{-1} \frac{-n}{\sqrt{x-n^2}} - \\ &\quad - n^2 \left[\left(\frac{3}{10}x - \frac{7}{20} - \frac{13}{60}n^2 \right)p - \left(\frac{3}{4} - \frac{3}{2}n^2 + \frac{13}{60}n^4 \right)\tan^{-1}z + \right. \\ &\quad \left. + \frac{4}{n} \left(\frac{1}{5} - \frac{1}{3}n^2 \right) \tan^{-1} nz \right] \\ B_2 &= 4n \left[(1 - \frac{2}{3}n^2)(1-x)^{3/2} - \frac{3}{5}(1-x)^{5/2} \right] \tan^{-1} \frac{1/2}{\sqrt{x-1/4}} - \\ &\quad - n \left[\left(\frac{3}{10}x - \frac{19}{80} - \frac{2}{3}n^2 \right)p' - \left(\frac{27.1}{320} - \frac{11}{6}n^2 \right)\tan^{-1} z' + \right. \\ &\quad \left. + 8 \left(\frac{1}{5} - \frac{1}{3}n^2 \right) \tan^{-1} \frac{1}{2}z' \right] \end{aligned} \right\} \quad (5)$$

The integration of the four parts of $\int C dx$ is already more difficult. Comparatively easy are C_1 and C_2 :

$$\left. \begin{aligned} C_1 &= -n^2(3-n^2) \int \frac{3x^2-1/2}{x^2} \tan^{-1} z \cdot dx = \\ &= -n^2(3-n^2) \left[-\frac{3}{2}p + \left\{ 3x + \frac{1}{2x} - \frac{3}{2}(1+n^2) \right\} \tan^{-1}z - \frac{1}{2n} \tan^{-1} nz \right] \\ C_2 &= 2n \int \frac{3x^2-1/2}{x^2} \tan^{-1} nz \cdot dx = \\ &= 2n \left[\frac{p}{4nx} - 3n \tan^{-1} z + \left(3x + \frac{1}{2x} - \frac{1+n^2}{4n^2} \right) \tan^{-1} nz \right] \end{aligned} \right\} \quad (6)$$

To integrate C_3 we add to this integral :

$$- n (3 - n^2) \int V \frac{x^{-1/4}}{x - n^2} \tan^{-1} z \cdot dx ,$$

which afterwards will be subtracted from D_3 . So C_3 becomes :

$$C_3' = - n (3 - n^2) \int \frac{4 x^2 + 1/2 x (1 - 4 n^2) - n^2}{x^2} V \frac{x^{-1/4}}{x - n^2} \tan^{-1} z \cdot dx .$$

But now we find :

$$\frac{4 x^2 + 1/2 x (1 - 4 n^2) - n^2}{x^2} V \frac{x^{-1/4}}{x - n^2} dx = d \left[\frac{4 (x^{-1/4})}{x} V \frac{(x^{-1/4}) (x - n^2)}{(x - n^2)} \right] ,$$

so that we get (the various developments — as was said before — will be published elsewhere) :

$$C_3' = - n (3 - n^2) \left[\frac{4 (x^{-1/4})}{x} V \frac{(x^{-1/4}) (x - n^2)}{(x - n^2)} \tan^{-1} z - 2 \left\{ p' + 1/4 \tan^{-1} z' + 1/4 \tan^{-1} 1/2 z' \right\} \right]. \quad (7)$$

Likewise to the integral C_4 is added :

$$2 \int V \frac{x^{-1/4}}{x - n^2} \tan^{-1} nz \cdot dx ,$$

which shall directly be subtracted from D_4 . For

$$C_4' = 2 \int \frac{4 x^2 + 1/2 x (1 - 4 n^2) - n^2}{x^2} V \frac{x^{-1/4}}{x - n^2} \tan^{-1} nz \cdot dx$$

we find in quite the same way as for C_3' :

$$C_4' = 2 \left[\frac{4 (x^{-1/4})}{x} V \frac{(x^{-1/4}) (x - n^2)}{(x - n^2)} \tan^{-1} nz + \right. \\ \left. + 2 n \left\{ \frac{1}{4} \frac{p'}{x} - 2 \tan^{-1} z' + \frac{11}{8} \tan^{-1} 1/2 z' \right\} \right]. \quad (8)$$

As $\int \frac{3x-2n^2}{\sqrt{x-n^2}} dx$ is equal to $2x\sqrt{x-n^2}$, we shall find after various reductions successively for the 4 parts of $\int D dx$:

$$\begin{aligned} D_1 &= -n(3-n^2) \int \frac{3x-2n^2}{\sqrt{x-n^2}} \tan^{-1} \frac{-n}{\sqrt{x-n^2}} \tan^{-1} z \cdot dx = \\ &= -n(3-n^2) \left[2x \sqrt{x-n^2} \tan^{-1} \frac{-n}{\sqrt{x-n^2}} \tan^{-1} z - \right. \\ &\quad \left. - \frac{2}{3}(x+2)\sqrt{1-x} \tan^{-1} \frac{-n}{\sqrt{x-n^2}} + \frac{5}{6}np - n(x-\frac{3}{2}) - \frac{5}{6}n^2 \right] \tan^{-1} z - \frac{4}{3} \tan^{-1} nz \end{aligned} \quad (9)$$

Likewise:

$$\begin{aligned} D_2 &= 2 \int \frac{3x-2n^2}{\sqrt{x-n^2}} \tan^{-1} \frac{-n}{\sqrt{x-n^2}} \tan^{-1} nz \cdot dx = \\ &= 2 \left[2x \sqrt{x-n^2} \tan^{-1} \frac{-n}{\sqrt{x-n^2}} \tan^{-1} nz - 2n \sqrt{1-x} \tan^{-1} \frac{-n}{\sqrt{x-n^2}} + \right. \\ &\quad \left. + 3n^2 \tan^{-1} z - n(x+2) \tan^{-1} nz \right] \dots \quad (10) \end{aligned}$$

The integral D_3 , viz.

$$D_3 = 2n(3-n^2) \int \frac{3x-2n^2}{\sqrt{x-n^2}} \tan^{-1} \frac{1/2}{\sqrt{x-1/4}} \tan^{-1} z \cdot dx$$

can first be reduced to

$$\begin{aligned} D_3 &= 2n(3-n^2) \left[2x \sqrt{x-n^2} \tan^{-1} \frac{1/2}{\sqrt{x-1/4}} \tan^{-1} z + \right. \\ &\quad \left. + \frac{1}{2} \int \sqrt{\frac{x-n^2}{x-1/4}} \tan^{-1} z \cdot dx + \int \frac{x}{\sqrt{1-x}} \tan^{-1} \frac{1/2}{\sqrt{x-1/4}} \cdot dx \right]. \end{aligned}$$

But as $\int \sqrt{\frac{x-n^2}{x-1/4}} \tan^{-1} z \cdot dx$ can be transformed into

$$-\int \sqrt{\frac{x-1/4}{x-n^2}} \tan^{-1} z \cdot dx + 2\sqrt{(x-1/4)(x-n^2)} \tan^{-1} z - p' - \frac{3}{4} \tan^{-1} z',$$

we subtract from D_3 the part $2n(3-n^2) \times -1/2 \int V \frac{x-1/4}{x-n^2} \tan^{-1} z \cdot dx$,

which part, as we saw above, was already added to C_3 . After various lengthy developments we find at last:

$$D_3' = 2n(3-n^2) \left[2x \sqrt{x-n^2} \tan^{-1} \frac{1/2}{\sqrt{x-1/4}} \tan^{-1} z - \right. \\ - \frac{2}{3}(x+2) \sqrt{1-x} \tan^{-1} \frac{1/2}{\sqrt{x-1/4}} + \sqrt{(x-1/4)(x-n^2)} \tan^{-1} z - \\ \left. - \frac{2}{3} p' - \frac{11}{12} \tan^{-1} z' + \frac{4}{3} \tan^{-1} 1/2 z' \right] \dots \quad (11)$$

And in the same way for

$$D_4 = -4 \int \frac{3x-2n^2}{\sqrt{x-n^2}} \tan^{-1} \frac{1/2}{\sqrt{x-1/4}} \tan^{-1} nz \cdot dx,$$

after transformation of the integral $\frac{1}{2} \int V \frac{x-n^2}{x-1/4} \tan^{-1} nz \cdot dx$ into

$$\frac{1}{2} \left[- \int V \frac{x-1/4}{x-n^2} \tan^{-1} nz \cdot dx + 2 \sqrt{(x-1/4)(x-n^2)} \tan^{-1} nz - \right. \\ \left. - 2n \tan^{-1} z' + n \tan^{-1} 1/2 z' \right],$$

and subtraction of the integral already added to C_4 :

$$2 \int V \frac{x-1/4}{x-n^2} \tan^{-1} nz \cdot dx,$$

we find:

$$D_4' = -4 \left[2x \sqrt{x-n^2} \tan^{-1} \frac{1/2}{\sqrt{x-1/4}} \tan^{-1} nz - 2n \sqrt{1-x} \tan^{-1} \frac{1/2}{\sqrt{x-1/4}} + \right. \\ \left. + \sqrt{(x-1/4)(x-n^2)} \tan^{-1} nz - 2n \tan^{-1} z' + \frac{5}{2} n \tan^{-1} 1/2 z' \right]. \quad (12)$$

If we now join all the similar terms, we shall find for

$$\int (A + B + C + D) \cdot dx$$

the following:

$$\begin{aligned}
 & \left[\frac{1+n^2}{2x} + \frac{6}{5}n^2x + \frac{2}{5}n^2(4-3n^2) \right] \sqrt{(1-x)(x-n^2)} + \\
 & + \left[\frac{n(1+n^2)}{x} + \frac{6}{5}n^3x + \frac{1}{5}n(9-10n^2) \right] \sqrt{(1-x)(x-\frac{1}{4})} + \\
 & + \left[-n^2(3-n^2)\left(2x + \frac{1}{2x}\right) - n^2(1-2n^2 + \frac{6}{5}n^4) + \right. \\
 & \quad \left. + n(3-n^2)\left(\frac{1}{x}-2\right)\sqrt{(x-\frac{1}{4})(x-n^2)} \right] \tan^{-1} \sqrt{\frac{1-x}{x-n^2}} + \\
 & + \left[2n\left(2x + \frac{1}{2x}\right) - \frac{1}{n}(1/2 - 7/10n^2) - \right. \\
 & \quad \left. - 2\left(\frac{1}{x}-2\right)\sqrt{(x-\frac{1}{4})(x-n^2)} \right] \tan^{-1}\left(n\sqrt{\frac{1-x}{x-n^2}}\right) - \\
 & - \frac{21}{5}n\tan^{-1}\sqrt{\frac{1-x}{x-\frac{1}{4}}} + \frac{39}{10}n\tan^{-1}\frac{1}{2}\sqrt{\frac{1-x}{x-\frac{1}{4}}} + \\
 & + 2n\sqrt{1-x}\left(-\frac{2}{5} + (\frac{4}{5}-n^2)x + \frac{3}{5}v^2\right)\left(\tan^{-1}\frac{-n}{\sqrt{x-n^2}} - 2\tan^{-1}\frac{1/2}{\sqrt{x-\frac{1}{4}}}\right) \\
 & + 2x\sqrt{x-n^2}\left(\tan^{-1}\frac{-n}{\sqrt{x-n^2}} - 2\tan^{-1}\frac{1/2}{\sqrt{x-\frac{1}{4}}}\right) \times \\
 & \times \left[2\tan^{-1}\left(n\sqrt{\frac{1-x}{x-n^2}}\right) - n(3-n^2)\tan^{-1}\sqrt{\frac{1-x}{x-n^2}} \right]
 \end{aligned} \tag{13}$$

Let us now introduce the limits for h . These are (see the paper of Prof. v. d. WAALS) for values of r lying between R and $R\sqrt{3}$:

$$\frac{R^2 - \frac{1}{2}r^2}{2\sqrt{R^2 - \frac{1}{4}r^2}} \text{ and } \sqrt{R^2 - \frac{1}{4}r^2},$$

whilst for values of r between $R\sqrt{3}$ and $2R$ they are:

$$-\sqrt{R^2 - \frac{1}{4}} \text{ and } \sqrt{R^2 - \frac{1}{4}r^2}.$$

So with our notations we get ($h = R\sqrt{x-n^2}$):

For $n = \frac{1}{2}$ to $\frac{1}{2}\sqrt{3}$:

$$\sqrt{x-n^2} \text{ from } \frac{1-2n^2}{2\sqrt{1-n^2}} \text{ to } \sqrt{1-n^2} \left(x \text{ from } \frac{1}{4(1-n^2)} \text{ to } 1 \right);$$

For $n = \frac{1}{2}\sqrt{3}$ to 1:

$$\sqrt{x-n^2} \text{ from } -\sqrt{1-n^2} \text{ to } \sqrt{1-n^2} (x \text{ from } 1 \text{ to } 1).$$

Substitution of these limits in (13) gives, paying attention to the circumstance that wherever $\sqrt{x-n^2}$ appears in \tan^{-1} , the value of \tan^{-1} is equal to π for $\sqrt{x-n^2} = -\sqrt{1-n^2}$, when at the same time $\sqrt{1-x}$ (which becomes 0) appears in the numerator:

$$\begin{aligned} I_a (n = \frac{1}{2} \text{ to } \frac{1}{2}\sqrt{3}) &= -(\frac{1}{2} + \frac{3}{2}n^2 - \frac{3}{5}n^4)\sqrt{3-4n^2} + \\ &+ n^2(7-4n^2 + \frac{6}{5}n^4)\tan^{-1} \frac{\sqrt{3-4n^2}}{1-2n^2} + \frac{1}{n} \left(\frac{1}{2} - \frac{47}{10}n^2 \right) \tan^{-1} \frac{n\sqrt{3-4n^2}}{1-2n^2} + \\ &+ \frac{21}{5}n\tan^{-1} \frac{\sqrt{3-4n^2}}{n} - \frac{39}{10}n\tan^{-1} \frac{\sqrt{3-4n^2}}{2n} \\ I_b (n = \frac{1}{2}\sqrt{3} \text{ to } 1) &= \pi \left[\frac{1}{2n} - \frac{57}{10}n + \frac{17}{2}n^2 - \frac{9}{2}n^4 + \frac{6}{5}n^6 \right] + \\ &+ \frac{1}{2}\pi\sqrt{3}(2-3n+n^3)\sqrt{1-n^2} + \\ &+ 2\pi(2-3n+n^3)\sqrt{1-n^2} \left(\tan^{-1} \frac{n}{\sqrt{1-n^2}} - \frac{1}{3}\pi \right) \end{aligned} \quad (14)$$

Here I have also availed myself of the circumstance that

$$\tan^{-1} \frac{-n}{\sqrt{x-n^2}} - 2\tan^{-1} \frac{\frac{1}{2}}{\sqrt{x-\frac{1}{4}}}.$$

disappears for $x = \frac{1}{4(1-n^2)}$ ($\sqrt{x-n^2} = \frac{1-2n^2}{2\sqrt{1-n^2}}$).

The expressions found for I_a and I_b have been verified by me in various ways and every time found true. They are both still to be multiplied by $\frac{1}{3}\pi R^6 \frac{N}{V}$. Before passing to the second integration with respect to n , we must calculate a *complementary term* for the cases

(not mentioned in the cited paper) in which point M falls outside the segment and an *entire* segment is included by a third sphere. It is easy to see that that complementary term is obtained out of

$$I'_b = 2 \iint \frac{N}{V} 2\pi (h + a \cos \theta)^2 dh d\theta \times \text{segment},$$

which produces after some reductions,

$$\text{Segment being equal to } \frac{1}{3}\pi(2R^3 - \frac{3}{2}R^2r + \frac{1}{8}r^3) = \frac{1}{3}\pi R^3(2 - 3n + n^3):$$

$$I'_b = \frac{1}{3}\pi^2 R^6 \frac{N}{V} (2 - 3n + n^3) \int \left[\frac{n(3x^{2-1/2})}{x^2} + \right. \\ \left. + \frac{3x^2 + 1/2x(1-4n^2) - n^2}{x^2} \sqrt{\frac{x^{-1/4}}{x-n^2} + \frac{3x-2n^2}{\sqrt{x-n^2}} \left(\tan^{-1} \frac{-n}{\sqrt{x-n^2}} - 2 \tan^{-1} \frac{1/2}{\sqrt{x-n^{1/4}}} \right)} \right] dx.$$

Of this integral we mention only the result taken between the limits $\sqrt{x-n^2} = \frac{1-2n^2}{2\sqrt{1-n^2}}$ to

$$\sqrt{x-n^2} = -\sqrt{1-n^2} \quad (x \text{ from } \frac{1}{4(1-n^2)} \text{ to } 1).$$

It is evident that this integral relates only to the second tempo ($n = 1/2 \sqrt{3}$ to 1).

We find :

$$I'_b = \frac{1}{3}\pi^2 R^6 \frac{N}{V} (2 - 3n + n^3) \left[\frac{1}{2}n - \frac{1}{2}\sqrt{3(1-n^2)} - \right. \\ \left. - 2\sqrt{1-n^2} \left(\tan^{-1} \frac{n}{\sqrt{1-n^2}} - \frac{1}{3}\pi \right) \right], \dots \quad (15)$$

by which most remarkably the above named value of I_b is considerably simplified and, with the omission of the factor $\frac{1}{3}\pi R^6 \frac{N}{V}$, passes into :

$$I_b + I''_b = \pi \left[\frac{1}{2n} - \frac{47}{10}n + 7n^2 - 4n^4 + \frac{6}{5}n^6 \right] \dots \quad (16)$$

If we now multiply by

$$4 \pi r^2 dr \times \frac{N}{V} = 32 \pi R^3 \frac{N}{V} \times n^2 dn,$$

we have still to integrate:

$$I = \frac{32}{3} \pi^2 R^9 \frac{N^2}{V^2} \left[\int_{1/2}^{1/2\sqrt{3}} I_a n^2 dn + \int_{1/2\sqrt{3}}^1 (I_b + I''_b) n^2 dn \right]$$

This integration we break up into parts again.

$$I_1 = \int_{1/2}^{1/2\sqrt{3}} (-\frac{1}{2} n^2 - \frac{3}{2} n^4 + \frac{3}{5} n^6) \sqrt{3 - 4 n^2} dn$$

furnishes:

$$I_1 = \left[\left(\frac{735}{8192} n - \frac{89}{1024} n^3 - \frac{83}{320} n^5 + \frac{3}{40} n^7 \right) \sqrt{3 - 4 n^2} + \frac{2205}{16384} \tan^{-1} \frac{\sqrt{3 - 4 n^2}}{2n} \right]_{1/2}^{1/2\sqrt{3}},$$

as is to be verified by means of the relations:

$$d\sqrt{3 - 4 n^2} = \frac{-4 n dn}{\sqrt{3 - 4 n^2}} \quad d\tan^{-1} \frac{\sqrt{3 - 4 n^2}}{2n} = \frac{-2 dn}{\sqrt{3 - 4 n^2}}.$$

Introduction of the limits gives further:

$$I_1 = -\frac{2169}{5 \times 16384} \sqrt{2} - \frac{2205}{16384} \tan^{-1} \sqrt{2} . . . \quad (17)$$

In the second place, see (14) and (16):

$$I_2 = \int_{1/2}^{1/2\sqrt{3}} (7 n^4 - 4 n^6 + \frac{6}{5} n^8) \tan^{-1} \frac{\sqrt{3 - 4 n^2}}{1 - 2 n^2} dn + \int_{1/2\sqrt{3}}^1 (7 n^4 - 4 n^6 + \frac{6}{5} n^8) \pi .$$

As $\tan^{-1} \frac{\sqrt{3-4n^2}}{1-2n^2}$ takes for $n = \frac{1}{2}\sqrt{3}$ the value π , we find by means of

$$d \tan^{-1} \frac{\sqrt{3-4n^2}}{1-2n^2} = \frac{2n}{1-n^2} \frac{dn}{\sqrt{3-4n^2}},$$

and, paying attention to the relation $\tan^{-1} 2\sqrt{2} = \pi - 2 \tan^{-1} \sqrt{2}$:

$$\begin{aligned} I_2 &= \left(\frac{5719}{15.256} - \frac{127}{7.32} \right) \pi + \left(\frac{169}{15.128} - \frac{1}{7.16} \right) \tan^{-1} \sqrt{2} - \\ &\quad - \int_{\frac{1}{2}}^{\frac{1}{2}\sqrt{3}} \frac{\frac{14}{5}n^6 - \frac{8}{7}n^8 + \frac{4}{15}n^{10}}{(1-n^2)\sqrt{3-4n^2}} dn \quad . . \quad (18) \end{aligned}$$

In the third place:

$$I_3 = \int_{\frac{1}{2}}^{\frac{1}{2}\sqrt{3}} \left(\frac{1}{2}n - \frac{47}{10}n^3 \right) \tan^{-1} \frac{n\sqrt{3-4n^2}}{1-2n^2} \cdot dn + \int_{\frac{1}{2}\sqrt{3}}^1 \left(\frac{1}{2}n - \frac{47}{10}n^3 \right) \pi \cdot dn.$$

With $d \tan^{-1} \frac{n\sqrt{3-4n^2}}{1-2n^2} = \frac{3-2n^2}{1-n^2} \frac{dn}{\sqrt{3-4n^2}}$ we easily find:

$$I_3 = -\frac{37}{40}\pi + \frac{7}{640} \tan^{-1} \sqrt{2} - \int_{\frac{1}{2}}^{\frac{1}{2}\sqrt{3}} \frac{\frac{3}{4}n^2 - \frac{161}{40}n^4 + \frac{47}{20}n^6}{(1-n^2)\sqrt{3-4n^2}} dn \quad . \quad (19)$$

For

$$I_4 = \int_{\frac{1}{2}}^{\frac{1}{2}\sqrt{3}} \left[\frac{21}{5}n^3 \tan^{-1} \frac{\sqrt{3-4n^2}}{n} - \frac{39}{10}n^3 \tan^{-1} \frac{\sqrt{3-4n^2}}{2n} \right] dn$$

we find by means of

$$d \tan^{-1} \frac{\sqrt{3-4n^2}}{n} = \frac{-dn}{(1-n^2)\sqrt{3-4n^2}}, \quad d \tan^{-1} \frac{\sqrt{3-4n^2}}{2n} = \frac{-2dn}{\sqrt{3-4n^2}};$$

$$I_4 = \left(-\frac{21}{320} \pi + \frac{21}{160} \tan^{-1} \nu^2 \right) + \frac{39}{640} \tan^{-1} \nu^2 + \\ + \int_{1/2}^{1/2} \frac{\frac{21}{20} n^4 - \frac{39}{20} n^4 (1 - n^2)}{(1 - n^2) \sqrt{3 - 4 n^2}} dn,$$

where again in the first part I made use of the relation $\tan^{-1} 2 \nu^2 = \pi - 2 \tan^{-1} \nu^2$.

If we then join the terms obtained, we shall get besides the foremost factor

$$I = -\frac{2169}{5.16384} \nu^2 + \left(\frac{383}{3.256} - \frac{127}{7.32} \right) \pi + \\ + \left(-\frac{2205}{16384} + \frac{559}{15.128} - \frac{1}{7.16} \right) \tan^{-1} \nu^2 + \\ + \int_{1/2}^{1/2} \frac{-\frac{3}{4} n^2 + \frac{25}{8} n^4 - \frac{16}{5} n^6 + \frac{8}{7} n^8 - \frac{4}{15} n^{10}}{(1 - n^2) \sqrt{3 - 4 n^2}} dn \quad \left. \right\} (20)$$

For the integrating of this last integral we again refer to the more lengthy paper; it is sufficient to mention the result for I . I only draw attention to the fact that after having successively determined

$$\int \frac{n^{2k} dn}{\sqrt{3 - 4 n^2}},$$

where $k = 1, 2, 3, 4$, besides

$$\int \frac{n^2 dn}{(1 - n^2) \sqrt{3 - 4 n^2}} = \pi - \frac{5}{2} \tan^{-1} \nu^2,$$

all the above-named integrals are found by parts. So is e.g.

$$\int \frac{n^4 dn}{(1 - n^2) \sqrt{3 - 4 n^2}} = \int \frac{n^2 dn}{(1 - n^2) \sqrt{3 - 4 n^2}} - \int \frac{n^2 dn}{\sqrt{3 - 4 n^2}},$$

etc. The result now becomes, after multiplication by the foremost

factor and by N for all N spheres:

$$\frac{32}{3} \pi^2 R^9 \frac{N^3}{V^2} \left[\frac{73}{7.45.64} V^2 - \frac{153}{7.5.256} \pi + \frac{153}{7.5.64} \tan^{-1} V^2 \right] . \quad (21)$$

If for a moment we call the expression between brackets ω , this may be written :

$$\left(\frac{4}{3} \pi R^3 \right)^3 \frac{N^3}{V^2} \times \frac{9}{2} \frac{\omega}{\pi} .$$

For the double volume of the N spheres of distance remains also, after paying attention to the 1st and 2nd corrections:

$$N \cdot \frac{4}{3} \pi R^3 - \left(\frac{4}{3} \pi R^3 \right)^2 \frac{17}{64} \frac{N^3}{V} + \left(\frac{4}{3} \pi R^3 \right)^3 \cdot \frac{9}{2} \frac{\omega}{\pi} \frac{N^3}{V^2} = \\ = 8 b \left[1 - \frac{17}{64} \frac{8 b}{V} + \frac{9}{2} \frac{\omega}{\pi} \frac{64 b^2}{V^2} \right] ,$$

$N \cdot \frac{4}{3} \pi R^3$ being equal to $8 b$. If now $4 b = b_\infty$, then in

$$2 b_\infty \left[1 - \frac{17}{32} \frac{b_\infty}{V} + 18 \frac{\omega}{\pi} \left(\frac{b_\infty}{V} \right)^2 \right]$$

the 2nd correction sought for evidently becomes equal to $18 \frac{\omega}{\pi}$, so:

$$\beta = \frac{9}{35.32 \pi} \left[\frac{73}{9} V^2 + 153 \left(\tan^{-1} V^2 - \frac{1}{4} \pi \right) \right] ,$$

or

$$\beta = \frac{1}{1120 \pi} \left[73 V^2 + 81.17 \left(\tan^{-1} V^2 - \frac{1}{4} \pi \right) \right] , \quad (22)$$

this being our definite result. The value of this is, exact in 4 decimals,

$$\underline{\underline{\beta = 0.0958}} ,$$

so almost $1/_{10}$, whereas the 1st correction was fully $1/_{2}$.

Physics. — „Measurements on the system of isothermal lines near the plaitpoint, and especially on the process of the retrograde condensation of a mixture of carbonic acid and hydrogen”.
By Dr. J. VERSCHAFFELT. (Communication N°. 45 from the Physical Laboratory at Leiden by Prof. H. KAMERLINGH ONNES).

(Read in the Meeting of December 24th 1898).

§ 1.

In one of his first communications on the process of the retrograde condensation KUENEN has given some measurements, from which the course of the ratio of the volume of the liquid-phase to that of the vapour-phase along one definite α -line (VAN DER WAALS' ψ -surface) may be deduced. It was desirable to get a general view of the process of condensation along different α -lines on one and the same ψ -surface (especially between the α at the critical plaitpoint and the α at the critical point of contact) or, what comes to the same in this case, to get to know, for a definite composition of the mixture, the process of the condensation along lines belonging to ψ -surfaces for different temperatures, near and especially between the temperature of the plaitpoint and that of the critical point of contact for this definite α .

An opportunity for making measurements on this subject was offered by a more ample investigation on isotherms of mixtures of carbonic acid and hydrogen which was undertaken by myself¹).

§ 2. Method.

The pure hydrogen was prepared by means of the apparatus used in filling the hydrogen-thermometers for the measurement of low temperatures, and formerly described²).

The pure carbonic acid was obtained from the commercial liquid gas, following the method first applied by KUENEN³) and now regularly used in the Physical Laboratory at Leiden. The way of preparing the mixture and the apparatus used were mainly the same as those formerly used bij KUENEN in a similar investigation (see the annexed figure).

¹) Compare KAMERLINGH ONNES, Versl. Kon. Akad., Dec. 1894, pg. 179.

²) KAMERLINGH ONNES, Versl. Kon. Akad., May 30th 1896; June 27th 1896. Comm. fr. the Phys. Lab. of Leyden, N°. 27.

³) Diss. Leiden, 1892.

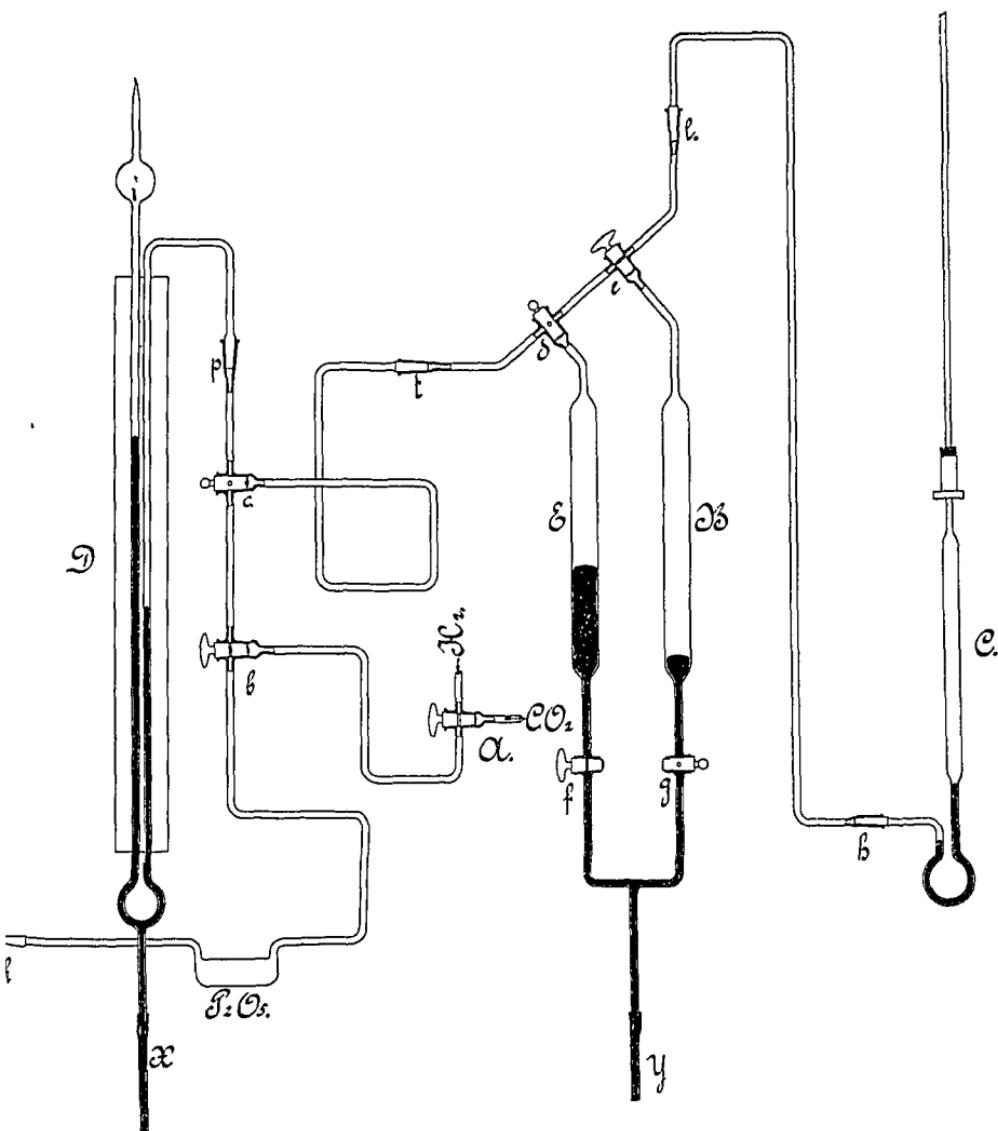


Fig. I.

Even small quantities of hydrogen, added to the carbonic acid cause a great increase of pressure when the latter gas is condensed. In order that the end of the condensation can be reached without danger to the glass tube, the experiments were made with a mixture containing 5 mol. hydrogen on 100 mol.

The volumes of the mixture were measured in a manometer-tube of about 0,05 c.m.² in section, on which a graduated scale in m.m. had been etched tested carefully by a kathetometer. The calibration was made by filling the tube with mercury and by causing this liquid to flow out in small quantities; the columns of mercury thus

run out were weighed and afterwards measured by the kathetometer.

Perfect equilibrium of the phases was secured by stirring with an electro-magnetic stirring apparatus, first used and described by KUENEN¹⁾, afterwards applied by VAN ELDIK²⁾ and HARTMAN³⁾. In measuring the volumes, the contents of the stirring-rod had to be taken into account. This volume was calculated from the shape of the stirring rod; this was very regular, and consisted in an almost perfectly cylindrical part 28,5 m.m. long and on an average of 1,450 m.m. in diameter, and two globules, one on each end, of 2,300 m.m. and 2,383 m.m. diameter respectively. From these data the volume of the stirring-rod was found to be 60.4 m.m.³.

The compressibility of the mixture was compared with that of pure hydrogen. To this end the experimental tube was connected with another manometer-tube on which, like on the former, a graduated scale in m.m. had been etched. The scale was tested by a kathetometer and the manometer was carefully calibrated. This tube however was much narrower than the first, its section being only about 0,006 c.m.². It was filled with a known quantity of pure hydrogen.

From the specific volumes of the hydrogen, the pressures were calculated by means of the formula

$$p(v-0,000690) = 1 + 0,00370 t^4$$

borrowed from AMAGAT's latest observations on the compressibility of hydrogen⁵⁾.

Volumes and pressures were carefully calculated to within $\frac{1}{1000}$, as in these experiments no higher degree of accuracy was to be expected.

The two glass manometers were screwed in the usual way into two steel cylinders, which were partly filled with mercury, and for the rest with glycerine which transferred the pressure from a compression-pump. The lower ends of the cylinders were connected by a steel tube filled with mercury; at their higher ends they communicated by means of a brass tube with T-piece for joining on to the compression-pump.

By connecting the cylinders at their lower ends, a contrivance

¹⁾ loc. cit.

²⁾ Versl. Kon. Akad., 29 May 1897; 21 June 1897. Comm., N°. 39.

³⁾ Versl. Kon. Akad., 25 June 1898. Comm., N°. 43.

⁴⁾ For explanation see a following communication.

⁵⁾ Annales de Chim. et de Phys., 6e série, t. XXIX.

recently introduced by HARTMAN¹⁾ a difference of the levels of the mercury in the two cylinders is avoided, a difference which we should have to take into account without being able to determine it with sufficient certainty. The only thing now left to be taken into account was the hydrostatical difference of pressure caused by a different height of the mercury in the two tubes, and of the difference in capillary depression caused by the difference in diameter of the two tubes. An experiment made on purpose showed this difference to be about 7 m.m., a quantity which could be neglected as being smaller than the possible error of observation in the measurement of the pressures.

Suitable temperatures ranging between that of the room to a little above the critical temperature of carbonic acid, were obtained by means of an apparatus first used by VAN ELDIK, and at the same time with me by HARTMAN. The bath was constructed in a way as described by me in a former paper and as it was used afterwards by VAN ELDIK. Whenever small fluctuations of temperature occurred the observations were reduced to one and the same mean temperature by making use of an approximated coefficient of temperature which could be deduced from the uncorrected experiments.

§ 3. *The isothermals.*

In table I the results of the determination of isothermals are communicated. The pressure is expressed in atmospheres, the volumes have been measured by taking as unit the theoretical volume, viz.: the volume that would be occupied by the same quantity of the mixture at a temperature of 0° C. and under a pressure of 1 atm., if it followed the laws of a perfect gas²⁾.

The reduction to 0° C. was made by means of the coefficient of expansion of carbonic acid 0,00371³⁾, the reduction to 760 m.m. by means of BOYLE's law which in the range of pressures here coming into account was sufficiently accurate.

¹⁾ loc. cit.

²⁾ For further explanation see a following communication.

³⁾ See a following communication.

T A B L E I¹⁾

Mixture 0,0494 Isotherms

Temp	Volume 0.0	Pressure.	Temp	Volume. 0.0	Pressure.	Temp	Volume 0.0	Piess
15°.30	2719	31.41	26°.80 (cont.)	06480	79.30	27.30 (cont.)	04063	92.15
	2471	34.19		06050	80.95		03750	95.10
	2074	30.12		05704	82.35		03890	100.6
	1688	45.08		05330	83.90		02995	111.8
	1302	52.75		04197	90.25	27°.50 (or t.)	06180	79.95
	1187	55.35		04163	90.55		06095	81.55
	1152	56.25		04028	91.55		05710	83.20
	1110	57.20		03979	92.05		05540	83.90
	1074	57.55		03950	92.25		05380	84.60
	09180	59.25		03403	99.05		04975	86.00
	07240	62.15		03013	109.4		01645	88.45
	05975	67.75	27°.10 (pl. D. temp.)	05710	82.70		04573	88.95
	03539	80.55		05345	84.20		04415	89.60
	02543	102.9		04205	90.65		04410	89.80
	02500	103.1	(pl. p.)	04063	91.85		04306	89.95
	02471	111.3		04028	92.20		04297	90.55
	02428	117.0		03992	92.15		04255	90.85
21°.50	2071	40.63		03963	92.60	27°.90	04573	89.35
	1880	43.56		03928	93.00		04184	92.05
	1691	46.97		03915	93.20		03788	96.00
	1495	50.90		03716	95.10		03319	103.4
	1302	55.45	27°.30	2079	41.87		03051	111.2
	1112	60.40		1688	48.78		02952	115.5
	09190	65.95		1805	57.80	32°.10	2074	42.98
	08450	68.00		09190	69.85		1880	46.36
	07940	70.30		06470	79.80		1686	50.25
	05285	76.00		06120	81.25		1494	51.65
	03629	88.00		05700	83.05		1302	59.90
	02832	102.9		05330	84.55		1106	66.05
	02782	105.2		04946	86.50		09190	72.95
	02732	107.7		01523	88.05		07235	81.35
	02675	112.6		01255	90.55		05310	90.90
	02612	118.4		01219	90.85		04190	99.40
	00125	60.80		04197	91.05		03377	111.9
	07220	76.60		01105	91.65			

¹⁾ By corrections afterwards made, and which will be discussed in a following

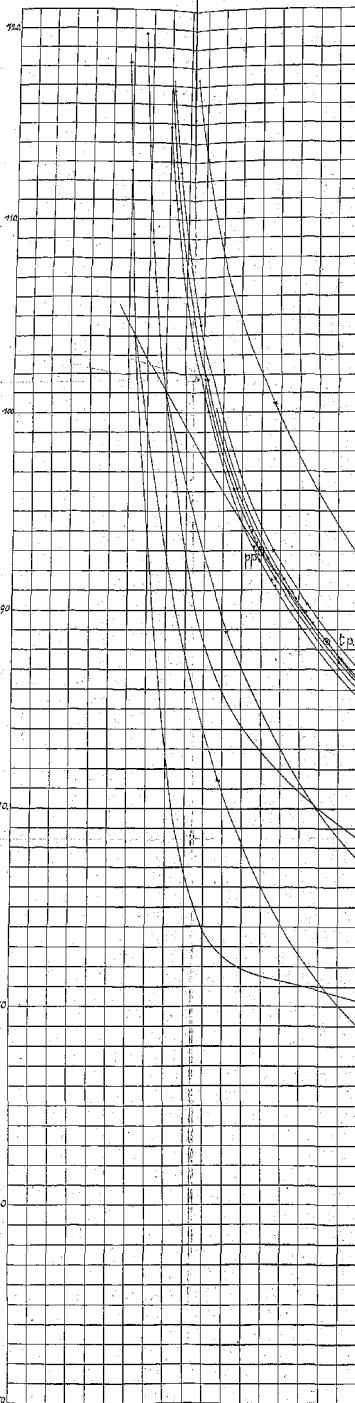


Fig. 1.

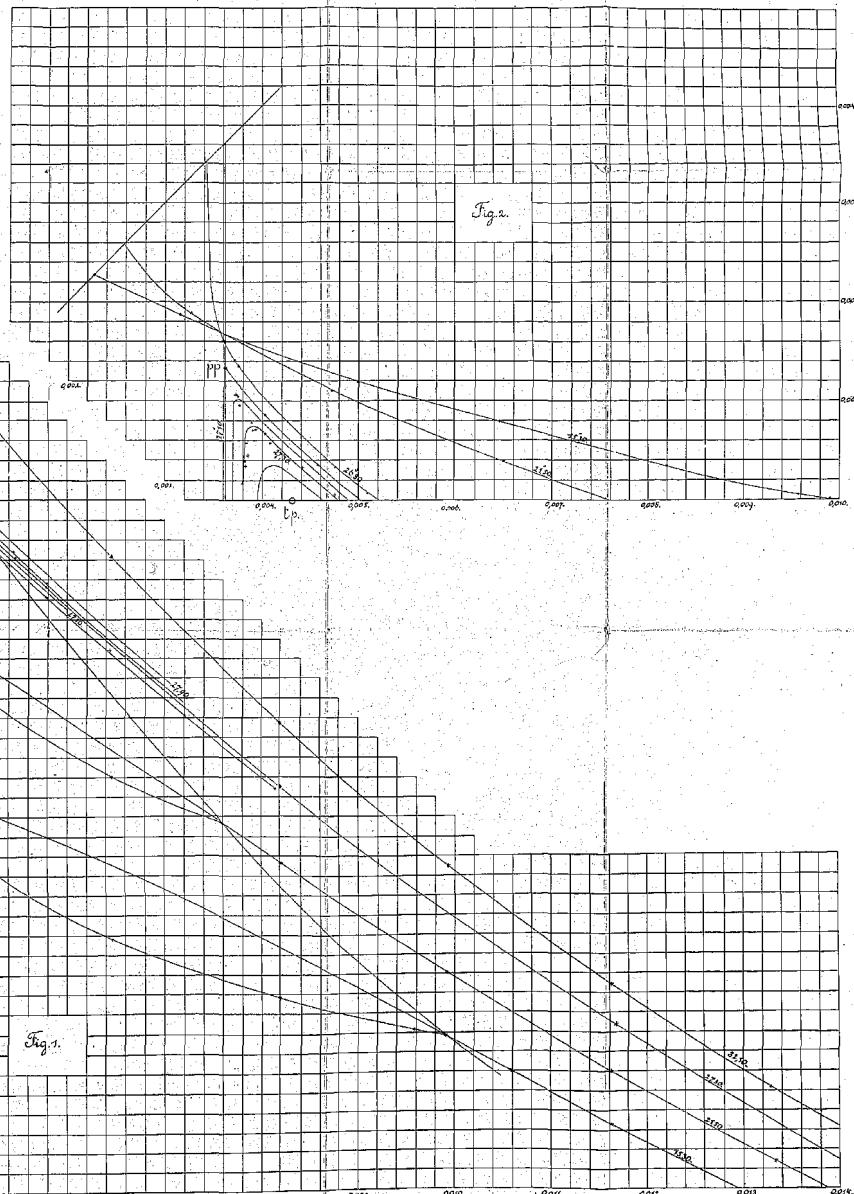


Fig. 2.

Fig. I shows the system of isothermal lines¹⁾) At temperatures below 27°.50 I observed separation into two phases; 27.50 is the temperature of the critical point of contact of the mixture. The isothermals of 15°.30 and 21°.50 show distinctly a discontinuity in slope resulting from the separation into two phases.

In the diagram the points where condensation begins and ends are connected by a curve, which forms the limit between the area where there is only one phase and the area where there are two. The common tangent to the border-curve and the isothermal is not horizontal as in the case of a pure substance. As for the course of the border-curve at still smaller volumes, it is bound to show a point of inflection somewhere and to be reversed finally towards the axis of the volumes. In the figure however the convex side is turned towards this axis, so that the reversing will probably occur only at a very high pressure. I observed that at a temperature of 27°.10 at a decrease of volume the meniscus became more and more indistinct and disappeared at last in the middle of the tube as a mist when the volume 0,004063 and the pressure 91.85 atm. were reached. And so this point is the plaitpoint for the composition $x = 0,0494$. The critical point of contact cannot be deduced with accuracy from the experiments themselves; the best way is to find out from the figure the point of contact of the critical isothermal and the border curve. In doing so we find with pretty great certainty the elements of the critical point of contact for the composition $x = 0,0494$; $t = 27°.50$; $v = 0.0048$; $p = 87.4$ atm.

The value of the coefficient of pressure for different volumes can be determined from the isothermals above the critical isothermal. These coefficients of pressure within the narrow limits of the temperature may be considered as constant, and the observations at large volumes point to this fact. These coefficients of pressure are:

$v = 0.0338$	$\left(\frac{dp}{dt}\right)_v = 0.138$
281	0.170
225	0.216
203	0.245
180	0.283
158	0.336

communication, the numbers here given and others occurring in this communication do not exactly agree with those published in the dutch communication. The volumes underlined are those where separation in two phases takes place.

¹⁾ Fig. I has been constructed at an arbitrary scale. The exact one we obtain by multiplying the abscissae by 1,127, and the ordinates by 0,99

= 0.0338	$\left(\frac{dp}{dt}\right)_v = 0.138$
135	0.409 .
112	0.515
101	0.58
090	0.68
079	0.83
068	1.00
056	1.22
045	1.50
034	2.40

With the aid of these coefficients of pressure the unstable parts of the isothermal lines were extrapolated and mapped on the figure. Only for $15^{\circ}.30$ and $21^{\circ}.50$ the extrapolated parts deviate sufficiently from those observed, to be clearly distinguished. As in the case of a pure substance the two curves intersect and the areas thus included must be equal as in the case of a pure substance¹⁾. We can immediately see on the figure that this condition is satisfied, in so far as we can judge by examination only. A more accurate test by measuring the areas is useless on account of the uncertainty of the extrapolation we have made.

§ 4. *The course of the condensation.*

Below the temperature of the plaitpoint the process of the condensation was normal: the quantity of fluid was constantly increasing while the volume was decreasing, so that finally the whole space was filled with liquid. Between the temperature of the critical point of contact and the plaitpoint-temperature retrograde condensation was distinctly observed.

In order that we might represent the process of the condensation graphically, I measured volumes of liquid belonging to definite volumes. The results of these measurements are given in Table II.

This table is graphically represented in fig. 2. There we can distinctly see the normal course of the condensation below the plaitpoint-temperature $27^{\circ}.10$; the lines of condensation are constantly rising and end in points lying on a straight line with a slope of 45° : at these points the volume of the liquid is equal to the total volume. Above the plaitpoint-temperature the condensation-line rises first but finally returns with great rapidity to the axis of abscissa; the course of these condensation-lines is therefore very asymmetrical, and even more so the closer we get to the plaitpoint; in approaching

¹⁾ See BLUMCKI, Zeitschr. f. physik. Chem., VI, p. 157.

T A B L E II.
Mixture $x = 0.0494$. Condensation lines.

Temp.	Total vol. 0.0	Lq. vol 0.0	Temp.	Total vol 0.0	Lq. vol. 0.0	Temp.	Total vol. 0 0	Lq. vol. 0.0
15°.30	1111	00015	26°.80	05700	00076	27°.30 (cont.)	04573	00644
	1074	00056		05330	00369		04523	00758
	09190	00388		04197	01522		04418	00799
	07235	00875		04163	01585		04374	00786
	05275	01428		04028	01798		04331	00877
	03539	02093		03979	01936		04298	00644
	02543	02543		27°.10 (pl. p. temp.)	05120		04318	00174
	08450	00021		04524	00966		04283	00438
	07285	00446		(pl. p.)	04073		04297	00383
	C5285	01231		27°.30	05330		04255	00190
	03660	02115		04948	00376		.	.

the plaitpoint from below the condensation-lines in the last part become more and more steep. As for the condensation-line of the plaitpoint-temperature itself, this terminates in the point with the abscissa 0,004073, and the ordinate 0,001486. We may however also consider the perpendicular through that point as forming part of this condensation-line, and then the steepness of the acclivity on the one side and the steepness of the declivity on the other side are easier to be understood.

Finally I give the volumes and pressures at the beginning and the end of the condensation as read on figures 1 and 2.

Beginning of the condensation. End of the condensation.

Temperature.	Volume.	Pressure.	Volume.	Pressure.
15°.30	0.01111	57.20	0.002543	102.9
21°.50	0.008545	67.90	2892	100.0
26°.80	5850	81.75	3833	93.20
27°.10 (pl. p. t.)	5625	83.00	4063	91.85
27°.30	540	84.6	427	90.5
27°.50 (cr. t.)	48	87.4	48	87.4

These data when brought in connection with those for other mixtures may serve for mapping the binodal curves on the ψ -surfaces, and for constructing the p - t - x -diagram.

Physics. — „*Measurements on the magnetic rotatory dispersion of gases.*” By Dr. L. H. SIERTSEMA. (Communication N°. 46 from the Physical Laboratory at Leiden by Prof. H. KAMERLINGH ONNES.)

(Read in the meeting of December 24th 1898.)

1. *Description of the apparatus.* The figures given here are intended to complete and to illustrate the description¹⁾ of the apparatus used in the investigation.

Fig. 1 shows a general view of the whole apparatus and of the different electric circuits, fig. 2—5 show the two nicolbearers in section. For the description of these I refer to the communication of March 1896, for the coils to that of June 1893, for the optical apparatus and several other details to that of January 1895. The description of the mirror-reading of which fig. 6 gives a general view, and fig. 7 and 8 represent some details, may also be found in the latter communication. Fig. 9 represents the high-pressure tubes with the manometer, fig. 10 shows one of the high-pressure stopcocks that have been used. Fig. 11 represents a section of the system of tubes for the circulation of water together with the thermometers.

EXPLANATION OF THE FIGURES.

Fig. 1. *General arrangement of the apparatus.* A collimator, B smaller nicol-bearer, C and D coils, E greater nicol-bearer, FF experimental tube, G screw for the rotation of the smaller nicol-bearer, which is connected with it by the steel wire HH, guided by the pulley O; I weight for pulling back the nicol-bearer. L vertical graduated glass scale on which the rotation of the mirror N is read in the telescope K by means of the intermediate mirror M, P prism and Q telescope of the spectrometer, X switchboard where the currents are supplied, γ arc-lamp, the motor of which is fed through a thinner wire originating from terminal 3, α resistance and β ammeter in the arclamp-circuit, ε switch for transferring the current from the coils to the resistance Y or for breaking the current, δ commutator for reversing the current in the coils, Z switch by means of which the whole apparatus can be switched out except the shunt T, which sends a derived current to the galvanometer V with the stop-commutator U.

Fig. 2—5. *Section of the two nicol-bearers.* a nicol, mounted in a cylinder q, and kept in its place in the nicol-bearer m by means of a ring b with adjusting screws, f experimental-tube, on to which the tube with flange n has been soldered and which is fastened to the nicol-bearer by the nut g with the stuffing ring t, e connecting nut for the high-pressure conduit, h level, o flange with six bolts and nuts p, c glass plate with nut d and leaden-stuffing ring r, which is prevented from being forced out by small rings s, u handle to which the steel wire h (comp. H fig. 1) is fastened.

¹⁾ Verslag Kon. Akad. 1893/94, p. 31; 1894/95 p. 230; 1895/96 p. 291, 317; 1896/97 p. 131, 132; Comm. Phys. Lab. Leiden N°. 7, 15, 24, 31.

Fig. 1.

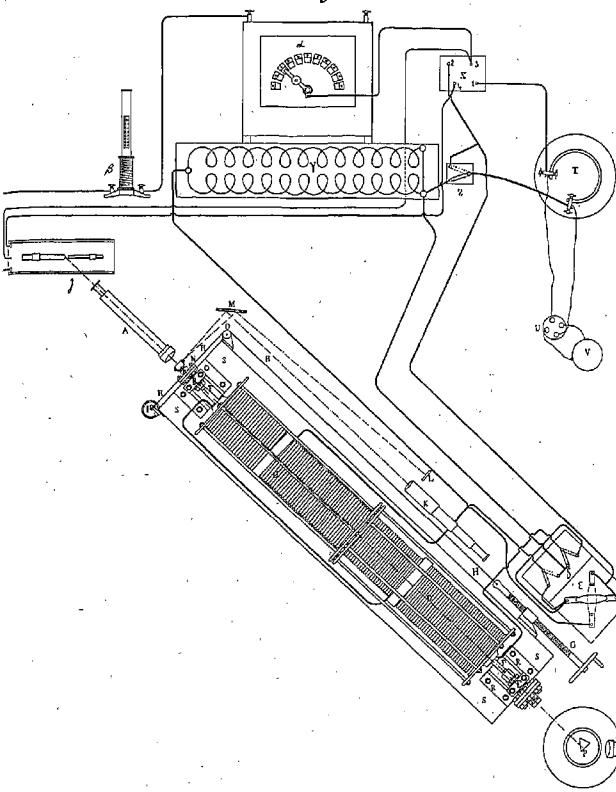


Fig. 2.

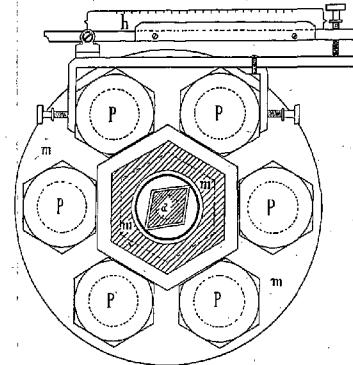


Fig. 3.

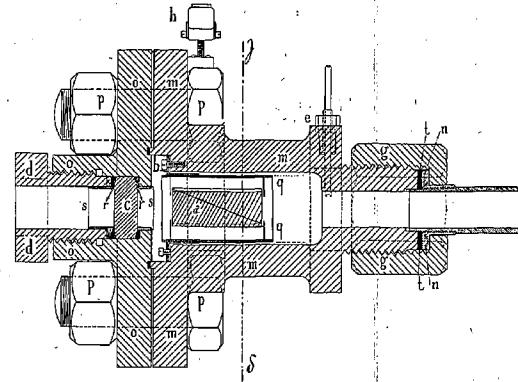


Fig. 4.

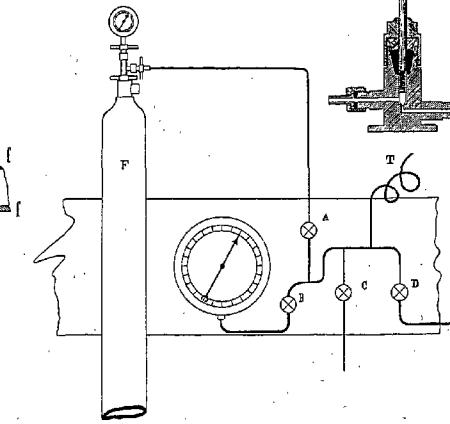


Fig. 10.

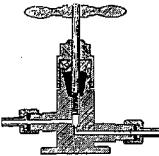


Fig. 6.

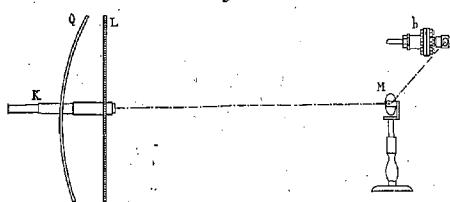


Fig. 11.

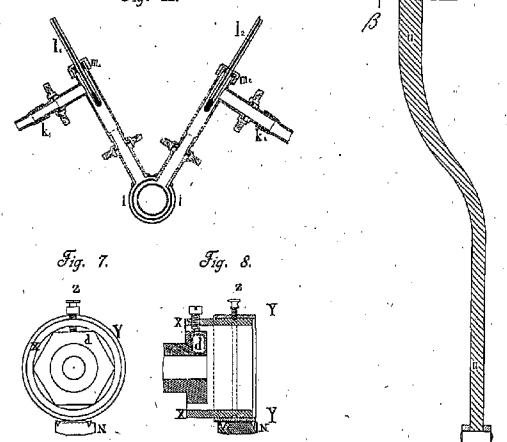


Fig. 7.



Fig. 8.

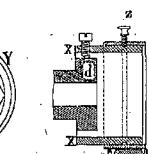


Fig. 5.

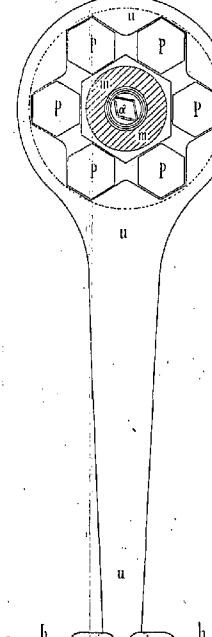


Fig. 12.

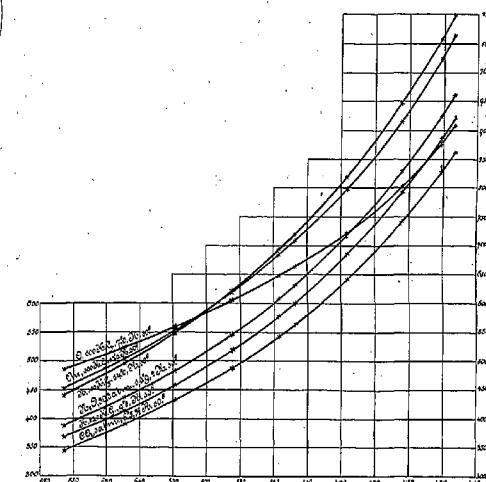


Fig. 6. General arrangement of the mirror-reading. *K* telescope, *L* graduated glass scale, illuminated by the mirror-strips *Q*, *M* intermediate mirror, *N* reading mirror.

Fig. 7 and 8. Mounting of the mirror on the nut in the flange of the smaller nicol-beaver. *d* nut, *X* and *Y* two rings, *Z* fastening-screws, *N* reading-mirror.

Fig. 9. - High-pressure conduit. *F* cylinder with compressed gas, *A* high-pressure stopcock for closing the former, *B* stopcock for closing the manometer, *C* and *D* stopcocks for drawing off the gas, *T* connection with the experimental tube.

Fig. 10. High-pressure stopcock. See *A*, *B*, *C* and *D* of the preceding figure.

Fig. 11. Sections of the tubes for the circulation of water together with thermometers. *i i* tubes in sections, *k*₁ supply, *k*₂ outlet, *l*₁ and *l*₂ thermometers, *m*₁ and *m*₂ nuts.

Fig. 12. Curves of the rotatory constants. See further on.

2. Formulae for interpolation of different forms. It has been stated in a previous communication¹⁾ that the different theories of the magnetic rotation lead to two forms of formulae for the rotatory constants of ω , viz.

$$\omega = \frac{C_1}{\lambda} + \frac{C_3}{\lambda^3} + \dots \quad \text{(I)}$$

$$\omega = \frac{C_2}{\lambda^2} + \frac{C_4}{\lambda^4} + \dots \quad \text{(II)}$$

With a view to the fact that in the case of oxygen we obtained with the first form a better accordance with the rotatory constants observed than with the second form²⁾, we have published in the later communications only formulae for interpolation of the form (I) with two terms. In the meantime the continuation of the calculations showed that for nearly all gases the form (II) with two terms was more satisfactory. The contrary proved to hold only for oxygen and for the mixtures of oxygen and nitrogen, among which we may mention in the first place air, further a mixture with 87.8 percent oxygen, on which some observations have been made in the be-

¹⁾ Versl. Kon. Akad 1894/95 p. 237; Comm. Phys. Lab. at Leiden N°. 15, p. 27.

²⁾ " " " " p. 238; " " " " N°. 15, p. 28.

ginning¹⁾, and a mixture with 26.0 percent, on which only a few observations have been made not published hitherto. For these mixtures form (II) required three terms, as was the case with oxygen.

For judging of the accordance of the formulae we have always calculated the probable error of an observation of the weight one, according to the rules of the method of least squares. In the two first columns of the following table these probable errors are to be found. They are computed by taking three terms in the form II for oxygen and mixtures containing oxygen as said above, and two terms for the other gases, and belong to the rotatory constants expressed in minutes, multiplied by 10^6 and for the pressure and the temperature mentioned in the observations.

P R O B A B L E E R R O R S.

Form of formula for interpolation:	I	II	III
Oxygen	5.8	5.4	5.0
Mixture with 87.8 percent O	6.9	7.8	
" " 26.0 " O	3.9	3.0	
Air	3.2	2.7	4.4
Nitrogen	9.8	7.4	7.7
Hydrogen	8.9	4.7	4.2
Carbonic acid I	0.147	0.122	0.106
" " II ²⁾	0.184	0.081	0.047
Nitrogen monoxide	5.4	3.0	2.2

An investigation of the errors, which are to be anticipated in the different adjustments and readings, always yielded for the error to be expected in the rotatory constant numbers larger than those of the table given above, e. g. for oxygen 11 in the violet and 6 in the red. With a view to this fact the accordance for all formulae mentioned may be considered satisfactory. Yet in the gases without oxygen the errors of the form (II) with two terms are always smaller than those of (I), so that if in choosing between the two forms we had to consider this only, we should prefer for these gases the form (II). But it is not excluded that other forms of formulae for interpolation should yield a still smaller error. This appeared to be the case with the form

¹⁾ Versl. Kon. Akad. 1894/95, p. 236. Comm. Phys. Lab. at Leiden №. 15, p. 26.

²⁾ Observations with gas from two cylinders of unequal purity.

$$\omega = a + \frac{b}{\lambda^2} \dots \dots \dots \quad (\text{III})$$

which gave the errors given in the third column of the table.

3. *Law of mixtures.* When we wish to apply the corrections for impurities in the gases investigated, we have to assume a law, from which the magnetic rotatory constant of a mixture can be calculated from the constants of its constituents. In a former communication¹⁾ a simple law has been assumed, without any consideration of its theoretical basis.

We can deduce this law from the supposition that the magnetic rotation is an additive property, and that the rotatory constant of a mixture is therefore equal to the sum of a number of terms, to which each molecule contributes one. Let the condition of the mixture be determined by the pressure p , by the temperature t and for the rest by assuming that in the unit of volume μN grammolecules of one component are mixed with $(1 - \mu) N$ of the other. Let $\varrho'_{p, t, \mu}$ be the amount which a grammolecule of the first component contributes to the rotatory constant, and $\varrho''_{p, t, 1-\mu}$ the same for the other component and let $N \varrho_{p, t, \mu}$ be the rotatory constant of the mixture, then we find that

$$N \varrho_{p, t, \mu} = N \mu \varrho'_{p, t, \mu} + N(1 - \mu) \varrho''_{p, t, 1-\mu}$$

and therefore

$$\varrho_{p, t, \mu} = \mu \varrho'_{p, t, \mu} + (1 - \mu) \varrho''_{p, t, 1-\mu}.$$

The quantities $\varrho_{p, t, \mu}$, introduced here, which we might call molecular rotatory power, will depend on the condition of the molecules determined by p, t and μ . The way in which they depend on these variables cannot be deduced from the observations without the aid of a molecular theory, except in the case when $\mu = 0$ and we have therefore to deal with simple substances.

In applying this law we assume these quantities ϱ to be constants and then we can deduce them from those for the simple gases. Let r' and r'' be the rotatory constants for the two gases at the pressure p and the temperature t . Let further x and y be the volumes of the two substances which we should have to mix in order to obtain the unit of volume of the mixture, all these volumes being measured

¹⁾ Verslag Kon. Akad. 1894/95, p. 236. Comm. Phys. Lab. Leiden N°. 15 p. 26.

at the pressure p and the temperature t . Then we may deduce from the foregoing law that

$$r = x r' + y r''$$

which expression agrees with the one assumed before. It may be remarked that the quantities r , r' , r'' , x and y are taken at one and the same pressure p and one and the same temperature t , and moreover that on account of discrepancies from DALTON's law $x+y$ may differ from 1.

This law of mixtures deduced for gases, agrees, if the molecular rotatory power is considered to be constant, with that which has been adopted by JAHN¹⁾, WACHSMUTH²⁾ and others for mixtures of liquids and saline solutions.

4. *Results.* Bij applying the above-mentioned law of mixtures we can calculate the rotatory constants of the pure gases, as has been shown before. The following table gives the coefficients of the formulae

$$\omega = \frac{C_1}{\lambda} + \frac{C_3}{\lambda^3} = \frac{C_1}{\lambda} \left(1 + \frac{d_1}{\lambda^2} \right) \dots \dots \quad (I)$$

$$\omega = \frac{C_2}{\lambda^2} + \frac{C_4}{\lambda^4} \left(+ \frac{C_6}{\lambda^6} \right) = \frac{C_2}{\lambda^2} \left(1 + \frac{d_2}{\lambda^2} \right) \dots \dots \quad (II)$$

which represent the rotatory constants ω , expressed in minutes, and multiplied by 10^6 .

	pressure	temp.	C_1	C_3	d_1	C_2	C_4	C_6	d_2
Oxygen	100 Kg.	7° 0	272.2	19 15	0.0704	296.7	- 48.03	4 294	
mixt with 87 8% O	" "	17° 0	240.6	23.30	0.097	249.1	- 30.57	2.521	
" " 26 0% O	" "	17° 6	200.7	40.93	0.204	210.1	- 7.71	0.805	
air	" "	13° 0	191.5	46.19	0.241	216.3	- 12.57	1.410	
nitrogen	" "	14° 0	171.2	52.86	0.309	183.6	2.27		0.0124
hydrogen	85.0 "	9° 5	138.8	45.19	0.325	151.5	2.38		0.0157
carbonic acid	1 atm.	6° 5	2.682	0.8305	0 310	2.894	0.0337		0.0116
nitrogen monoxide	30.5 atm	10° 9	75.85	22.95	0.303	81 26	0.820		0.0101

1) JAHN. Wied. Ann. 43, p. 284.

2) WACHSMUTH. Wied. Ann. 44, p. 380.

The numbers given differ slightly from those communicated before¹⁾ on account of a general revision of the calculations.

The law of mixtures may further be applied, as has been done before²⁾, to the mixtures of oxygen and nitrogen and the observed rotatory constants may be compared with those computed from the components. This comparison will prove very satisfactory in the case of air, but less so in the case of the two other mixtures.

In fig. 12 curves are given showing the relation between magnetic rotatory constant and wave-length, which are drawn by making use of the values calculated from some of the communicated formulae for interpolation. They prove clearly that the dispersion of the magnetic rotation for all gases, oxygen excepted, is pretty much the same. If we consider the large coefficient of magnetisation of oxygen, it becomes evident³⁾ that the magnetic rotatory dispersion depends on the magnetic properties of the substance, a fact to which H. BECQUEREL has drawn the attention⁴⁾.

It is a striking fact that the order of the gases examined, when they are arranged according to the values of the coefficients d_1 and d_2 , should be the same as the one found for the coefficients of magnetisation. This appears from the annexed table:

	d_1	d_2	$Q \cdot 10^{10} \cdot 2)$
oxygen	0.0704	-0.056 ¹⁾	0.662
nitrogen monoxide	0.303	+0.0101	-0.158
nitrogen	0.309	0.0124	-0.165
carbonic acid	0.310	0.0116	-0.172
hydrogen	0.325	0.0157	-0.176

¹⁾ Verslag Kon. Akad. 1896/97, p. 132. Comm. Phys. Lab. Leiden №. 31, p. 56.

²⁾ Verslag Kon. Akad. 1895/96, p. 301; 1896/97, p. 132; Comm. Phys. Lab. Leyden, №. 24 p. 10, №. 31 p. 6.

³⁾ H. BECQUEREL. Ann. de Ch. et de Ph. (5) 12 p. 85. Comp. also Verslag Kon. Akad. 1893/94 p. 31; Comm. Phys. Lab. №. 7 p. 9.

⁴⁾ The extraordinary large dispersion, observed by BECQUEREL in substances with negative magnetic rotation drew my attention to the negative constant which QUINCKE [Wied. Ann. 24. p. 615 (1885)] found in amber. A fresh measurement with a piece of transparent, light brown amber of 1.78 cm. thickness actually showed negative rotation. Yet it appeared that the emerging ray of light, even without the action of the electro-magnet, was strongly elliptically polarized. In connection with the theoretical investigations on the magnetic rotation in doubly refracting bodies (GOUY,

The fact that d_1 is almost the same for all gases with small coefficients of magnetisation seems favourable to MASCART's formula which leads to a formula for interpolation of the form (I); coefficient γ in this form is related in a simple way to d_1 , and is according to JOUBIN's investigation³⁾ in a high degree dependent on the coefficient of magnetisation.

If we express the rotatory constants in circular measure instead of minutes, we obtain the following coefficients

$$10^{10} \omega' = \frac{C_1'}{\lambda} + \frac{C_3'}{\lambda^3} = \frac{C_1'}{\lambda} \left(1 + \frac{d_1}{\lambda^2} \right) \quad (I)$$

$$10^{10} \omega' = \frac{C_2'}{\lambda^2} + \frac{C_4'}{\lambda^4} \left(+ \frac{C_6'}{\lambda^6} \right) = \frac{C_2'}{\lambda^2} \left(1 + \frac{d_2}{\lambda^2} \right) . . . \quad (II)$$

	pressure	temp.	C_1'	C_3'	d_1	C_2'	C_4'	C_6'	d_2
Oxygen	100 Kg.	7°.0	792	55.7	0.0704	863	— 139.7	12.49	
mixture with 87.8% O	" "	17°.0	700	67.8	0.097	725	— 88.9	7.33	
" " 26.9% O	" "	17°.6	584	119.1	0.204	611	— 22.43	2.342	
air	" "	13°.0	557	134.4	0.241	629	— 36.57	4.10	
nitrogen	" "	14° 0	498	153.8	0.309	534	— 6.60		0.0124
hydrogen	85.0 Kg.	9° 5	404	131.5	0.325	441	— 6.92		0.0157
carbonic acid	1 atm.	6° 5	7.80	2416	0.310	8.42	— 0.0980		0.0116
nitrogen monoxide	30.5 atm	10°.9	220.6	66.8	0.303	236.4	— 2.385		0.0101

For practical use the following table may serve, which gives the magnetic rotatory constants for different wave-lengths, expressed in minutes and multiplied by 10^6 .

WIENER) this result may easily be explained without assuming a negative rotatory constant. It is not improbable that QUINCKE's result may be accounted for in a similar way.

¹⁾ According to a formula for interpolation (II) with two terms, which is not mentioned in this paper.

²⁾ QUINCKE's constants relative to air. TÖPLER and HENNIG. Wied. Ann. 34, p. 790 (1880).

³⁾ JOUBIN. Thèse Paris 1888, p. 24. Ann. de Ch. et de Ph. (6) 16, p. 78 (1889).

λ	oxygen	mixture with 87.8% O	mixture with 26.0% O	air	nitrogen	hydrogen	oxygen	nitrogen monoxide
pressure	100 Kg.	100 Kg.	100 Kg.	100 Kg.	100 Kg.	85.0 Kg.	1 atm.	30.5 atm.
temp.	7°.0	17°.0	17°.6	13°.0	14°.0	9°.5	6°.5	10°.9
0.423	908	877	1074	1062	1097	921	17.23	480
431	875	849	1033	1020	1054	884	16.56	461
454	799	779	930	914	944	791	14.83	413
486	721	698	812	797	818	684	12.86	359
517	663	634	720	707	719	600	11.30	315
527	646	616	694	682	691	576	10.86	303
555	604	570	628	618	620	517	9.75	272
589	559	522	561	553	548	456	8.62	241
619	523	487	510	504	495	412	7.78	218
656	484	449	457	452	439	365	6.91	193
$\frac{p}{p_w} =$	0.531			0.559	0.563	0.570	0.838	0.616

In the last line of this table the values of the molecular rotatory power for Na-light divided by that of water, are given according to the example of PERKIN¹⁾. Although it appears from his investigations and those of others that the calculation of the molecular rotatory power of a chemical compound as the sum of that of its atoms does not hold good throughout, it may be interesting to compare PERKIN's atomic rotations with those resulting from our observations. From the above-mentioned values of the molecular rotatory power follows for 1 atom:

$$O = 0.265$$

$$N = 0.281$$

$$H = 0.285.$$

PERKIN finds for oxygen 0.194 or 0.261 according to the way of composition, and for hydrogen 0.254, numbers as we see of the same order. PERKIN's number 0.717, calculated for nitrogen in amines

¹⁾ PERKIN. J. of Chem. Soc. 45, p. 421 (1884), and following volumes.

does not agree with our number; neither is our value of N₂O to be composed from PERKIN's atomic values.

If following PERKIN we take in CO₂ for 1 atom carbon 0.515, we find O₂ = 0.323, which would be more agreeable to the value 0.39, which HINRICH¹⁾s deduced from organic acids. Taking everything together the agreement is fairly well, except for nitrogen.

¹⁾ HINRICH^s, C. R. 113, p. 500 (1891).

KONINKLIJKE AKADEMIE VAN WETENSCHAPPEN
TE AMSTERDAM.

PROCEEDINGS OF THE MEETING
of Saturday February 25th 1899.

(Translated from: Verslag van de gewone vergadering der Wis- en Natuurkundige Afdeeling van Zaterdag 25 Februari 1899 Dl. VII).

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The following papers were read:

Mathematics. — „On the orthoptical circles belonging to linear systems of conics.” By Prof. J. DE VRIES.

1. The locus of the points through which we can draw orthogonal tangents to the conic

$$\frac{x^2}{A} + \frac{y^2}{B} = C,$$

is the circle represented by

$$x^2 + y^2 = (A + B) C.$$

This orthoptical circle (MONGE's circle) is real for the ellipses and the hyperbolae situated in the acute angles of their asymptotes, and imaginary if the hyperbola lies in the obtuse asymptote-angles; it degenerates into a point for the rectangular hyperbola ($A+B=0$) and for the system of two right lines ($C=0$). With the parabola it is represented by the directrix.

The orthoptical circle being concentric with the conic to which it belongs, the centres of the orthoptical circles ω of a pencil of conics lie on the conic γ , containing the centres of the pencil.

If now, following Mr. FIEDLER, we represent each circle by the vertices of the two right cones of which it is the base-circle, the system (ω) is represented by a skew curve of the 4th order Ω_4 . Indeed, each plane perpendicular to the plane V of the orthoptical circles intersects the conic in two points and bears two pairs of points representing orthoptical circles.

The skew curve Ω_4 being intersected in four points by a plane parallel to V , there are in (ω) four circles with given radius.

So the system (ω) contains four point-circles; it is evident that we find these in the double points of the three degenerated conics and in the centre of the orthogonal hyperbola belonging to the pencil.

2. The system (ω) contains two orthoptical right lines, viz. the directrices of the two parabolae in the pencil. Each of these lines is represented by two planes inclined to the plane V at angles of 45°, i.e. parallel to four generatrices of each rectangular cone Π_2 having its vertex in a point P of V and placed symmetrically with respect to this plane.

The cone Π_2 bearing the images of all the circles through P , and the above named four points at infinity representing two right lines not passing through P , the remaining four points of intersection of Π_2 with Ω_4 will represent two circles (ω) intersecting in P . Therefore:

The orthoptical circles of a pencil of conics form a system with index two.

This can also be shown as follows. The tangents through P to the conics of the pencil are arranged in a (2,2)-correspondence, each ray through P being touched by two curves. This system has

two pairs of rays in common with the involution of the lines, intersecting in P at right angles; so through P pass two circles ω .

Two planes intersecting the plane V in the right line l at angles of 45° have four pairs of points in common with the curve Ω_4 . This shows that l is touched by four orthoptical circles. Their points of contact are the coincidences of the $(2,2)$ -correspondence, determined on l by the system (ω) .

3. If we represent the form

$$x^2 + y^2 + ax + by + c$$

by C , the system (ω) is represented by the equation

$$C_1 + 2\lambda C_2 + \lambda^2 C_3 = 0.$$

The power of the point (x, y) with regard to the circle indicated by a definite value of λ is then represented by

$$\frac{C_1 + 2\lambda C_2 + \lambda^2 C_3}{1 + 2\lambda + \lambda^2}.$$

This expression becomes independent of λ if we assume the radical centre of C_1, C_2, C_3 for (x, y) . So all the circles ω have a common radical centre or, in other words, all orthoptical circles cut a fixed circle A at right angles.

As A must bear the point-circles of (ω) we may conclude to the following theorem:

The circle through the diagonal points of a complete quadrangle contains the centre of the orthogonal hyperbola determined by the vertices of the quadrangle. Through its centre pass the directrices of the parabolae which can be drawn through those vertices.

Considering the obtained results as a property of the rectangular hyperbola it can be expressed in the following terms:

The diagonal points of each complete quadrangle inscribed in a rectangular hyperbola can be connected by a circle with the centre of that curve.

4. All circles ω being orthogonal to the fixed circle A their images lie on the orthogonal hyperboloid of revolution with one sheet cutting V in A at right angles.

So the image Ω_4 is the section of this hyperboloid with the cylinder, which is orthogonally cut by V in the conic γ .

When the base points of the circle form an orthocentric group¹⁾ all the conics are rectangular hyperbolae; so the system (ω) consists entirely of point-circles.

The orthogonal circle A , bearing them, passes through the diagonal points of the complete orthogonal quadrangle and coincides with the circle of FEUERBACH (nine-points circle), containing as is known the centres of all the orthogonal hyperbolae through the vertices and the orthocentre of a triangle.

5. According to the method of FIEDLER the orthoptic circles of a net of conics will be represented by a surface, intersecting the plane V of the net in the locus of the orthoptical point-circles.

Any point P determines a pencil belonging to the net, one of the base points of which is P ; so through P passes an orthogonal hyperbola. From this follows, that the orthogonal hyperbolae form a pencil situated in the net. As was said above the orthoptical point circles of this pencil lie on the circle A , passing through the diagonal points and the middle points of the six sides of the quadrangle of the base points.

The remaining point-circles ω are centres of pairs of lines and form the cubic curve, called the HESSIAN of the net.

Three pairs of lines of the net belong to the pencil of orthogonal hyperbolae and consist of orthogonal right lines. The double points must lie on the HESSIAN as well as on the circle A ; so these curves must touch each other in three points.

Both loci of point-circles forming together a curve of the 5th order, the image of the system of orthoptical circles is a surface of the 5th order S_5 .

Each right line of V is touched in each of its points by *one* conic of the net; each right line determines the direction of the axis of a parabola belonging to the net. From this follows, that the point at infinity of each right line cutting V at angles of 45° is to be regarded as the image of an orthoptical right line.

So the points of contact of the asymptotes of the rectangular hyperbola representing all the circles through two points belong to the ten points common to S_5 and that hyperbola. The remaining eight points of intersection representing four circles it is evident from this that we can draw four orthoptical circles of the system through any two points.

¹⁾ That is to say: four points, each of which is the orthocentre of the triangle having the others for vertices.

6. If U_x and U_y represent the differential quotients of the quadratic function U with regard to x and y , the centres of the conics of the net

$$U + \lambda V + \mu W = 0$$

are indicated by the relations

$$\begin{aligned} U_x + \lambda V_x + \mu W_x &= 0, \\ U_y + \lambda V_y + \mu W_y &= 0. \end{aligned}$$

So in general any point (x, y) is the centre of one conic. On the other hand any of the four points determined by

$$U_x : U_y = V_x : V_y = W_x : W_y$$

is the common centre of an infinite number of conics. For each of these points the above mentioned linear equations are dependent of each other, so that λ and μ are connected by a linear relation; so in this way a pencil of conics is characterized.

Consequently the surface S_5 contains four right lines perpendicular to V .

7. Let us now consider the orthopatical circles belonging to the conics with four common tangents. Any right line through the point P determining a conic of the tangential pencil, the tangents through P form an involution. This contains only one pair of rectangular rays; so P lies on one orthopatical circle.

The wellknown property according to which the centres of the tangential pencil lie in a right line, also proves that the orthopatical circles of a tangential pencil form a pencil.

According to the circles of that pencil passing through two fixed points or intersecting a fixed circle orthogonal or touching each other in a fixed point, the system (ω) will be represented by a rectangular hyperbola with its real axis perpendicular to V or situated in V , or by two right lines cutting V at angles of 45° .

So the system (ω) contains no more than two point-circles or, in other words, the tangential pencil can contain only two rectangular hyperbolae.

A conic of the tangential pencil degenerating into two points, the line joining these points is the diameter of the corresponding ω . In connection with the statement above the wellknown property results from this:

The circles described on the diagonals of a complete quadrilateral as diameters belong to a pencil.

8. Finally we consider a tangential net.

Two right lines through P which intersect at right angles are touched by one conic of the net. If the right angle included by these lines turns about the vertex P the pairs of tangents drawn through Q to the variable conic form an involution. For every ray s , drawn through Q , determines a tangential pencil belonging to the net, having the pairs of an involution (r, r') in common with the point P . The conic touched by the orthogonal pair (r, r') has a second tangent s' through Q , forming with s a pair of the involution indicated above. The orthogonal pair (s, s') determining a conic for which r and r' are at right angles to each other, we can draw but one orthoptical circle through the points P and Q .

According to the circles (ω) possessing an orthogonal circle or intersecting a fixed circle in two diametrically opposite points or passing through a fixed point, the obtained net of circles is represented by an orthogonal hyperboloid of revolution with one or two sheets or by an orthogonal right cone.

Chemistry. — “*On solubility and meltingpoint as criteria for distinguishing racemic combinations, pseudoracemic mixed crystals and inactive conglomerates.*” By Prof. BAKHUIS ROOZEBOOM.

Though several times attention has been drawn to the phenomena of solubility and melting in order to distinguish between the types mentioned above, no certainty has been attained as yet.

1. *Solubility.* We only get a clear insight in the phenomena of solubility by drawing attention to the number of solubility curves obtainable at a given temperature.

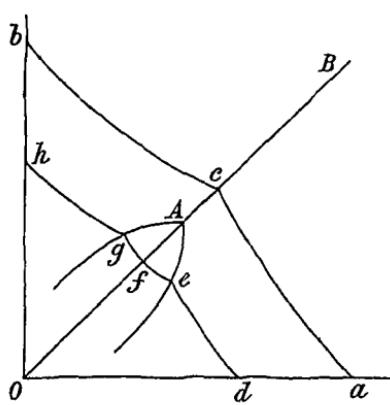


Fig. 1.

If Oa is the proportion of the saturated solution of the dextrosubstance D , and Ob the same for the laevosubstance L , these two are equal at the same temperature.

By adding L to the D -solution and vice-versa we now get, if no racemic combination appears at the temperature used, nothing else than two solubilitycurves, starting from the points a and b and meeting in c . Their precise direction depends on

the manifold actions that can take place in the solution. From the perfect equivalence of L and D it results however necessarily that c must always lie on the line OB , which halves the angle of

the axes, so that the solution which is equally saturated with *L* and with *D* is inactive, no matter how great the excess of *L* or *D* with which it is in contact.

If at another temperature a racemic combination is possible, we get three solubility-curves *de*, *efg* and *gh*. The second regards the solutions of the combination; here *f* represents the pure solution, *e* and *g* those which are acquired in presence of an excess of *D* or *L*.

At the transition temperature the curve in the middle disappears (point *A*). Here too the solution is always inactive. If therefore an inactive substance is a conglomerate (by which I mean a solid mixture in which the components lie side by side separately) of *L* and *D*, we shall never get as a saturated solution anything else than point *c*; if the substance is a combination we can get three solutions, as we dissolve it alone or with an excess of *L* or *D*.

Phenomena observed at evaporation. On this subject experiments have lately been made by KIPPING and POPE. From the figure of the solubility-curves it is easy to deduct, in the graphic manner first instituted by SCHREINEMAKERS, that in case of the evaporation of a solution containing an excess of *D* or *L*, the solution arrives at last in point *c* — that is to say becomes inactive — in case no racemic combination exists at the temperature used. If on the other hand this combination does exist — we arrive with an excess of *D* to a final point in *e*, with an excess of *L* in *g*.

In this manner it would be equally possible to make out what is deposited: conglomerate or combination — if in these evaporation experiments care were always taken that the necessary crystallisationnuclei were present — a circumstance to which K. and P. have paid no attention.

Partially racemic combinations. These have lately been discovered by LADENBURG and been studied with a view to the solubility in *D*- and *L*-strychninetartrate.

The symmetry of fig. 1 disappears in that case owing to the unequal solubility of the two components. Consequently point *A* will generally lie no longer on *OB*. It results therefrom that the combination, even before its transition-temperature is reached, already possesses a temperature-interval of partial decomposition. So Mr. LADENBURG is wrong in the opinion that this combination in its transition-temperature must furnish a solution, containing an equal amount of *D*- and *L*-tartrate.

At temperatures situated in the decomposition interval, we now only get two kinds of solutions in case the combination is dissolved alone and with an excess of *D* or *L*.

Pseudo-racemic mixed crystals. As I demonstrated on a former occasion, mixed crystals must be considered as one solid phase. For mixed crystals formed of optical antipodes, if they exist, it will perhaps be possible to appear in every proportion. In such a case we should only get one solubility-curve in fig. 1, connecting the points *a* and *b* and being symmetrical again.

If at lower temperatures, a racemic combination should appear, the continued mixing-series would be broken by it. The sketch of the solubility curves would consequently be altered in so far that the lines *EA* and *CA* would pass into one continuous curve. The evaporation phenomena would equally be altered.

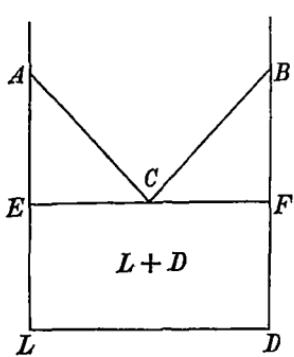


Fig. 2.

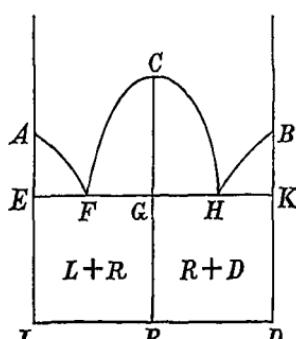


Fig. 3.

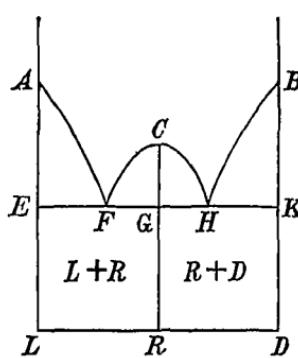


Fig. 4.

II. *Melting points.* The opinion exists that a higher melting point is a proof of a racemic combination when compared with the melting point of the antipodes. Uncertainty exists when the melting point of the inactive substance is equally high or lower.

Here too, it is only the study of the melting- and congealingcurves along their entire lengths, that can give us sufficient certainty.

If no combination, nor mixed crystals exist, fig. 2 must give the groundplan for the congelation. The mixing proportions of *L* and *D* are indicated on the horizontal axis, the temperature on the vertical. *A* and *B* are the melting points of *L* and *D*. *AC* is the congealing-line for liquids depositing *L*, *BC* for such as deposit *D*. Every mixture congeals at last in *C* to an inactive conglomerate of *L* and *D*. If on the other hand there is a racemic combination, two types are possible, represented by fig. 3 and 4. In these *C* is the melting point of the combination, which is higher or lower than the melting points *A* and *B*. Independent of this fact, the congelation now leads to three curves, the one in the middle representing the case that the combination is deposited. It has two branches.

If, to end with, there is a continual mixing-series, there will be but one congealing curve. But the latter need not be, in a repre-

sentation as the one used here, a horizontal line connecting A and B -as was the opinion of KIPPING and POPE, but may quite as well show a maximum or a minimum, which then however lies at 50 pCt. I even suspect such to be the case in camphersulfonic chlorid and carvontribromide.

According to these views, neither a higher nor a lower melting-point furnishes a proof on the nature of an inactive substance, but the study of the entire melting-line does.

A single curve serves in case of mixed crystals, two curves in case of an inactive conglomerate, three in case of a combination.

Other remarkable phenomena may still present themselves, in case transformations of the combination, mixture or conglomerate appear after the congelation.

Mathematics. — “*A geometrical interpretation of the invariant $II(a b)^2$ of a binary form a^{2n} of even degree*”. By Prof.
 $n+1$
 P. H. SCHOUTE.

With regard to the creation of the beautiful theory of the invariants undoubtedly very much is due to SYLVESTER as well as to ARONHOLD, BOOLE, BRIOSCHI, CAYLEY, CLEBSCH, GORDAN, HERMITE and others. As early as 1851, indeed, he developed in his treatise: “*On a remarkable discovery in the theory of canonical forms and of hyperdeterminants*” (*Phil. Mag.*, Vol. II of Series 4, p. 391—410) the foundation upon which the theory of the canonical forms is based. The principal contents consist of the proof of two theorems. According to the first the general binary form of the odd degree $2n-1$ can always be written in a single way as the sum of the $2n-1^{\text{st}}$ powers of n binary linear forms; according to the second the binary form of the even degree $2n$ can be written as the sum of the $2n^{\text{th}}$ powers of n binary linear forms — and in that case in a single way too — only when a certain invariant vanishes. For this invariant with which we shall deal here compare a.o. GUNDELFINGER’s treatise in the “*Journ. f. Math.*”, Vol. 100, p. 413—424, 1883, and SALMON’s “*Modern higher algebra*”, 4th ed., p. 156, 1885. So the theory of invariants of a certain form of any kind is ruled by the question about the minimum number of homonymous powers of linear forms by which it can be represented. (Compare a.o. REYE in the “*Journ. f. Math.*”, Vol. 73rd, p. 114—122). With this the theory of involutions of a higher dimension and order are closely allied. Likewise theorems are deduced from it relating to

hypergeometry. For all this we refer to two important papers by Mr. W. Fr. MEYER. The first, published at Tübingen 1883, is entitled; "*Apolarität und Rationale Curven*"; the second, inserted as "*Bericht über den gegenwärtigen Stand der Invariantentheorie*" in the 1st Vol. published in 1892 of the "*Jahresbericht der Deutschen Mathematiker-Vereinigung*", is an invaluable report about this branch of Mathematics. On page 365 of the former work a theorem appears under γ_4 which is closely connected with our subject.

It goes without saying that it must be possible to reach conversely the above quoted theorems of SYLVESTER and the higher involutions connected with them starting from the theory of poly-dimensional space. Indeed, Mr. CLIFFORD has stated already in 1878 in his important treatise "*On the classification of loci*" (*Phil. Trans.*, Vol. 169, part 2nd, p. 663—681) that in this direction a geometrical interpretation of any invariant of a binary form is to be found. So in trying to determine a certain locus in space with an even number of dimensions I have fallen back upon a geometrical interpretation of the invariant of SYLVESTER; however examining the above mentioned literature I soon discovered that this interpretation had already been found.

Yet I wish to publish my study. In the first place because it may prove that the geometrical way is at least equally simple as the algebraical. Secondly on account of its containing a method of elimination I have as yet nowhere met with in this form. Thirdly because it is not quite impossible that entirely corresponding investigations may lead to a geometrical interpretation of other general invariants ¹⁾.

1. A curve allows of a twofold infinite number of chords, containing together a threefold infinite number of points. If this curve is situated in the space S^3 with three dimensions these points will fill the whole space and one or more of these chords will pass through any given point. If the curve is situated in the space S^4 with four dimensions the locus of the points through which chords pass, i.e. the locus of those chords themselves, is a curved space of the third order. The point from which we start here is the investigation of this curved space for the simplest possible case, namely

¹⁾ I think e.g. of the invariant $(ab)^{2n} (cd)^{2n} (ac)(bd)$ of a general binary form a_x^{2n+1} of odd degree (compare SALMON l. c., p. 129, problem 2nd) forming an extension of the discriminant of a_x^3 , of which as yet no general algebraical interpretation seems known.

that of the normal curve of the space S^4 that is of the rational C_4 of the fourth order, represented by the equations:

$$\frac{x_0}{1} = \frac{x_1}{\lambda} = \frac{x_2}{\lambda^2} = \frac{x_3}{\lambda^3} = \frac{x_4}{\lambda^4},$$

where x_0, x_1, x_2, x_3, x_4 are the homogeneous coordinates of the point L of the curve belonging to the parametervalue λ .

2. If λ_1 and λ_2 are the parametervalues of any two points L_1 and L_2 on C_4 , the coordinates of any point A of the line joining L_1 and L_2 are given by the relations

$$\left. \begin{array}{l} x_0 = p_1 + p_2 \\ x_1 = p_1 \lambda_1 + p_2 \lambda_2 \\ x_2 = p_1 \lambda_1^2 + p_2 \lambda_2^2 \\ x_3 = p_1 \lambda_1^3 + p_2 \lambda_2^3 \\ x_4 = p_1 \lambda_1^4 + p_2 \lambda_2^4 \end{array} \right\} \dots \dots \dots \quad (1)$$

and now by eliminating the four quantities $\lambda_1, \lambda_2, p_1, p_2$ we find the equation of the locus required.

The result of this elimination is the cubic curved space

$$\left| \begin{array}{ccc} x_0 & x_1 & x_2 \\ x_1 & x_2 & x_3 \\ x_2 & x_3 & x_4 \end{array} \right| = 0 \quad \dots \dots \dots \quad (2)$$

For if

$$\left| \begin{array}{ccc} p_1 + p_2 & p_1 \lambda_1 + p_2 \lambda_2 & p_1 \lambda_1^2 + p_2 \lambda_2^2 \\ p_1 \lambda_1 + p_2 \lambda_2 & p_1 \lambda_1^2 + p_2 \lambda_2^2 & p_1 \lambda_1^3 + p_2 \lambda_2^3 \\ p_1 \lambda_1^2 + p_2 \lambda_2^2 & p_1 \lambda_1^3 + p_2 \lambda_2^3 & p_1 \lambda_1^4 + p_2 \lambda_2^4 \end{array} \right|$$

is written down, it is immediately evident that every combination of partial columnus vanishes after easy simplifications, two of the three columns being equal to each other¹⁾.

¹⁾ As I already stated above, I nowhere met with this simple deduction of the invariant of SYLVESTER which can be pursued through all spaces S^{2n} . In the original

3. The osculating space belonging to the point L of C_4 has for equation

$$x_0 \lambda^4 - 4 x_1 \lambda^3 + 6 x_2 \lambda^2 - 4 x_3 \lambda + x_4 = 0 \quad \dots \quad (3)$$

So the coordinates x_0, x_1, x_2, x_3, x_4 are the coefficients of the form

$$x_0 (-\lambda)^4 + 4 x_1 (-\lambda)^3 + 6 x_2 (-\lambda)^2 + 4 x_3 (-\lambda) + x_4$$

of the fourth order in $(-\lambda)$, bare of the binomial factors.

So the result (2) can be written as $j = 0$ and represented symbolically by $(bc)^2 (ca)^2 (ab)^2 = 0$ (see a. o. CLEBSCH-LINDEMANN's "Vorlesungen über Geometrie", I, page 229). From this ensues at the same time that any point of the obtained locus is distinguished moreover from any point taken at random in the space S^4 by the property that the four osculating spaces of C_4 passing through it belong to four harmonic points of the curve. We shall point this out more directly. We suppose in the formulae (1) the quantities $\lambda_1, \lambda_2, p_1, p_2$ to be given; then by substituting in (3) the values ensuing from this for x_k we shall find

$$p_1 (\lambda - \lambda_1)^4 + p_2 (\lambda - \lambda_2)^4 = 0 \quad \dots \quad (4)$$

as the equation which gives us the parametervalues of the four points L of C_4 , whose osculating spaces intersect in the point A of the line $L_1 L_2$ given by (1). If for convenience' sake we take the points L_1 and L_2 as base points with the parametervalues 0 and ∞ , this equation can be reduced to $\lambda^4 - 1 = 0$ and the roots $1, -1, \sqrt{-1}, -\sqrt{-1}$ show immediately that the pairs of points belonging to $(1, -1)$ and $(\sqrt{-1}, -\sqrt{-1})$ separate each other harmonically, whilst each of these pairs behaves in the same way with reference to the pair of base points $L_1 L_2$ belonging to $(0, \infty)$. By this, not only the harmonic position of the four points (4) has been indicated but moreover the following theorem has been proved:

"Any two points L_1, L_2 on C_4 determine on this curve a quadratic involution I_2 of which they are the double points. If of this I_2 we join two pairs separating each other harmonically we get the biquadratic involution I_4 represented by the equation (4)

work (*Phil. Mag.* 1. c.) first a binary form of odd degree is discussed and the invariant belonging to forms of even degree are reached at the end by a round-about way. In SALMON (l. c.) and elsewhere the determinant is regarded as an extension of the covariant of HESSE built up out of second differentialquotients, here differentialquotients of the 4th order, etc.

"characterized by the particularity that each of the two points " L_1 , L_2 counted four times represents a quadruple of it. The osculating spaces belonging to the points of any quadruple intersect in "a point A of $L_1 L_2$. And if this quadruple describes I_4 , the point " A generates on $L_1 L_2$ a series of points in projective correspondence "with I_4 ."

Moreover we easily find ¹⁾:

"If A and A' are two points of the line $L_1 L_2$ harmonically separated by L_1 and L_2 , the quadruple of I_4 belonging to A has the combination of L_1 and L_2 with the quadruple of I_4 belonging to " A' for its sextic covariant T . And combination of the quadruples "belonging to A and A' generates an involution of the eighth order."

The indicated I_8 is represented by the equation

$$p_1^2 (\lambda - \lambda_1)^8 - p_2^2 (\lambda - \lambda_2)^8 = 0;$$

so it is characterized by the particularity that each of the two points L_1 and L_2 counted eight times represents an octuple of it.

4. We now pass to the space S^6 and there we determine the locus of the planes having three points $L_1 L_2 L_3$ in common with the normal curve C_6

$$\frac{x_0}{1} = \frac{x_1}{\lambda} = \dots = \frac{x_5}{\lambda^5} = \frac{x_6}{\lambda^6}$$

of that space. This is obtained by eliminating the six quantities $\lambda_1, \lambda_2, \lambda_3, p_1, p_2, p_3$ between the seven relations

$$x_k = p_1 \lambda_1^k + p_2 \lambda_2^k + p_3 \lambda_3^k \dots \dots \dots \quad (5)$$

where λ must take the values 0, 1, ..., 5, 6. In quite the same way as above we find here the curved space S_4^5 with five dimensions of the fourth order represented by

$$\left| \begin{array}{cccc} x_0 & x_1 & x_2 & x_3 \\ x_1 & x_2 & x_3 & x_4 \\ x_2 & x_3 & x_4 & x_5 \\ x_3 & x_4 & x_5 & x_6 \end{array} \right| = 0.$$

¹⁾ In fact, the form T' of $x_1^4 \pm x_2^4$ is $x_1 x_2 (x_1^4 \mp x_2^4)$.

The first member of this equation is again an invariant of the sextic

$$x_0(-\lambda)^6 + 6x_1(-\lambda)^5 + 15x_2(-\lambda)^4 + \dots + 6x_5(-\lambda) + x_6,$$

which made equal to nought indicates the osculating space belonging to the point L .

Now we find in the ordinary manner by passing to the use of symbolic coefficients and by noticing their mutual equivalence that the indicated invariant may be represented by $(ab)^2(ac)^2(ad^2)(bc)^2(bd)^2(cd)^2$. Naturally if the invariant ¹⁾ vanishes there is a connection between the six points of C_6 whose osculating spaces pass through the point A of the plane $L_1 L_2 L_3$ indicated by the formulae (5), for substitution of the values following out of (5) for the seven coordinates x_k in the equation of the osculating space of the point L gives

$$p_1(\lambda - \lambda_1)^6 + p_2(\lambda - \lambda_2)^6 + p_3(\lambda - \lambda_3)^6 = 0 \dots \quad (6)$$

So we have the following theorem :

"Any three points L_1, L_2, L_3 on C_6 determine on this curve an "involution I_6^2 of the second dimension and the sixth order of which "each of the three points counted six times represents a sextuple. "The osculating spaces belonging to the points of any sextuple intersect in a point A of the plane $L_1 L_2 L_3$; if the sextuple describes " I_6^2 , the point A generates in the plane $L_1 L_2 L_3$ a plane system in "projective correspondence with I_6^2 ."

The considered invariant of a_x^6 is indicated by SYLVESTER as "catalecticant" because its vanishing is the condition under which a_x^6 can be represented as the sum of three sextic powers; in connection with this an a_x^6 allowing this reduction is called a "meio-catalectic" sextic (*Phil. Mag.* l. c. page 408).

5. Finally we examine in the space S^{2n} the locus of the linear space S^{n-1} having n points L_1, L_2, \dots, L_n in common with the normal curve C_{2n} represented by

¹⁾ If according to the common notation $f \equiv a_x^6$ and $k \equiv (f, f)^4 = (ab)^4 a_x^2 b_x^2$ the fourth transvectant of f with itself, then the indicated invariant is the fourth transvectant $(k, k)^4$ of k with itself (see GORDAN-KIRSCHENSTEINER "Vorlesungen über Invariantentheorie", Vol 2, page 286).

For the following case $f \equiv a_x^8$ we have got to deal with an invariant of the fifth order in the coefficients. There being (see a.o. VON GALL's two papers in the "Math. Ann." Vol 17, p. 31—51 and 139—152, 1890 on "Das vollständige Formensystem einer binären Form achter Ordnung") but one invariant of this kind, our invariant must in this case correspond to the one indicated by the sign $f_{k, k}$.

$$\frac{x_0}{1} = \frac{x_1}{\lambda} = \dots = \frac{x_{2n-1}}{\lambda^{2n-1}} = \frac{x_{2n}}{\lambda^{2n}}.$$

For this the $2n$ quantities $\lambda_1, \lambda_2, \dots, \lambda_n, p_1, p_2, \dots, p_n$ must be eliminated between the $2n+1$ equations

$$x_k = \sum_{l=1}^{l=n} p_l \lambda_l^k, \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (7)$$

where successively $0, 1, \dots, 2n-1, 2n$ has to be substituted for k .

In the indicated way we shall get the result

$$\left| \begin{array}{ccccc} x_0 & x_1 & x_2 & \dots & x_{n-1} & x_n \\ x_1 & x_2 & x_3 & x_n & x_{n+1} & \\ x_2 & x_3 & x_4 & \dots & x_{n+1} & x_{n+2} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ x_{n-1} & x_n & x_{n+1} & \dots & x_{2n-2} & x_{2n-1} \\ x_n & x_{n+1} & x_{n+2} & \dots & x_{2n-1} & x_{2n} \end{array} \right| = 0. \quad (8)$$

Likewise in this general case the left hand member of this equation represents an invariant of the binary form of the $2n^{\text{th}}$ degree in $(-\lambda)$, which made equal to nought indicates the osculating space belonging to the point L of λ . In symbols this invariant is indicated by $\prod_{n+1} (ab)^2$ where \prod is the general sign of multiplication and where the index $n+1$ points to the fact, that the multiplication must be extended to the $\frac{1}{2}n(n+1)$ factors $(ab)^2$ which can be formed of $n+1$ set of coefficients a, b, c, \dots ¹⁾.

By substituting the values (7) in the equation of the osculating space we find

$$\sum_{l=1}^{l=n} p_l (\lambda - \lambda_l)^{2n} = 0. \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (9)$$

that is to say :

¹⁾ Probably the general notation $\prod_{n+1} (ab)^2$ makes its first appearance here. At least I found everywhere the notation in the form of a determinant and nowhere a symbolic representation nor a reduction to transvectants.

"By taking on the normal curve C_{2n} the n points L_1, L_2, \dots, L_n "arbitrarily we determine on it an involution I_{2n}^{n-1} of the dimension " $n-1$ and the order $2n$, of which each of those n points taken $2n$ "times forms a group. The osculating spaces belonging to the $2n$ "points of any group of that involution intersect in a point A of "the linear space S_1^{n-1} , containing the n given points; if this group "describes the involution I_{2n}^{n-1} the point A generates in S_1^{n-1} a linear "system in projective correspondence with I_{2n}^{n-1} ."

As far as I am aware of up to now a polydimensional interpretation suiting all values of n is known of three general invariants, namely of the discriminant D , of the invariant $(ab)^{2n}$, and of the invariant of SYLVESTER dealt with here. If $x_{-\lambda}^n$ is again the equation of the osculating space of the normal curve $x_k = \lambda k$, ($k = 0, 1, \dots, n$) in the space with n dimensions, corresponding to the parameter value λ , then $D = 0$ represents as is known the curved space $S_{2(n-1)}^{n-1}$ with $n-1$ dimensions of the order $2(n-1)$ which is enveloped by the osculating space if λ varies; leaving alone the supposition $n=2$, which has no sense, we get that $n=3$ gives in the ordinary space the developable surface having the cubic normal curve of that space as cuspidal line. According to CLIFFORD (l. c.) $(ab)^{2n} = 0$ is in the space S_1^{2n} the quadratic curved space S_2^{2n-1} with $2n-1$ dimensions representing the locus of the point lying in a space S_1^{2n-1} with the points of contact of the $2n$ osculating spaces of the normal curve of that space S_1^{2n} passing through this point; whilst the corresponding invariant $(ab)^{2n-1}$ of the normal curve of the space S_1^{2n-1} vanishes identically and the indicated particularity presents itself there, compare the case of the skew cubic in our space, for any point.

For the case $n=4$ the invariant $(ab)^4 = 0$ is identical with $i=0$ (CLEBSCH-LINDEMANN l. c.) and at the same time the condition that the four points of contact of the osculating spaces through any point of the locus form an equianharmonic quadruple. Moreover D is a linear combination of i^3 and j^2 , from which finally ensues that any plane cuts the space $D=0$ according to a curve of the sixth order, having the six points of intersection with the surface of intersection

of both spaces $i = 0, j = 0$ for cusps; in each of these points the section of the plane with the osculating space of $j = 0$ form the cuspidal tangent. As is known the space $D = 0$, also by the aid of its double surface $i = 0, j = 0$, divides the space S^4 into three parts containing the points for which the number of the real osculating spaces passing through them is successively 4, 2 and 0.

Physics. — Prof. HAGA made, both on behalf of himself and Dr. C. H. WIND a communication: „*On the deflexion of X-rays*”.

Deflexion of X-rays was proved on the experiment being arranged as follows:

The RÖNTGEN-tube was placed behind a slit 1 cm. high and 14 microns wide; at 75 cm. from the latter was the diffraction slit, which gradually diminished in width from 14 to about 2 microns. The photographic plate was placed at 75 cm. from the diffraction slit. Time of exposure from 100 to 200 hours. The image of the slit first became narrower and then showed an unmistakable broadening. From the width of the part of the diffraction slit corresponding to this broadening and the character of the broadening an estimation can be made of the wavelength. It appeared that X-rays exist of about 0.1 to $2\frac{1}{2}$ ANGSTRÖM units, comprising 4 octaves.

(A detailed paper will appear in the Proceedings of the next meeting).

Physiology. — Prof. STOKVIS presented for the Library the inaugural dissertation of Dr. G. BELLAAR SPRUYT: „*On the physiological action of methylnitramine in connection with its chemical constitution*.”

At different occasions our member Prof. FRANCHIMONT exposed in our meetings his views about the chemical structure of nitramines, especially of methylnitramine. Till yet the question about the intimate chemical constitution of these compounds, in reference to the manner, in which their nitrogen is linked with the other elements, is an open one. Whereas some authors believe, that the nitrogen of nitramines is linked with hydroxyle, so that the whole compound is a species of nitrite: $H—O—N=O$, Prof. FRANCHIMONT rejects this view, and considers it linked in a cyclical

way, for instance $H—N\begin{array}{l} \diagup \\ O \\ \diagdown \\ O \end{array}$ or $H—N\begin{array}{l} \diagup \\ O \\ \diagdown \\ O \end{array}=O$. As Prof. FRANCHI-

MONT considered it of some value to study the physiological action of nitramines, to the aim of throwing more light on the open

chemical question, Dr. SPRUYT took the matter in hand in the Pathological Laboratory of Amsterdam. In applying to nitramines and nitrites the well known law of the relation between chemical constitution and physiological action, which is best expressed in this formula: "compounds of homologous chemical structure possess homologous physiological action," we may conclude, that if nitramine should belong really to the group of nitrites, its physiological action must be also that of a nitritecompound. The physiological action of nitrites is a well known one. They are all toxic substances. Dr. SPRUYT considered it as a first step in his researches to state the physiological effects common to all nitrites without exception, to the nitrites as well derived from alcoholic radicals, as to the simple alkali-nitrites. As such he found invariably:

1st the formation of methaemoglobin out of the haemoglobin of the blood in the living body as well as in the blood "in vitro";

2nd dilatation of the arteriolae, and rapid sinking of the arterial bloodpressure;

3rd injurious effects on the intensity of the contraction of the isolated frog's heart, as fast as the nitrites circulate with the blood through it;

4th paralysis of the nervous system in frogs, convulsions in mammalians.

In experimenting on frogs and rabbits with methylnitramine-natrium, dissolved in a physiological salt-solution, Dr. SPRUYT never found one of the essential phenomena, which are produced by nitrites. The methyl-nitramine-natrium compound behaved itself in the animal body and its liquids on the contrary as a fast indifferent substance.

If we consider, that nitrite of amyle $C_5H_{10}O_2N_2$ is one of the best known nitrites, with an eminent toxic action, in which all the physiological effects of nitrites are represented in the most typical way, and if we pay attention at the same time to the remarkable fact, that SCHADOW in his study of the physiological action of nitropentan, which is an isomeric compound of nitrite of amyle never met with one of the essential phenomena, produced in animals by nitrites, the conclusions, which Dr. SPRUYT arrived at, are easily conceived. These conclusions are: that the study of the physiological action of methylnitramine makes highly probable the opinion of FRANCHIMONT about its chemical structure (nitrogen linked in a cyclical way), and is in direct contradiction with HANTSCH's hypothesis, that it should contain nitrogen linked to hydroxyle, and belong to the nitrite-group.

Physics. — „On a simplified theory of the electrical and optical phenomena in moving bodies”. By Prof. H. A. LORENTZ.

(Will be published in the Proceedings of the next meeting).

Physics. — „STOKES’ aberration theory presupposing an ether of unequal density”. By Prof. H. A. LORENTZ.

(Will be published in the Proceedings of the next meeting).

Physics. — „Measurements on the system of isothermal lines near the plaitpoint, and especially on the process of the retrograde condensation of a mixture of carbonic acid and hydrogen”. By Dr. J. VERSCHAFFELT. (Continued). (Communication n°. 47 from the Physical Laboratory at Leiden by Prof. H. KAMER LINGH ONNES).

§ 5. The course of the plaitpoint curve.

Two other mixtures $x = 0,0995$ and $x = 0,1990$ were investigated in the way described in § 2. But in the case of these two mixtures the investigation could not be made so completely as in the case of the first mixture. For $x = 0,0494$ we could trace the isothermals pretty far above the plaitpoint-pressure; but for $x = 0,0995$ the plaitpoint was situated towards the end of the series of observations, and for $x = 0,1990$ the observations could only be made till near the critical point of contact. By means of these observations, the results of which are communicated in the tables III and IV, we can derive some information regarding the course of the plaitpoint curve in mixtures of hydrogen and carbonic acid in the neighbourhood of pure carbonic acid.

T A B L E III.

Mixture $\alpha = 0.0995$. Isothermal lines

Temp.	Volume. 0.0	Pressure.	Temp	Volume. 0.0	Pressure.	Temp	Volume. 0.0	Press.
16°.90	2774	32.24	22°.80 (cont.)	07185	81.60	25°.00	03974	110.8
	2490	35.25		06495	85.05	25°.45 (C.R.D. of C. temp.)	06685	86.85
	2199	38.90		05955	87.80		06070	90.50
	2066	40.80		05325	91.95		04918	99.45
	1874	43.91		05260	92.45		04580	102.9
	1683	47.51		04171	104.1		04196	108.2
	1490	51.65		03612	114.6		03776	115.3
	1296	56.60	24°.20 (pl. p. temp.)	09145	73.45	26°.05	2069	42.82
	1106	62.25		07245	82.50		1873	46.27
	1067	63.50		06700	85.40		1685	50.20
	1029	64.85		06470	86.65		1486	54.90
	09940	66.00		06120	88.60		1300	60.25
	09565	67.30		05685	91.60		1107	66.85
	09145	68.30		05270	94.30		09185	74.50
	08800	69.10		04902	97.45		07240	84.20
	08360	70.25		04566	101.1		05300	96.90
	07990	71.35		04171	106.3		04566	103.9
	07165	74.00		03904	110.8		03806	115.8
	05240	84.20		03821	112.2	32°.30	2052	44.58
	03551	106.9	25°.00	03738	114.2		1870	47.93
	03284	113.6		07260	83.25		1675	52.20
22°.80	2061	42.20		06685	86.40		1479	57.15
	1877	45.40		06090	89.95		1297	62.85
	1683	49.26		05700	92.60		1109	96.95
	1480	53.90		05380	95.30		09145	78.60
	1293	59.10		04948	98.45		07220	89.70
	1110	65.15		04551	102.5		05255	105.4
	09145	72.55		04376	104.7		04249	118.1

T A B L E IV.
Mixture $x = 0,1990$. Isothermal lines.

Temp	Volume 0,0.	Pressure.	Temp	Volume 0,0.	Pressure	Temp	Volume 0,0	Press.
15°.35	2960	31.34	15° 35 (cont.)	05180	105 9	22°.80 (cont)	1316	63.20
	2700	33.84		04552	117 4		1083	72.85
	2423	37.27	20°.90	07810	89 15		09025	82.45
	2142	41.26		07465	91.75		07115	96.05
	2031	43.03		07105	94.55		05255	116.0
	1844	46.58		06735	97 70		05140	117.6
	1653	50.75		06335	101 5		05070	118.8
	1461	55 70		05965	105.1		04965	120.6
	1276	61.55		05575	109.6	31° 80	2018	47.00
	1090	68.90		05210	114 2		1838	50 90
	1051	70.55	22°.20	06715	99.05		1651	55.50
	1016	72.10		06340	102.5		1465	61.15
	09745	73.95		05965	106.4		1275	68.20
	09385	75.80		05585	110 9		1090	77.00
	09020	77.75		05240	115 4		09020	88.10
	08605	79 75	22° 80	2030	44.79		07105	104.1
	08275	81 40		1833	48.78		05965	117.1
	07860	83.60		1651	53.00			
	07125	88.20		1461	58.85			

For the three mixtures the volumes and the pressures at the beginning and the end of the condensation were read on the $p-v-t$ diagrams, constructed by means of the data in the tables I, III and IV, and of which only one was given in the previous communication; these volumes and pressures are given in table V. Only for $x = 0,0494$ the end of the condensation was observed for temperatures at some distance from the critical point of contact; for $x = 0,0995$ the increase of pressure during the condensation was so great that for temperatures a little below the plaitpoint-temperature observations of the end of the condensation could not be made. In the case of the mixture $x = 0,1990$ no observations could be made on the liquid-branch of the border-curve.

T A B L E V.

Border-curves.

Temp.	Begin. condensation.		End condensation.	
	Volume.	Pressure.	Volume.	Pressure.
$x = 0,0494$				
15°.30	0.01111	57.20	0.002543	102.9
21°.50	0.008545	67.90	2892	100.0
26°.80	5850	81.75	3833	93.20
27°.10 (pl. p. t.)	5625	83.00	4063	91.85
27°.30	540	84.6	427	90.5
27°.50 (r.p.of cont. temp.)	48	87.4	48	87.4
$x = 0,0995$				
16°.90	0.009440	67.80		
22°.80	6890	83.20		
24°.20 (pl. p. t.)	6255	87.90	0.003737	114.3
25°.00	565	92.9	412	108.4
25°.45 (r.p.of cont. temp.)	47	101	47	101
$x = 0,1990$				
15°.35	0.008795	79.10		
20°.90	6335	101.5		
22°.20	560	110.9		
± 22°.8 (r.p.of cont. temp.)	± 50	± 120		

Fig. 4 shows a graphical representation of table V. In this diagram t is abscissa, p ordinate and x parameter. As will be known this diagram consists of the vapour-pressure curves of the two pure substances, connected by the plaitpoint curve; in between are the loop-

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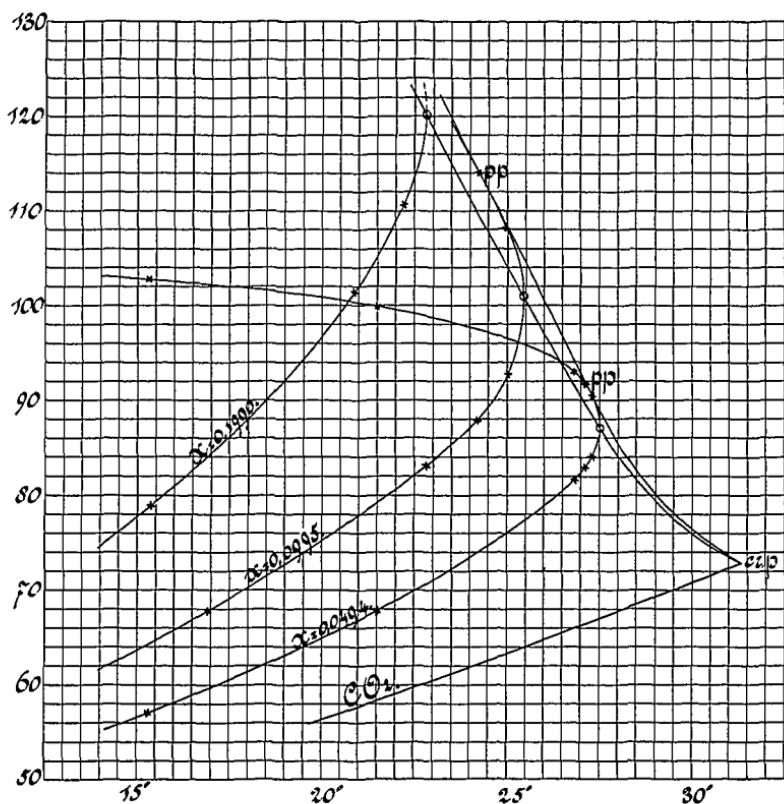


Fig. 4.

shaped border-curves of the mixtures which touch the plaitpoint curve in the plaitpoint. In the critical point of contact the tangent is parallel to the p -axis.

In this figure I have also drawn the vapour-pressure curve of pure carbonic acid, as determined by AMAGAT¹⁾. By connecting the plaitpoints with the critical point of carbonic acid: $t = 31^{\circ}, 35$, $p = 72,9$ atm., we obtain a part of the plaitpoint curve. This plaitpoint curve rises steeply. It is probable that it also rises steeply from the critical point of hydrogen: $t = -234^{\circ}, 5$, $p = 20$ atm.²⁾.

The course of the plaitpoint curve found thus agrees with KUNDT's³⁾ observations on the influence of the pressure of compressed hydrogen on the surface-tension of liquids in contact with it, if they are understood as VAN ELDIK⁴⁾ has explained in his doctor-paper. VAN ELDIK points out that the pressure at which the surface-tension would become zero is the plaitpoint pressure which corresponds to the temperature of observation. Moreover he has investigated the law of the surface-tension as a function of the pressure. He concludes that the plaitpoint pressure for hydrogen and ether at the ordinary temperature would be no less than 750 atm, from which follows a steep rising of the plaitpoint curve for ether and hydrogen on the etherside.

The experiments communicated in this paper are not the first that have been made on the critical phenomena in mixtures of hydrogen and carbonic acid. CAILLETET⁵⁾ has made experiments with a mixture containing about 5 mol. of CO_2 on 1 mol. of H_2 . They were undertaken in order to show, with a view to JAMIN's explanation of the critical phenomena, that by increase of hydrogen-pressure the carbonic acid is bound to disappear. CAILLETET has found that this really occurred and this at a higher pressure as the temperature was lower; for instance at 245 atm. at 15° , and at 153 atm. at 25° . But when we have to construct the border-curves and the plaitpoint-curve we can set no value upon these observations, as CAILLETET did not secure the equilibrium of the phases by stirring; and only since KUENEN⁶⁾ has avoided the appearing of

¹⁾ Comptes rendus, 114, p. 1093, 1892.

²⁾ See OLSZEWSKI, Wied. Ann., 56, p. 133, 1895.

³⁾ Ber. d. Kon. Acad. v. Wiss. Berlin, 21 Oct. 1880.

⁴⁾ VAN ELDIK, Dissertation, Leiden 1898. Communic. Leiden N°. 39.

⁵⁾ Comptes rendus, 96, p. 1448, 1883.

⁶⁾ KUENEN, Dissertation, Leiden 1892. Communic. Leiden N°. 4.

phenomena of retardation, the experimental investigation of the critical phenomena in mixtures has led to reliable results. That phenomena of retardation give rise to important deviations is shown by the fact that CAILLETET has observed condensation at 25° , whereas my experiments show that the critical point of contact of his mixture after equilibrium of the phases has been obtained, ought to be found at about 23° .

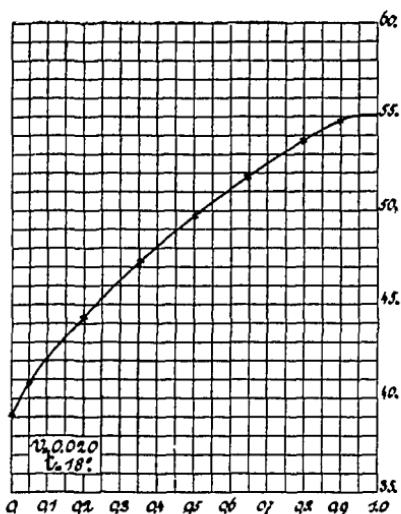
Physics. — „*Measurements on the change of pressure by substitution of one component by the other in mixtures of carbonic acid and hydrogen*“. By Dr. J. VERSCHAFFELT. (Communication N°. 47 from the Physical Laboratory at Leiden by Prof. H. KAMERLINGH ONNES. (*Continued.*))

§ 1. *Change of pressure by substitution.*

Tables VI—X contain the results of the determination of isotherms in mixtures with a still larger quantity of hydrogen than those treated of in the former communications. In these mixtures no condensation-phenomena appeared in the area observed; and so they cannot reveal to us anything more about the further course of the plaitpoint-curve, but in connection with the results communicated before, they show us in what way, at a given temperature and a given volume, the pressure of the mixture depends on its composition.

With the aid of all the determinations communicated we have first calculated the coefficients of pressure, the values of which for different compositions and volumes are given in Table XI. Then we have calculated for one and the same temperature (18° C.) the isotherms of the different mixtures and represented them in a new diagram, the $p-v-x$ diagram. On this diagram we have read the pressures belonging to one and the same volume for different mixtures: table XII contains the values read in this way for some volumes.

As will be explained in § 2 we have chosen the units of volume for the different mixtures so that 1 c.m.³ of each mixture contains the same number of molecules, when the volumes of these mixtures expressed in the units accepted for this purpose, have the same value. Table XII therefore shows the change in pressure when, starting from one of the two substances in pure condition, we gradually substitute the molecules of this substance for an equal number of molecules of the other substance.



The character of this „change of pressure by substitution” is illustrated in the annexed figure: in this diagram the abscissae represent the compositions, and the ordinates the pressures for the volume 0,020 and at the temperature 18° C. From this figure it appears that this change of pressure is not proportional to that of the composition, the pressure is always greater than would follow from a linear relation; if only a small fraction of the number of hydrogen molecules is substituted

by an equal number of carbonic acid molecules hardly any change in the pressure takes place.

Table XII shows clearly that neither the coefficients of pressure undergo any material change, when the number of hydrogen molecules substituted by carbonic acid molecules is only a small fraction of the whole number of hydrogen molecules.

§ 2. *The unit of volume.*

It was desirable to express the volume for each mixture in a special unit of volume, chosen so that one c.m.³ of each of the mixtures contains the same number of molecules when its volume, expressed in this unit, has the same value. We can do this by taking for each mixture as unit of volume: the volume which the quantity of substance used would occupy at 0° C. and 1 atm., if the mixture behaved like a perfect gas. This volume I will call the *theoretically normal volume*; it is obtained by multiplying the volume generally called the *normal volume* viz. the volume which is actually occupied by the substance at 1° C. and 1 atm., by a factor expressing the deviation from AVOGADRO's law.

In connection with AVOGADRO's law this measure for pure substances has been proposed by KAMERLINGH ONNES in 1881. At the same time he pointed out the desirability of expressing henceforth the molecular quantities also in absolute measure. In order to determine the deviation of the normal volume of hydrogen from that which is to be used in applying AVOGADRO's law, he employed VAN DER WAALS's law and the value 0,00050, deduced by VAN DER

WAALS for $a-b$ (Continuiteit p. 91) from REGNAULT's observations.
And so he obtained in K.G. M. S. the equation

$$\left(p + \frac{A}{v^2} \right) (v-b) = 2262000 (1+\alpha t),$$

which he thought fit for determining the deviation of the gases from AVOGADRO's law¹⁾.

In the case of hydrogen we shall however use AMAGAT's observations below 200 atm., which we have also used in determining the pressures. With sufficient approximation they yield for the isothermal of 0° C.

$$p(v - 0,000690) = 0,99931^2),$$

when the normal volume is chosen as unit. Hence in order to obtain the theoretically normal volume we must multiply the volume at 0° and 1 atm. by 0,99931.

From the weight of 1 liter of hydrogen: 0,08955 gramme at 0° and 760 m.m. mercury, 0° C. and 45° latitude and the molecular weight of CO₂: 43,89, we deduce the weight of a liter carbonic acid to be 1,9652 gramme under the same circumstances; REGNAULT found for this 1,9771, which means a deviation of 0,0060 from the theoretical value with respect to hydrogen. The density of carbonic acid is therefore 0,0053 higher than the theoretically normal density, so that in order to find the theoretically normal volume of pure carbonic acid we must multiply the volume at 0° and 1 atm. by 1,0053.

An observation made by BRAUN³⁾ gives us some information on the deviations of AVOGADRO's law shown by mixtures of carbonic acid and hydrogen. He has found that when we mix two equal volumes of hydrogen and carbonic acid under a pressure of about 71 c.m. mercury, the pressure rises 0,097 c.m. thus 0,0014. And so we may accept that when taking two equal volumes of hydrogen and carbonic acid we obtain by mixing a

¹⁾ The problem in general has been treated by v. d. WAALS in his communications "On the accurate determination of the molecular weight of gases from their density." Proc. Nov. '98 and "Variation of volume and of pressure in mixing II." Proc. Dec. '98

²⁾ See KAMERLINGH ONNES. Verh. Kon. Akad. v. Wet. 1881. Algemeene theorie der vloeistoffen, p. 5—7. Abstract: Arch. Néerl. t. XXX.

³⁾ Wied. Ann., 34, p. 948, 1888. KUENEN has pointed out a similar change of pressure in mixing compressed gases by stirring.

double volume under a pressure of 1.0014 atm. Let n be the number of molecules which one volume of a perfect gas would contain at 0° and 1 atm., so that 0,99931 n is the number of molecules of hydrogen and 1,0053 n the number of molecules of carbonic acid, then it is easy to see that one volume of the mixture at 0° and 1 atm. must contain 1,0009 n mol. And so in order to find the theoretically normal volume of a mixture consisting of an equal number of molecules of hydrogen and carbonic acid we must multiply the volume at 0° and 1 atm. by 1,0009.

Let, as in v. d. WAALS's paper, x be the proportion of the number of hydrogen-molecules to the whole number of molecules, then we can represent the deviation from AVOGADRO's law by the numbers: 1,0053 for $x=0$, 1,0009 for $x=0,5$, and 0,99931 for $x=1$. The deviation for intermediate mixtures can be found with sufficient approximation by applying a parabolic formula of the form $y=a+bx+cx^2$; and then we find that the deviation may be represented by

$$y = 0,99931 + 0,0060 (1-x)^2 .$$

According to this formula the influence of small admixtures of carbonic acid with the hydrogen is very small, a fact to which the attention will be drawn later on.

For the reduction to 0° C. we had to use the coefficients of expansion of the mixtures. As a first approximation I might have calculated the coefficients of expansion with the aid of a linear formula from the coefficients of expansion of the pure substances: 0,00366 for H₂, 0,00371 for CO₂. But led by the previous result regarding the deviations from AVOGADRO's law I thought it probable that the dependence of the coefficient of expansion on the composition x would show the same characteristics; and so I was obliged to put it thus:

$$a_p = 0,00366 + 0,00005 (1-x)^2 .$$

In determining the theoretically normal volume of hydrogen it was mentioned that by starting from different experimental data we arrive at different deviations from AVOGADRO's law.

This is even more so in the case of carbonic acid. If by the side of the number 0,99950 for hydrogen formerly accepted by KAMERLINGH ONNES we also borrow from v. d. WAALS's Continuiteit (p. 76) the number 1,00646 for carbonic acid (deduced from REGNAULT's isothermal lines) and if we substitute this for REGNAULT's determination

of the density of carbonic acid we should find for the mixture $\frac{1}{2}$ hydrogen— $\frac{1}{2}$ carbonic 1,00158, whence:

$$y = 0.99950 + 0.00136(1-x) + 0.00560(1-x)^2.$$

From the point of view of a consistent application of VAN DER WAALS's law a calculation of the theoretically normal volume of the carbonic acid from the normal volume and the compressibility would be preferable to the method of calculation followed by me. But then the proportion of the weights of one volume of carbonic acid and hydrogen in the state of perfect gases and the molecular weights would be discordant, and it are the latter which we want as a basis for our choice of units of volume (with a view to v. d. WAALS's theory of mixtures). The influence of the differences yielded by the calculations from the different data is not such as to render doubtful the character of the change of pressure when substitution takes place, but still in my determinations the discrepancies differ little from the errors of observation. From this we see once more the necessity of accurate observations with perfectly pure substances in order to make progress in the knowledge of the laws which govern the gaseous state.

TABLE VI. $x = 0.3528$.

Volume 0,0	$p_{16^{\circ} \cdot 00}$	$p_{31^{\circ} \cdot 60}$
2810	34.44	36.85
2585	37.17	39.80
2442	39.15	41.96
2299	41.38	44.38
2159	43.77	46.95
2014	46.54	49.99
1869	49.70	53.45
1724	53.35	57.40
1587	57.35	61.80
1480	60.90	65.75
1340	66.35	71.75
1207	72.40	78.45
1067	80.30	87.25
09255	89.90	98.30
07940	101.60	112.0
06510	115.3	

TABLE VII. $x = 0.4993$.

Volume 0,0	$p_{18^{\circ} \cdot 00}$	$p_{33^{\circ} \cdot 00}$
3168	32.11	33.84
2975	34.09	35.95
2825	35.78	37.78
2667	37.85	39.97
2512	40.08	42.36
2355	42.60	45.07
2206	45.34	47.97
2045	48.71	51.55
1889	52.36	55.50
1736	56.80	60.20
1624	60.45	64.05
1468	66.35	70.40
1323	73.05	77.65
1172	81.85	87.10
1017	93.15	99.50
08694	107.3	114.9

TABLE VIII.

 $x = 0.6445.$

Volume 0,0	$p_{16^{\circ} \cdot 80}$	$p_{24^{\circ} \cdot 20}$	$p_{31^{\circ} \cdot 90}$
2979	34.85	35.82	36.85
2746	37.76	38.79	39.95
2600	39.90	41.00	42.13
2446	42.35	43.53	44.80
2294	45.10	46.34	47.88
2143	48.21	49.60	51.00
1989	51.90	53.35	54.95
1886	56.05	57.60	59.35
1686	60.90	62.65	64.55
1576	65.20	67.05	69.05
1433	71.45	73.60	75.80
1284	79.50	82.00	84.55
1140	89.25	92.05	94.85
09935	102.2	105.5	108.9
08495	119.2		

TABLE IX.

 $x = 0.7963.$

Volume 0,0	$p_{18^{\circ} \cdot 30}$
3293	32.61
2990	35.81
2723	39.47
2474	43.49
2253	47.78
2032	53.00
1840	58.80
1630	66.20
1484	72.95
1335	81.35
1212	89.65
1090	100.1
09650	113.2

TABLE X.

 $x = 0.8972.$

Volume 0,0	$p_{17^{\circ} \cdot 10}$
3234	33.46
2989	36.29
2840	38.15
2692	40.31
2532	42.94
2384	45.72
2217	49.22
2076	52.65
1904	57.50
1764	62.25
1635	67.40
1485	74.55
1336	83.15
1173	95.20
1028	109.5

TABLE XI. Coefficients of tension. (15° — 30° C.)

Volume 0,0	Compositions $x =$									
	0.0000	0.0494	0.0995	0.1990	0.3528	0.4993	0.6445	0.7963	0.8972	1.0000
30	0.162	0.159	0.156	0.150	0.142	0.134	0.130	0.127	0.126	0.126
28	175	172	168	163	155	146	141	138	136	135
26	190	186	183	177	168	159	154	150	148	146
24	209	204	199	193	183	174	168	163	161	159
22	229	223	218	211	201	191	185	179	176	173
20	257	249	242	234	223	212	204	197	194	192
18	295	285	276	265	250	237	227	221	216	214
16	346	333	321	306	286	270	256	249	244	242
14	412	395	381	360	331	312	296	286	281	278
12	503	481	461	431	395	373	353	339	331	327
10	620	590	566	433	495	464	436	415	405	397

TABLE XII. Pressures at 18°.

Volume	Compositions $\pi =$									
	0.0000	0.0494	0.0995	0.1990	0.3528	0.4993	0.6445	0.7963	0.8972	1.0000
30	28.90 ¹⁾	29.68	30.37	31.33	32.70	33.81	34.77	35.80	36.19	36.31
29	29.73	30.43	31.33	32.25	33.75	34.92	35.95	37.09	37.46	37.59
28	30.58	31.35	32.25	33.24	34.85	36.12	37.19	38.43	38.83	38.97
27	31.45	32.32	33.23	34.34	36.04	37.39	38.56	39.81	40.29	40.45
26	32.40	33.33	34.28	35.53	37.32	38.80	40.05	41.38	41.87	42.04
25	33.36	34.40	35.37	36.77	38.64	40.23	41.64	42.98	43.62	43.77
24	34.38	35.53	36.54	38.04	40.12	41.80	43.33	44.80	45.48	45.65
23	35.47	36.73	37.78	39.43	41.72	43.57	45.17	46.73	47.48	47.70
22	36.55	38.00	39.11	40.91	43.40	45.46	47.18	48.90	49.67	49.94
21	37.75	39.40	40.55	42.50	45.26	47.50	49.39	51.20	52.15	52.40
20	39.08	40.85	42.10	44.26	47.30	49.65	51.80	53.75	54.80	55.10
19	40.40	42.40	43.76	46.15	49.49	52.10	54.45	56.65	57.80	58.10
18	41.90	44.02	45.56	48.22	51.90	54.05	57.45	59.85	61.15	61.50
17	43.43	45.70	47.47	50.40	54.60	58.00	60.75	63.45	64.90	65.25
16	45.00	47.52	49.55	52.80	57.50	61.30	64.55	67.45	69.10	69.50
15	46.64	49.45	51.85	55.45	60.80	65.00	68.75	72.10	73.85	74.35
14	48.35	51.65	54.30	58.40	64.60	69.30	73.35	77.50	79.25	79.95
13	50.05	54.00	56.00	61.75	68.80	74.30	78.80	83.55	85.65	86.45
12	51.85	56.40	59.85	65.60	73.60	80.00	85.30	90.85	93.10	94.10
11	53.65	59.00	63.05	69.75	79.10	86.75	93.00	99.25	102.0	103.1
10	55.50	61.65	66.55	74.25	85.40	94.40	102.1	109.6	112.9	114.3

Chemistry. — „On the velocity of electrical reaction”. By Dr. ERNST COHEN, (Communicated by Prof. H. W. BAKHUIS ROOZEBOOM).

1. If two elements, arranged as follows.

Electrode, reversible with respect to the anion.	Saturated solution of a salt S in presence of the stable solid phase of the salt.	Electrode, reversible with respect to the cathion.
--	---	--

and

¹⁾ This column has been deduced from isothermal determinations of pure carbonic acid, not yet published.

Electrode, reversible with respect to the anion.	Saturated solution of the salt S in presence of the metastable solid phase of the salt.	Electrode, reversible with respect to the cathion.
--	---	--

are coupled up in opposition to each other, a transition element of the third kind¹⁾ is obtained.

If the salt S is zinc sulphate, the combination in question may be composed of two CLARK-cells; in the one $\text{Zn SO}_4 \cdot 7 \text{ H}_2\text{O}$, in the other $\text{Zn SO}_4 \cdot 6 \text{ H}_2\text{O}$ forms the solid phase provided that the temperature lies between the cryohydratic temperature of $\text{Zn SO}_4 \cdot 6 \text{ H}_2\text{O}$ and the transitionpoint (39°).

2. The electromotive force E of this transition element is the measure of the maximum work which the reaction occurring in the element at the temperature T can perform.

In a later communication I shall show how E may be calculated by thermodynamics.

Experimentally, E may be directly determined or it may be calculated from the measurements of JAEGER²⁾, who has measured the *E. M. F.* of CLARK-cells at different temperatures, $\text{Zn SO}_4 \cdot 7 \text{ H}_2\text{O}$ (the stable phase) or $\text{Zn SO}_4 \cdot 6 \text{ H}_2\text{O}$ (the metastable phase) being present as the solid phase between 0° and 39° .

In this way the following numbers are obtained :

Temperature.	<i>E.M.F.</i> of the transition element.
— $5^\circ, 0$	16.2 Millivolts.
$0^\circ, 0$	14.9 „
+ $5^\circ, 0$	13.5 „
$9^\circ, 0$	12.3 „
$15^\circ, 0$	10.3 „
$25^\circ, 0$	6.4 „
$30^\circ, 0$	4.2 „
$35^\circ, 0$	1.9 „
$39^\circ, 0$	0 „

3. The velocity with which the reaction which occurs in the

¹⁾ See COHEN, Zeitschrift für phys. Chemie, Bd. 25 (1898). S. 300.

²⁾ WIEDEMANN's Annalen, Bd. 68 (1897), 354.

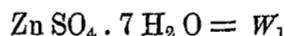
transition element proceeds at the temperature T is represented by the equation

$$K = \frac{E}{\Sigma(W)} \quad \dots \quad (1)$$

where $\Sigma(W)$ is the sum of the internal resistances (at T^0) of the elements of which the transition element is composed, and E the electromotive force of the transition element at T^0 .

I have shown ¹⁾ that the internal resistance of a CLARK-cell at T^0 is proportional to that of the zinc sulphate solution, saturated at T^0 .

Let the measured resistance of a saturated solution of



and that of a saturated solution of $\text{Zn SO}_4 \cdot 6 \text{H}_2\text{O}$ at the same temperature $= W_2$. Then

$$\Sigma(W)_{T^0} = (p_1 W_1)_{T^0} + (p_2 W_2)_{T^0},$$

where p_1 and p_2 are constants depending on the capacities of the CLARK-cells used and of the vessels in which the resistances of the saturated solutions were determined.

If the same vessel is used for all the measurements, the equation may be written

$$\Sigma(W)_{T^0} = p(W_1 + W_2).$$

If we call Ω_1 and Ω_2 the specific resistances of the saturated solutions of $\text{Zn SO}_4 \cdot 7 \text{H}_2\text{O}$ and $\text{Zn SO}_4 \cdot 6 \text{H}_2\text{O}$ at T^0 and \varkappa the resistance-capacity of the vessel employed in measuring W_1 and W_2 , then

$$\Omega_1 = \varkappa W_1$$

$$\Omega_2 = \varkappa W_2$$

and

$$K = \frac{E}{\frac{p}{\varkappa}(\Omega_1 + \Omega_2)}.$$

Or

$$K_1 = \frac{E}{\Omega_1 + \Omega_2}.$$

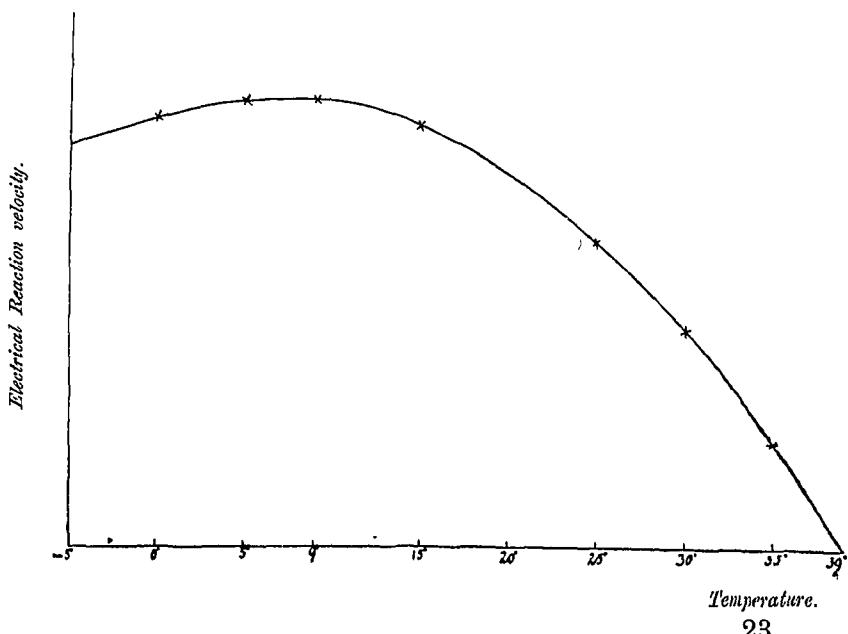
¹⁾ The paper relating to this will appear shortly in the Zeitschrift für physikalische Chemie.

The values of the *electrical reaction velocity constant*, K_1 , are placed in the last column of the following table, which contains the results of the observations.

T	E (Millivolts)	\mathcal{W}_1	\mathcal{W}_2	$K_1 = \frac{E}{\Omega_1 + \Omega_2}$
— 5°	16.2	445.9	524.1	0,0167
0°,0	14.9	384.2	452.2	0,0178
+ 5°,0	13.5	337.0	396.3	0,0184
9°,0	12.3	305.75	360.85	0,0185
15°,0	10.3	271.60	315.50	0,0175
25°,0	6.4	236.40	274.25	0,0125
30°,0	4.2	225.10	248.85	0,0088
35°,0	1.9	218.50	228.85	0,0042
39°,0	0	215.0	215.0	0

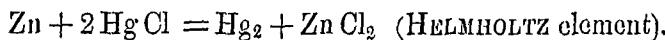
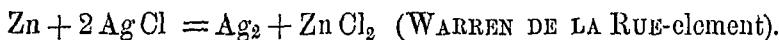
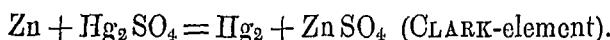
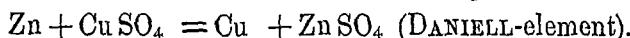
The observations cannot be continued below — 5° because the cryohydratic temperature of $Zn SO_4 \cdot 7 H_2 O$ lies at about — 6°.

Representing the values of K_1 graphically as a function of the temperature the following curve is obtained.



From this it is clearly seen that starting from 39° the *velocity of electrical reaction* rapidly increases, reaching a maximum at about 9° and then diminishing again.

It is worthy of note that the curve, which here represents the *velocity of electrical reaction* at different temperatures, possesses the same form as that representing the rate of crystallisation of many substances at temperatures below the melting-point¹⁾. I shall take up this subject more fully later, as also the study of the velocity of the following reactions:



Amsterdam, February 1899.

Chemistry. — Prof. FRANCHIMONT presents to the library of the Academy the dissertation of Mr. L. T. C. SCHEY entitled: „*On synthetically prepared neutral glyceryl-etheral salts — triacylins — of saturated monobasic acids with an even number of C-atoms*” and elucidates it in the following words :

Since CHEVREUL's experiments in the first quarter of this century, fats, at least animal fats, are considered as mixtures of glyceryl-etheral salts, on account of the products formed by them after treatment with solutions of bases; but it is extremely rare that a glyceryl-ether has been extracted from it in a pure form. The difficulties attached to such separations have as yet not been sufficiently overcome.

About the middle of this century some of these glyceryl-etheral salts have been made synthetically by BERTHELOR and others, but generally they did not obtain them in a pure condition. Now Mr. SCHEY has, with a view to the acids said to have been obtained from butter, made synthetically eight glycerides, called by him, in order to prevent confusion with polyglycerinderivatives, *triacylins* and has determined

¹⁾ See GERNLZ, Journal de physique (2) 4, (1885) p. 349. TAMMAN, Zeitschrift für phys. Chemie, o. a. 23, 226 (1888). v 't HOOF, Vorlesungen über theor. und phys. Chemie (1898). S. 226.

their density, refractive power and meltingpoint. Of these eight, three were entirely unknown viz caproin, caprylin and caprin; three: butyrin, palmitin and stearin had formerly been prepared synthetically, but the former two certainly not pure; two: laurin and myristin had only been obtained from natural products, but neither quite pure.

The method of preparation adopted by Mr. SCHEY consists simply in heating glycerin with an excess of the acid up to a temperature at which neither the glycerine, nor the acid, nor the ethereal salt suffer decomposition and under circumstances in which the water formed is immediately drawn off as completely as possibly e.g. by a slight current of air and under diminished pressure in the apparatus, this being followed by fractional distillation in vacuo or crystallization from different solvents, until one of the properties mentioned underwent no further noticeable alteration by this treatment. This fact led to the conclusion of the purity of the product; Mr. SCHEY attaching little value to elementary analyses in these cases, they have not been made.

The determination of the specific gravity was always done by weighing a certain volume of the substance and an equal volume of water at the same temperature; the exactness acquired in this way holds good up to one or two unities of the 4th decimal.

The refraction-index was determined for sodium light with a refractometer of PULFRICH with which generally the 4th decimal is certain. The determination of the meltingpoint has not been so exact, but that differences of one or two tenths degrees may occur.

Mr. SCHEY has not only determined the three properties of his products, but also of the original substances (the acids), which were purified to the highest possible degree, except the two highest acids, which Dr. L. E. O. DE VISSER of Schiedam had kindly procured him in a very pure condition. Of the eight triacylins prepared, tricaprin offers the highest guarantee of purity because it crystallizes best of all and was obtained in great, clear, well-formed crystals; the lower terms are at the ordinary temperature liquids of which tributyryin alone has an intensely bitter taste.

Finally Mr. SCHEY treats of the methods by which it is possible to calculate, approximatively at least, density and refractive power of these substances. The calculation with the numbers determined by others for the atomicvolumes (even TRAUBE's with the co-volumes) and for the atomicrefractions, gives results differing pretty much from those found by him. He here points out the little certainty offered by the generally accepted values for atomicvolumes and atomicrefraction.

Another method of calculation already indicated by BERTHELOT in 1856 viz. from the molecular-volumes and the molecular-refractions of the substances that have interacted, at equal temperatures, led to better results. It has already been remarked by others, that the ethereal salts of the fatty acids in general often seem to be formed without great change of volume and this seems equally to be the case with triacylins. For caprin the result of this calculation of the molecular-volume even perfectly agrees with the one really found; it does not result therefrom that this should necessarily be the case with the other terms as well, and in the subjoined table given by Mr. SCHLEY their deviations are shown.

A third method of calculation starts from what is found for one of the terms and as all of them rise or fall with an equal difference of composition, it takes into account the average value of this difference. Starting from caprin, as the purest product, the values calculated on this base and those for molecular-volumes and molecular-refraction concurred pretty well, as will be seen from subjoined tables.

As to the meltingpoints it was found that they were quite or nearly quite equal for capric acid and for tricaprin, while for the lower terms of the triacylins the meltingpoint is below that of the acid, for the higher ones on the other hand above it, which does not agree with what BERTHELOT thought to have found.

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Physics. — „*On the Vibrations of Electrified Systems, placed in a Magnetic Field*“. By Prof. H. A. LORENTZ.

(Read in the meeting of January 28th 1899).

§ 1. Many spectral lines show the ZEEMAN-effect according to the well known elementary theory, and are thus changed into triplets or, if viewed along the lines of force, into doublets, yet there are a rather large number of cases, in which the phenomena are more complicated. CORNU¹⁾ found that the line D_1 becomes a quartet, whose outer and inner components are polarized, the first parallel and the latter perpendicularly to the lines of force. Similar quartets have been observed in other cases. Sometimes²⁾, in triplets and quar-

¹⁾ CORNU, Comptes rendus, T. 126.

²⁾ BECAUDEREL and DESLANDRES, Comptes rendus, T. 127, p. 18.

tets, the inner and outer lines have interchanged their ordinary states of polarization. Finally MICHELSON, PRESTON¹⁾ and other physicists have seen a division of some lines into 5, 6 or even more components.

I shall examine in this paper, to what extent such multiple lines may be explained by appropriate assumptions concerning the way in which light is emitted. Of course I am perfectly aware of the possibility that my interpretation of the facts will have to be replaced by a more adequate one. The special form of my hypotheses has the less value, as in the only case in which I have endeavoured to account for all the peculiarities of the phenomena, I have succeeded but poorly, at the best.

§ 2. Since the components, into which a line is broken up by the magnetic force, are in many cases as sharp as the original line itself, it must be admitted that the periods of all the luminous particles of the source of light are modified in the same way. This is only possible in two ways. Either, in the magnetic field, all the particles must take the same direction, or the modification of the periods must remain unchanged, into whatever position the particles may be turned. The first assumption leads however to some difficulties²⁾. I shall therefore suppose the luminous particles to be spherical bodies, having the same properties in all directions. This may be true, even though the chemical atoms be of a much less simple structure; indeed, the vibrating spherical ion may very well be only a part, perhaps a very small part, of the whole atom³⁾.

It has been shown in a former paper⁴⁾ that a triplet may be observed if, among the principal modes of vibration of the system, there be three, for which, outside the magnetic field, the time of vibration is the same, or, as we may say, if the system have three equivalent degrees of freedom. Afterwards Mr. PANNEKOEK⁵⁾ remarked that a quartet may appear if there be, in the same sense, four equivalent degrees of liberty, and in general, a n -fold line, if n of the principal modes have equal periods.

Now, spherical systems, vibrating in one of their higher modes, have indeed more than three equivalent degrees of freedom.

¹⁾ MICHELSON, Phil. Mag., Vol. 45, p. 348. PRESTON, *ibidem*, p. 325.

²⁾ See LORENTZ, *Verslag der Verg. Akad. van Wetenschapen VI*, p. 197, and *Arch. Néerl.*, Sér. 2, T. 2, p. 5.

³⁾ See LORENTZ, *Verslag der Verg. Akademie van Wetenschappen VI*, p. 514.

⁴⁾ Wied. Ann. Bd. 63, p. 278.

⁵⁾ *Verslag der Verg. Akademie van Wetenschappen VII*, p. 120. *Proceedings Royal Acad.* Vol. I, p. 96

§ 3. I shall consider in the first place an infinitely thin spherical shell of radius a , charged, in the state of equilibrium, to a uniform surface-density σ . The surface-density of the ponderable matter itself will be denoted by ϱ . We shall suppose that the points of the shell can only be displaced along its surface, that the elements carry their charge along with them, and finally that, after a displacement, each element is acted on by an elastic force, which is brought into play merely by the displacement of the element itself, and not by the relative displacement of adjacent elements.

When the motions are infinitely small, the elastic force may be taken proportional to the displacement a . Let it be

$\equiv h^2 \eta$

per unit area, the constant k^2 having the same value all over the sphere.

The only connexion between the different parts of the shell will consist in their mutual electric forces. If the wave-length of the emitted radiations be very large in comparison with the radius of the sphere, we have merely to consider the ordinary *electrostatic* actions, depending solely on the configuration of the system. Hence there will be no resistances proportional to the velocities, and consequently no damping. In fact, it is well known that the damping which, in some degree, must always be caused by the loss of energy, accompanying the radiation, may be neglected when the wave-length is very much larger than the dimensions of the vibrating system.

§ 4. In the absence of magnetic forces the shell can vibrate in the following way.

Let Y_h be a surface harmonic of order h . Then the displacement of a point of the sphere is

$$p \frac{\partial Y_h}{\partial t} \cdot \dots \cdot \dots \cdot \dots \quad (1)$$

Here l is the direction in the surface in which Y_h increases most rapidly, and $\frac{\partial Y_h}{\partial l}$ is to be regarded as a vector in this direction l .

The coefficient p is the same all over the sphere; it has the form

$$q \cos(n_h t + c), \dots \dots \dots \dots \dots \quad (2)$$

so that n_h is the frequency of the vibrations.

In consequence of the displacements (1), the electric density will have changed from σ to

$$\sigma + h(h+1) \frac{\sigma}{a^2} p Y_h.$$

Hence, there will be an electric force

$$- 4\pi V^2 \frac{h(h+1)}{2h+1} \cdot \frac{\sigma}{a} p \frac{\partial Y_h}{\partial t}$$

(V = velocity of light) along the surface and, as the density differs from σ by an infinitely small amount, we may write for the force per unit area

$$- 4\pi V^2 \frac{h(h+1)}{2h+1} \cdot \frac{\sigma^2}{a} p \frac{\partial Y_h}{\partial t}.$$

The equation of motion becomes

$$\varrho \ddot{p} \frac{\partial Y_h}{\partial t} = - k^2 p \frac{\partial Y_h}{\partial t} - 4\pi V^2 \frac{h(h+1)}{2h+1} \frac{\sigma^2}{a} p \frac{\partial Y_h}{\partial t},$$

and the frequency n_h is determined by

$$\varrho n_h^2 = k^2 + 4\pi V^2 \frac{h(h+1)}{2h+1} \cdot \frac{\sigma^2}{a} \quad \dots \quad (3)$$

Thus, we see that the frequency is the same for all motions corresponding to a harmonic of order h , no matter what particular harmonic of this order may be chosen.

If we put $h=1$, we obtain the frequency of the slowest vibrations; $h=2$ corresponds to the first of the higher types of motion, and so on. However each of the different types includes a certain number of different modes of motion.

In the motion we have considered there is a kinetic energy

$$T = \frac{1}{2} \varrho \dot{p}^2 \int \left(\frac{\partial Y_h}{\partial t} \right)^2 d\omega,$$

$d\omega$ being an element of the sphere, and the integration extending all over the surface.

In virtue of the properties of spherical harmonics we may also write

$$T = \frac{1}{2} h(l+1) \frac{\varrho}{a^3} p^2 \int Y_h^2 d\omega .$$

The potential energy is given by

$$\begin{aligned} U &= 2\pi V^2 \frac{h^2(h+1)^2}{2h+1} \frac{\sigma^2}{a^3} p^2 \int Y_h^2 d\omega + \frac{1}{2} k^2 p^2 \int \left(\frac{\partial Y_h}{\partial l} \right)^2 d\omega = \\ &= \left[2\pi V^2 \frac{h^2(h+1)^2}{2h+1} \frac{\sigma^2}{a^3} + \frac{1}{2} h(h+1) \frac{1}{a^2} k^2 \right] p^2 \int Y_h^2 d\omega . \end{aligned}$$

If we put

$$A_h = 4\pi V^2 \frac{h^2(h+1)^2}{2h+1} \frac{\sigma^2}{a^3} + h(h+1) \frac{k^2}{a^2}$$

and

$$B_h = h(h+1) \frac{\varrho}{a^2} ,$$

these formulae may be replaced by

$$T = \frac{1}{2} B_h p^2 \int Y_h^2 d\omega$$

and

$$U = \frac{1}{2} A_h p^2 \int Y_h^2 d\omega .$$

§ 5. We shall now take for h a determinate number and consider only vibrations corresponding to harmonics of this order. These motions of the common frequency η_h may differ from one another by the position of the poles of the harmonic Y_h . Moreover, vibrations depending on different functions Y_h may be superposed with any amplitudes and phases we like.

However, not all of these modes of motion are mutually independent. Since any surface harmonic of order h may be decomposed into $2h+1$ particular harmonics of the same order, there are only $2h+1$ equivalent degrees of freedom, for which the frequency is η_h . As for those $2h+1$ fundamental harmonics, as we shall call them, they need only satisfy the condition that none of them can be expressed in terms of the other ones. After having chosen these functions, which I shall denote by

$$Y_{h1}, \quad Y_{h2}, \quad Y_{h3} \text{ etc. ,}$$

we may write for the displacement in the most general case we shall have to consider

$$a = p_1 \frac{\partial Y_{h1}}{\partial l} + p_2 \frac{\partial Y_{h2}}{\partial l} + p_3 \frac{\partial Y_{h3}}{\partial l} + \text{etc.}, \dots \quad (4)$$

where each term represents a vector along the surface in the manner indicated in § 4, so that l has different meanings in the different terms.

The potential and the kinetic energy will now be found to be

$$U = \frac{1}{2} a_{11} p_1^2 + \frac{1}{2} a_{22} p_2^2 + \frac{1}{2} a_{33} p_3^2 + \text{etc.} \dots + \\ + a_{12} p_1 p_2 + a_{13} p_1 p_3 + \text{etc.},$$

$$T = \frac{1}{2} b_{11} \dot{p}_1^2 + \frac{1}{2} b_{22} \dot{p}_2^2 + \frac{1}{2} b_{33} \dot{p}_3^2 + \text{etc.} \dots + \\ + b_{12} \dot{p}_1 \dot{p}_2 + b_{13} \dot{p}_1 \dot{p}_3 + \text{etc.},$$

where

$$a_{\mu\mu} = A_h \int Y_{h\mu}^2 d\omega, \quad a_{\mu\nu} = A_h \int Y_{h\mu} Y_{h\nu} d\omega,$$

$$b_{\mu\mu} = B_h \int Y_{h\mu}^2 d\omega, \quad b_{\mu\nu} = B_h \int Y_{h\mu} Y_{h\nu} d\omega.$$

As long as we limit the investigation to the vibrations of order h , we may ignore the other degrees of freedom; we may then consider the $2h+1$ coefficients $p_1, p_2, p_3 \dots$ as the general coordinates. The equation of motion for the coordinate p_μ will be

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{p}_\mu} \right) = - \frac{\partial U}{\partial p_\mu};$$

it will take the form

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{p}_\mu} \right) = - \frac{\partial U}{\partial p_\mu} + Q_\mu, \dots \quad (5)$$

if, besides the forces which we have considered thus far, there are other forces whose general components are Q_μ .

§ 6. If an electrified system be vibrating in a magnetic field, its parts will be acted on by electromagnetic forces proportional to the

charges. Per unit charge these forces are given by the vector product of the velocity and the magnetic force in the field.

Let there be a mode of motion A , with frequency n , before there is any magnetic force, and let F be the electromagnetic forces arising from this motion as soon as the field is produced. The direction of these forces will obviously change with the frequency n , and to determine their action on the system is a problem of "resonance" or of "forced vibrations". In general, the system will respond to the forces F by a motion in several of its other fundamental modes. In fact, any particular motion B will certainly be excited if only the forces F do a positive or negative work in an infinitely small displacement corresponding to that mode B .

Since the electromagnetic forces are perpendicular to the velocities, the forces F will do no work if the infinitely small displacement belong to the mode A itself. A direct influence of the forces F on the motion A which gave rise to them is thereby excluded.

As to the other modes, all depends on their frequency. If the frequency n' of a motion B be considerably different from n , the forced vibration B , if it exist at all, will be very insignificant, for experience shows the forces F to be very feeble as compared with the other forces acting in the system. As well as the forces F themselves, the amplitude of the forced vibrations B will be proportional to the strength H of the field. Hence, the electromagnetic forces F' , which exist in consequence of the vibration B , will be of the order H^2 , and it will be permitted to neglect their reaction on the original motion.

The case is quite different as soon as the frequency of B is equal to that of A . The amplitude of the new motion B will then rise to a much higher value; as may be deduced from the equations of the problem, it will reach the same order of magnitude as the amplitude of A itself. The influence of the forces F' on the original motion will likewise be much greater than in the former case.

One may see by a simple reasoning that this influence will consist in a modification of the period. Since the forces F have the same phase as the velocities in the motion A , there will be a difference of phase of $1/4$ period between them and the displacements A . On the other hand, the displacements in the motion B have the same or the opposite phase as the forces F , and the phase of the forces F' will differ by $1/4$ period from that of the displacements B . These latter forces will therefore have the same or the opposite phase as the displacements A , and this is precisely what is required, if they are to change the frequency of A .

We see at the same time that the simultaneous motions A and B will differ in phase by $1/4$ period. This is the reason why circularly polarized light can be emitted in the direction of the lines of force.

§ 7. The foregoing reasoning shows that, in the magnetic field, the vibrations of order h will never be perceptibly modified by the vibrations of different orders. We may therefore continue to consider them by themselves. Now, the meaning of the term Q_μ in equation (5) is this, that $Q_\mu \delta p_\mu$ is the work of the electromagnetic forces corresponding to the infinitely small displacement determined by δp_μ . But the electromagnetic forces are linear functions of the velocities; consequently, Q_μ will take the form

$$Q_\mu = \sum_v \varepsilon_{\mu v} \dot{p}_v \dots \dots \dots \quad (6)$$

The coefficients ε are easily calculated. Let the centre of the shell be the origin of coordinates, the axis of z having the direction of the magnetic force H . Then, if r be the distance to the centre, and

$$W_{h\mu} = r^h Y_{h\mu}$$

the solid harmonic of degree h , corresponding to the surface harmonic $Y_{h\mu}$, I find

$$\varepsilon_{\mu v} = \frac{H\sigma}{a^{2h+2}} \int z \left| \begin{array}{ccc} x, & y, & z, \\ \frac{\partial W_{h\mu}}{\partial x}, & \frac{\partial W_{h\mu}}{\partial y}, & \frac{\partial W_{h\mu}}{\partial z} \\ \frac{\partial W_{h\nu}}{\partial x}, & \frac{\partial W_{h\nu}}{\partial y}, & \frac{\partial W_{h\nu}}{\partial z} \end{array} \right| d\omega \dots \dots \quad (7)$$

I shall suppose that the axis of y points to the place the observer occupies when viewing the phenomena across the lines of force.

It will sometimes be found convenient to distinguish the fundamental harmonics by suffixes, indicating the position of their poles. Thus Y_x will be the surface harmonic of the first order whose pole is determined by the intersection of the axis of x with the sphere; Y_{xy} the harmonic of the second order, having its poles on the axes of x and y , Y_{xx} the zonal harmonic, whose poles coincide on the axis of x . If these notations be used, the suffix which indicates the order of the harmonic may be omitted,

By (7) we see that

$$\varepsilon_{\mu\mu} = 0, \quad \varepsilon_{\nu\mu} = -\varepsilon_{\mu\nu}.$$

These relations would hold for all electrified systems, vibrating in the magnetic field.

§ 8. We shall begin by examining the vibrations, depending on harmonics of the first order.

Let the fundamental harmonics be

$$Y_{11} = Y_x, \quad Y_{12} = Y_y, \quad Y_{13} = Y_z, \\ W_{11} = x, \quad W_{12} = y, \quad W_{13} = z.$$

Then :

$$a_{11} = a_{22} = a_{33} = \frac{4}{3}\pi a^2 A_1,$$

$$a_{12} = a_{23} = a_{31} = 0,$$

$$b_{11} = b_{22} = b_{33} = \frac{4}{3}\pi a^2 B_1 = \frac{8}{3}\pi \varrho,$$

$$b_{12} = b_{23} = b_{31} = 0,$$

$$\varepsilon_{12} = \frac{4}{3}\pi H \sigma, \quad \varepsilon_{13} = \varepsilon_{23} = 0,$$

and, if we replace a_{11} , b_{11} and ε_{12} by α_1 , β_1 , ε_1 ,

$$\beta_1 \ddot{p}_1 = -\alpha_1 p_1 + \varepsilon_1 \dot{p}_2, \quad \dots \quad \dots \quad \dots \quad \dots \quad (8)$$

$$\beta_1 p_2 = -\alpha_1 p_2 - \varepsilon_1 \dot{p}_1, \quad \dots \quad \dots \quad \dots \quad \dots \quad (9)$$

$$\beta_1 \ddot{p}_3 = -\alpha_1 p_3.$$

From these equations we conclude that, for $H = 0$ and $\varepsilon_1 = 0$, all vibrations have the frequency n_1 , given by

$$n_1^2 = \frac{\alpha_1}{\beta_1} = \frac{A_1}{B_1},$$

which follows also from (3).

When there is a magnetic field, the vibrations corresponding to Y_z will still have this frequency n_1 , but besides these there will be two motions with a modified time of vibration. In order to find them, we may suppose that p_1 and p_2 contain the factor e^{int} multi-

plied by quantities independent of the time. Then, neglecting quantities of the order of H^2 , we may satisfy (8) and (9), by assuming either

$$p_2 = + i p_1, \quad n = n_1 + n'_1$$

or

$$p_2 = - i p_1, \quad n = n_1 - n'_1.$$

In these formulae

$$n'_1 = \frac{\epsilon_1}{2 \beta_1} = \frac{H \sigma}{4 \varrho},$$

or, writing e for the total charge $4 \pi a^2 \sigma$, and m for the total mass $4 \pi a^2 \varrho$,

$$n'_1 = \frac{H e}{4 m}.$$

In the two modes of motion, which correspond to $p_2 = + i p_1$, and $p_2 = - i p_1$, and the expressions for which are got in the ordinary way by taking only the real parts of the complex quantities, the coexisting Y_x - and Y_y -vibrations will show a difference of phase of $\frac{1}{4}$ period. This difference will have opposite signs for the two modes.

The vibrations corresponding to surface harmonics of the first order may be roughly described as oscillations of the entire charge in the direction of one of the axes of coordinates, or, to speak more correctly, in these vibrations there exists at every instant an „electric moment” parallel to one of the axes. Thus it appears that the mode of motion we have now examined closely resembles the one that is assumed in the elementary theory of the ZEEMAN-effect and it is but natural that we should again be led to the triplets and doublets of this theory. Only, for equal values of e and m the change n'_1 of the frequency is half of what it would be in the elementary explanation.

§ 9. In investigating the vibrations of the second order we shall introduce two new axes OX' and OY' , which are got by rotating OX and OY in their plane and the first of which bisects the angle YOY' . We shall take for the fundamental harmonies :

$$Y_{21} = Y_{xy}, \quad Y_{22} = Y_{x'y'}, \quad Y_{23} = Y_{xz}, \quad Y_{24} = Y_{yz}, \quad Y_{25} = Y_{zz}.$$

We may really do so, because any harmonic of the second order

may be decomposed into these five functions. Now we have the following solid harmonics

$$W_{21} = {}^3/{}_2 x y, \quad W_{22} = {}^3/{}_2 x' y' = {}^3/{}_4 (y^2 - x^2),$$

$$W_{23} = {}^3/{}_2 x z, \quad W_{24} = {}^3/{}_2 y z$$

$$W_{25} = {}^1/{}_2 (-x^2 - y^2 + 2z^2),$$

and, putting

$${}^1/{}_5 \pi a^2 A_2 = \alpha_2, \quad {}^1/{}_5 \pi a^3 B_2 = \beta_2, \quad {}^3/{}_5 \pi H \sigma = \varepsilon_2,$$

the following values of the coefficients:

$$a_{11} = a_{22} = a_{33} = a_{44} = {}^3 \alpha_2, \quad a_{55} = {}^4 \alpha_2$$

$$b_{11} = b_{22} = b_{33} = b_{44} = {}^3 \beta_2, \quad b_{55} = {}^4 \beta_2$$

$$\varepsilon_{12} = +2\varepsilon_2, \quad \varepsilon_{21} = -2\varepsilon_2$$

$$\varepsilon_{34} = +\varepsilon_2, \quad \varepsilon_{43} = -\varepsilon_2.$$

All coefficients that have not been mentioned here have the value 0.

The equations of motion become

$$3\beta_2 \ddot{p}_1 = -3\alpha_2 p_1 + 2\varepsilon_2 \dot{p}_2. \quad \dots \quad (10)$$

$$3\beta_2 \dot{p}_2 = -3\alpha_2 p_2 - 2\varepsilon_2 \dot{p}_1. \quad \dots \quad (11)$$

$$3\beta_2 \ddot{p}_3 = -3\alpha_2 p_3 + \varepsilon_2 \dot{p}_4. \quad \dots \quad (12)$$

$$3\beta_2 \ddot{p}_4 = -3\alpha_2 p_4 - \varepsilon_2 \dot{p}_3. \quad \dots \quad (13)$$

$$\beta_2 \ddot{p}_5 = -\alpha_2 p_5.$$

It appears that, in the absence of a magnetic force, all vibrations of the second order will have a common frequency n_2 , given by

$$n_2^2 = \frac{\alpha_2}{\beta_2} = \frac{A_2}{B_2}.$$

When the shell is placed in the magnetic field, it will be only for the Y_{zz} -vibrations that this frequency remains unchanged, and there will be four motions with a slightly increased or diminished frequency.

Operating again with expressions that contain the factor e^{int} , we can satisfy (10) and (11) by the values

$$p_2 = +ip_1, \quad n = n_2 + n_2',$$

and likewise by

$$p_2 = -ip_1, \quad n = n_2 - n_2',$$

the change in the frequency being given by

$$n_2' = \frac{\epsilon_2}{3\beta_2} = \frac{H\sigma}{6\rho} = \frac{He}{6m}.$$

In both cases we have to do with a combination of a Y_{2y} - and a $Y_{x'y'}$ -vibration, the two vibrations having equal amplitudes, and differing in phase by $\frac{1}{4}$ period.

From (12) and (13) we deduce the possibility of two similar combinations of a Y_{xz} - and a Y_{yz} -vibration; the frequency is

$$n_2 + \frac{1}{2}n_2',$$

for one combination, and

$$n_2 - \frac{1}{2}n_2',$$

for the other.

§ 10. Similar results are obtained by supposing that a charge is distributed with uniform volume-density σ over a spherical space and that each element of volume, after having undergone a displacement a from its position of equilibrium, is acted on by an elastic force, proportional to the displacement. Let $k^2 a$ be this force per unit volume, ρ the uniform volume-density of the ponderable matter, and let us suppose that this density is invariable and that, besides the charge σ , the sphere contains an equal charge of opposite sign that is *immovable*. Then, outside the magnetic field, a motion represented by

$$a = p \frac{\partial W_h}{\partial t} \dots \dots \dots \dots \quad (14)$$

may take place.

By t I have now indicated the direction in space in which the solid harmonic W_h increases most rapidly, and the differential coefficient is to be understood as a vector in that direction. The factor p is still of the form (2), and for the frequency I find

$$\rho n_h^2 = k^2 + 4 \pi V^2 \frac{h \sigma^2}{2h+1} .$$

This formula is of some interest in connection with an important phenomenon that presents itself in the series of spectral lines. If, namely, the number h is made to increase indefinitely, the frequency n_h approaches to a determinate limit.

It appears from (14) that in the present case, as well as in the former one, each type of motion corresponds to a certain spherical harmonic. Hence, all the reasonings of the foregoing articles may be repeated with only a slight modification.

I shall not dwell at length on this subject; suffice it to say, that in the magnetic field the vibrations of the first order have the three frequencies

$$n_1 \text{ and } n_1 \pm \frac{He}{2m} ,$$

whereas the frequencies of the motions of the second order are

$$n_2 , \quad n_2 \pm \frac{He}{2m} \text{ and } n_2 \pm \frac{He}{4m} .$$

In these expressions e again denotes the total charge, and m the total mass.

§ 11. The fundamental electromagnetic equations for the surrounding ether enable us to determine the vibrations emitted by the systems whose motion we have examined. The expressions for the components of the dielectric displacement will contain terms inversely proportional to the distance r , but also other terms varying as the second and higher powers of r^{-1} . Now, it is clear that only the terms of the first kind are to be taken into account when we treat of the emission of light. If these terms are calculated for the vibrations of the first and the second order, they are found in the latter case to contain the factor $\frac{a}{\lambda}$, a being, again the radius of the sphere, and λ the wave-length of the emitted radiations. If, therefore, the displacements on the sphere itself in the Y_2 -vibrations were of the same order of magnitude as those in the Y_1 -vibrations, the light produced by the first would be very much feebler than that which is due to the latter. All determinations of molecular dimen-

sions tend to show that $\frac{a}{\lambda}$ is an excessively small fraction; if it were otherwise, there would be so much damping that the spectral lines could not be as sharp as they are.

Now one might believe that on the sphere itself the amplitude of the Y_2 -vibrations were so much greater than that of the Y_1 -vibrations, that the motions of the second order could produce a perceptible amount of light, notwithstanding the smallness of the factor $\frac{a}{\lambda}$. Assuming this for an instant, improbable though it seemed, and determining by my formulae, for the shell as well as for the solid sphere, the properties of the emitted rays of light, I was led precisely to CORNU's quartet if I supposed the observations to take place across the lines of force, the middle line of the quintet vanishing altogether. This seemed very promising at first sight, but, considering the radiation along the lines of force, I found that in this case it ought to be the two inner lines of the quartet that remained, and not the outer ones, as observation has shown to be the case. This suffices to banish all idea that the influence of the factor $\frac{a}{\lambda}$ might be compensated by a large amplitude in the sphere. We cannot but take for granted that the vibrations corresponding to harmonics of the second order are incapable of radiating. This is due to the circumstance that in adjacents parts of the sphere there are equal and opposite displacements of equal charges.

Of course, the vibrations of still higher orders will be equally incapable of producing rays, and similar remarks will apply to systems of a totally different nature. Thus, the higher tones of a sounding body whose dimensions are very much smaller than the wave-length, will be very feebly heard, and it is for a similar reason that the tone of a tuning fork has to be reinforced by a resonance case. After all it seems very probable that the light of a flame is in every case caused by vibrations in which there is a variable electric moment in a definite direction, and which may in so far be called of the first order, though they need not depend precisely on a spherical harmonic. If this principle be admitted, it may be shown that, along the lines of force, only those components remain visible which are polarized in the direction of these lines, when viewed across the field.

§ 12. Seeking for some means by which the vibrations of the second order might be made to reveal themselves in the spectrum,

and by which therefore the multiple lines in the ZEEMAN-effect might perhaps be explained, I have been led to the assumption that in a source of light there exist not only the primary vibrations we have so far considered, but also secondary vibrations which are produced in the way of VON HELMHOLTZ's combination tones. This assumption is by no means a new one. Many years ago, Mr. V. A. JULIUS¹⁾ has remarked that the many equal differences existing between the frequencies of different lines of a spectrum, seem to indicate the presence of such secondary vibrations. Indeed, it seems difficult to conceive another cause for the constancy of the difference of frequencies which is found e.g. in the doublets of the alkali metals. It ought to be remarked that secondary vibrations, the word being taken in its widest sense, may arise in very different ways. The displacements may be so large that the elastic forces — and in our spheres also the electric forces — are no longer proportional to the elongations. Or, perhaps, the vibrations will cause the superficial density of the charged shell to vary to such a degree, that the convection current cannot be reckoned proportional to the velocity and the *original* density. Moreover, two vibrating particles may act upon each other and each or one of them may thus be made to vibrate as a whole. This case would present itself e.g., if there were *two* concentric spherical shells, each of them capable of vibrating in the way we have examined. They might have different frequencies, or even one of them might have the frequency 0; i. e., one sphere might be charged to an invariable density proportional to some surface harmonic.

It is not necessary to make any special assumption concerning the mechanism by which the secondary vibrations are produced. It will suffice to assume that the system is perfectly symmetrical all around the centre of a particle and that, if in one primary vibration we have to do with expressions of the form:

$$q \cos(nt + c), \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (15)$$

and in a second one with similar expressions:

$$q' \cos(n't + c'), \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (16)$$

the derived vibrations will depend on the product

¹⁾ V. A. JULIUS, De lineaire spectra der elementen. Verh. der Kon. Akad. v. Wetensch., Deel 26.

$$q q' \cos(nt + c) \cos(n't + c') = \frac{1}{2} q q' \cos[(n - n')t + (c - c')] + \\ + \frac{1}{2} q q' \cos[(n + n')t + (c + c')].$$

Of the two vibrations, I shall only consider the one whose frequency is $n - n'$.

§ 13. It is easily seen, and may be verified by working out some example, that we can obtain a secondary vibration of the first order, i. e. one which really emits light, by combining a vibration of the second order with one of the first order, these primary motions being executed either by the same sphere, or by two concentric shells.

Let us now imagine the three vibrations corresponding to the functions Y_x , Y_y and Y_z , and the five vibrations determined by Y_{xy} , $Y_{x'y'} = \frac{1}{2}(Y_{yy} - Y_{xx})$, Y_{xz} , Y_{yz} , Y_{zz} . Let the factor p that has been introduced in § 4 be of the form (15) for one of the former vibrations, and of the form (16) for one of the latter. By considering the symmetry of the system, it may be shown that a secondary vibration in the direction of one of the axes of coordinates can only be produced by the combination of these two, if, among the three indices of the two harmonics, the one that corresponds to the axis considered, appear an uneven number of times. Thus the mutual action of a Y_{xy} - and a Y_z -vibration will call forth only a vibration in the direction of OY .

Another question is to determine the amplitudes of the derived vibrations taking place along OX , OY and OZ . In every special case the amplitude must be proportional to qq' ; we may therefore denote it by multiplying qq' by a certain amplitude factor.

These factors are not independent of one another; they may all be expressed in terms of one of them. To understand this, it must be kept in mind that, if the first of the two combined primaries a and b be decomposed into some components, say into a_1 , a_2 , etc., the secondary vibration $\{a, b\}$ may be considered as the resultant of $\{a_1, b\}$, $\{a_2, b\}$, etc. If we denote the amplitude factors by $[Y_{xx}, Y_x]_x$, etc., the last index indicating the direction of the secondary vibration, we shall have

$$[Y_{yy}, Y_x]_x = [Y_{zz}, Y_x]_x,$$

and

$$[Y_{xx}, Y_x]_x + [Y_{yy}, Y_x]_x + [Y_{zz}, Y_x]_x = 0.$$

The last formula is a consequence of the relation

$$Y_{xx} + Y_{yy} + Y_{zz} = 0.$$

Let us put

$$[Y_{xx}, Y_x]_x = z.$$

Then

$$[Y_{yy}, Y_x]_x = -\frac{1}{2}z, \quad [Y_{zz}, Y_x]_x = -\frac{1}{2}z,$$

so that the amplitude factor is now known in all cases in which the harmonic of the second order is a zonal one whose pole coincides with that of the other harmonic or is 90° distant from it. All other cases may be reduced to these two by suitable decomposition of the harmonics. In this way I find the values of the amplitude factors inscribed in the following table; the letters x, y, z again serve to indicate the direction of the secondary vibration.

	Y_{xy}	$Y_{x'y'}$	Y_{xz}	Y_{yz}	Y_{zz}
Y_x	$+\frac{3}{4}z(y)$	$-\frac{3}{4}z(x)$	$+\frac{3}{4}z(z)$	0	$-\frac{1}{2}z(x)$
Y_y	$+\frac{3}{4}z(x)$	$+\frac{3}{4}z(y)$	0	$+\frac{3}{4}z(z)$	$-\frac{1}{2}z(y)$
Y_z	0	0	$+\frac{3}{4}z(x)$	$+\frac{3}{4}z(y)$	$+z(z)$

§ 14. In the magnetic field there are three modes of motion of the first order, whose frequencies are

$$n_1 + n'_1, \quad n_1 - n'_1, \quad n_1. \dots \dots \dots \quad (17)$$

We shall call the amplitudes of the variable p_1 (§ 8) in the first two motions, and that of the variable p_3 in the last one

$$q_1, \quad q_2, \quad q_3.$$

Then there are five motions of the second order, having the frequencies

$$n_2 + n'_2, \quad n_2 - n_2, \quad n_2 + \frac{1}{2}n'_2, \quad n_2 - \frac{1}{2}n'_2, \quad n_2 \dots \quad (18)$$

Let

$$q'_1, \quad q'_2, \quad q'_3, \quad q'_4, \quad q'_5$$

respectively be the amplitudes of p_1 (§ 9) for the two first motions, of p_3 for the third and fourth, and of p_5 for the last vibration.

We shall now take as an example the combination of the first of the vibrations (17) with the first of (18).

The motion of the second order consists in a Y_{xy} - and a $Y_{y'y'}$ -vibration for which we may respectively write

$$q'_1 \cos [(n_2 + n'_2) t + c']$$

and

$$q'_1 \cos [(n_2 + n'_2) t + c' + \frac{1}{2}\pi].$$

On the other hand, there are at the same time a Y_x -vibration

$$q_1 \cos [(n_1 + n'_1) t + c],$$

and a Y_y -vibration

$$q_1 \cos [(n_1 + n'_1) t + c + \frac{1}{2}\pi].$$

Consulting the small table of the last Art., I find a vibration

$$\begin{aligned} & \frac{3}{4} \times q_1 q_1' \cos [(n_2 - n_1 + n'_2 - n'_1) t + c' - c - \frac{1}{2}\pi] - \\ & - \frac{3}{4} \times q_1 q_1' \cos [(n_2 - n_1 + n'_2 - n'_1) t + c' - c + \frac{1}{2}\pi] = \\ & = \frac{3}{2} \times q_1 q_1' \cos [(n_2 - n_1 + n'_2 - n'_1) t + c' - c - \frac{1}{2}\pi] \end{aligned}$$

parallel to OX , and a vibration

$$\begin{aligned} & \frac{3}{4} \times q_1 q_1' \cos [(n_2 - n_1 + n'_2 - n'_1) t + c' - c] + \\ & + \frac{3}{4} \times q_1 q_1' \cos [(n_2 - n_1 + n'_2 - n'_1) t + c' - c] = \\ & = \frac{3}{2} \times q_1 q_1' \cos [(n_2 - n_1 + n'_2 - n'_1) t + c' - c] \end{aligned}$$

in the direction of OY . Hence, across the lines of force we shall see light whose vibrations are perpendicular to the lines of force and whose intensity may be put proportional to $\frac{9}{4} \times^2 q_1^2 q_1'^2$. Since there is a difference of phase of $\frac{1}{4}$ period between the two secondary vibrations, both together will produce circularly polarized light along the lines of force.

By similar reasoning it is found that the second of (17) and the second of (18) do not produce any secondary vibrations.

Examining all the 15 combinations, I find the following results, as regards the radiation across the lines of force.

A. There will be seen in the spectrum the following lines, whose vibrations are parallel to the lines of force.

1. A central line whose frequency is $n_2 - n_1$, and whose intensity is proportional to

$$q_3^2 q_5'^2 [12].$$

2. Two side lines, each at a distance of $\frac{1}{2} n_2' - n_1'$ from the preceding one. Their intensities are

$$\frac{9}{4} q_1^2 q_3'^2 [9] \text{ and } \frac{9}{4} q_2^2 q_4'^2 [9].$$

B. The following lines will be produced by vibrations perpendicular to the lines of force.

1. Two lines at distances $n_2' - n_1'$ from A, 1. Intensities:

$$\frac{9}{4} q_1^2 q_1'^2 [9] \text{ and } \frac{9}{4} q_2^2 q_2'^2 [9].$$

2. Two lines at distances $\frac{1}{2} n_2'$ from A, 1. Intensities:

$$\frac{9}{16} q_3^2 q_3'^2 \left[\frac{9}{2} \right] \text{ and } \frac{9}{16} q_4^2 q_4'^2 \left[\frac{9}{2} \right].$$

3. Two lines at distances n_1' from A, 1. Intensities:

$$\frac{1}{4} q_1^2 q_5'^2 \left[\frac{3}{2} \right] \text{ and } \frac{1}{4} q_2^2 q_5'^2 \left[\frac{3}{2} \right].$$

In the observations along the lines of force, the lines B only will be seen, with the same relative intensities. They will then be circularly polarized.

Of course, the source of light will contain innumerable molecules for which the quantities q and q' will have widely different values. Assuming that both the vibrations of the first and those of the second order take place indifferently in all directions, and that even a particular vibration of one kind may be equally accompanied by vibrations of the other kind in all possible directions, I find for the relative intensities the numbers inclosed in brackets.

Perhaps the way in which the ions are made to vibrate will be unfavourable to the existence at the same time of certain particular vibrations of the first and the second order; some of the derived vibrations would then have a smaller intensity than the one indicated. As to the middle line A, 1, it must always be weakened by absorption in the exterior parts of the source. Yet, in the case of luminous particles of a symmetrical structure, it seems impossible that this central line should ever vanish altogether.

§ 15. If there were no ZEEMAN-effect for the vibrations of the first order, we should have $n_1' = 0$, and the lines B, 3 would form

a single line in the middle, whose intensity would be 3. If in this case, for one reason or another, this line B_3 , and the lines A_1 and B_2 were to disappear or to become imperceptible, we should only see A_2 and B_1 and this would be a quartet as has been observed by CORNU.

The case $n'_2 = \frac{2}{3} n'_1$ (§ 9) is likewise of some interest. B_1 and B_2 would then form a single pair, each of whose components would have the intensity $\frac{2}{3}$. The distance of these strong lines would be half that of the lines A_1 , A_2 , and, if it were not for A_1 and B_3 we should have a quartet, the outer components of which would be polarized perpendicularly to, and the inner components in the direction of the lines of force. Quartets of this kind have been really observed.

§ 16. The following remarks remain to be made.

1. Since the frequency of the secondary vibrations is wholly determined by that of the primary ones, we need not trouble ourselves about a direct ZEEMAN-effect in these secondary vibrations.

2. Any explanation of the spectral lines must account for their *reversibility*. Consequently, the foregoing theory, which attributes some lines to derived vibrations, will hold only, if a system can be made to vibrate by the action of forces, whose period corresponds not to a primary, but to a secondary vibration of the system. I believe this to be really possible, but for want of space, I shall not now insist on this point.

3. If one wishes to apply the above considerations to vibrations of an order, higher than the second, one soon perceives that it is impossible to obtain a motion of the first order by combining these higher modes with vibrations of the first order.

Vibrations that are capable of radiating may however be derived from two vibrations whose order differs by unity. If now the primary motions showed the peculiarity that has been mentioned in § 10 and has been observed in the series of spectral lines, this peculiarity would also present itself in those derived vibrations whose frequency is the sum of the frequencies of the primaries; it would not exist in the secondary vibrations corresponding to the difference of these frequencies. I must acknowledge however that this conception of the series of spectral lines seems hardly reconcilable to the fact of so large a number of lines becoming simple triplets in the magnetic field.

KONINKLIJKE AKADEMIE VAN WETENSCHAPPEN
TE AMSTERDAM.

PROCEEDINGS OF THE MEETING
of Saturday March 25th 1899.

(Translated from: Verslag van de gewone vergadering der Wis- en Natuurkundige Afdeeling van Zaterdag 25 Maart 1899 Dl. VII).

CONTENTS: „The effect produced upon respiration by faradic excitation of some nerve-tracts”. By Prof. C. WINKLER and Mr. J. WIARDI BECKMAN, p. 361 (With one plate). — „The influence of salt-solutions on the volume of animal cells”. (2nd Communication.) By Dr. H. J. HAMBURGER, p. 371. — „An anomaly in the course of the plaitpointcurve in a mixture of anomalous substances”. By Prof. J. D. VAN DER WAALS, p. 385. — „On variation of volume and of pressure” (III). By Prof. J. D. VAN DER WAALS, p. 390. — „On the characteristic equation of v. d. WAALS.” By Prof. L. BOLTZMANN (communicated by Prof. J. D. VAN DER WAALS), p. 398. — „The galvano-magnetic and thermo-magnetic phenomena in bismuth” (2nd Communication.) By Dr. E. VAN EVERDINGEN JR. (communicated by Prof. H. KAMERLINGH ONNES), p. 404. — „On the velocity of electrical reaction” (II). By Dr. ERNST COHEN (communicated by Prof. H. W. BAKHUIS ROOZEBOOM), p. 417. — „Diffraction of RÖNTGEN Rays”. By Prof. H. HAGA and Dr. C. H. WIND, p. 420 (With 2 plates). — „Simplified Theory of electrical and optical phenomena in moving systems”. By Prof. H. A. LORENTZ, p. 427. — „STOKES’s Theory of aberration in the supposition of a variable density of the ether”. By Prof. H. A. LORENTZ, p. 443. — „On reducible hyperelliptic Integrals”. By Prof. J. C. KLUYVER, p. 448.

The following papers were read:

Physiology. — “*The effect produced upon Respiration by faradic excitation of some nerve-tracts*”. By C. WINKLER and J. WIARDI BECKMAN.

In one of the previous meetings¹⁾ of the Academy, we made a communication about the results of our experiments upon the changes of respiration, occasioned by faradic excitation of the cortex of the frontal brain in dogs. We then demonstrated, that a defined spot (our point 16)²⁾ answers to that excitation by increasing the func-

¹⁾ C. WINKLER, Attention and respiration. Proceedings of the royal Academy of Sciences. Amsterdam. Saturday, October 29th 1898. p. 121.

²⁾ Ibidem, cf. Fig. XX—XXIII.

ions of the inspiratory centres, which are supposed to exist in the medulla oblongata.

Strong currents call forth a forced permanent inspiration (occasionally a deep inspiration followed by lengthening (suppression) of the expiration). Weak currents cause acceleration of respiration. Often, both these effects, may be observed combined.

So with respect to this problem, we took the side of MUNK¹⁾ and SPENCER²⁾ against HITZIG³⁾ and FRANÇOIS FRANCK⁴⁾.

During our experiments we found that a regular respiration was to be considered extremely rare, with the animals at our disposal.

Acceleration of rhythm, variation of amplitude, superficial inspiration alternating with a deep or a very deep one, all these apparently spontaneous disturbances were often observed, and may occur without any electric stimulation of the cortex. They were a hindrance to our experiments.

Two circumstances could be considered to be the cause of the irregularity in the respiration of our animals; the *narcosis*, which could not be dispensed with, as a painful operation had to be performed, and the serious *operation*, necessary for the exposure of the cortex of the brain. Both influence the respiration, increasing the action of the inspiratory functions.

The influence of morphia-narcosis.

All the animals experimented on, were operated in ether-narcosis, after the injection of a small quantity of murias morphici (± 0.005 — 0.010 Gr. pro kilogramme weight of the animal).

It may be true, and we also stated the fact, that the irregular respiration may be made into a regular one, in rhythm and in amplitude, by means of ether, but we did not dare make use of this poison, considering the uncertain influence exercised by it on the excitability of the psycho-motor centres of the cortex.

On the other side it is a well-known fact, that ether-narcosis passes

¹⁾ HERMAN MUNK. Ueber die Functionen der Groszhirn-Rinde. Elfte Mittheilung. 1890. S. 189. (Gelesen in der Gesammt-Sitzung der Königlichen Akademie der Wissenschaften zu Berlin, am 20 Juli 1882).

²⁾ W. G. SPENCER. Philosophical Transactions of the Royal Society. London 1895. Vol. 185. p. 609. The effect produced upon Respiration by faradic excitation of the cerebrum in the monkey, dog, cat and rabbit.

³⁾ EDUARD HITZIG. Untersuchungen ueber das Gehirn. Berlin 1874 S. 84.

⁴⁾ FRANÇOIS FRANCK. Leçons sur les fonctions du cerveau. Paris 1887. p 126—162.

off quickly and our experiments began at the earliest, an hour after the cessation of the narcosis.

As for our experiments, a discussion of the influence of ether can therefore have no value.

It was a different case, with the small quantity of morphia which we injected four hours before the beginning of our experiments. Our reason for doing so was partly to keep the dogs a long time quiet (insensible to pain), partly to use its influence for the stimulation of the cortex as morphia increases the irritability of the psycho-motor centres to a certain extent.

The influence of morphia on the respiration was evident in all the tracings we obtained of our animals. In consequence of the very moderate dose of morphia which was used, the respiration may be somewhat slower and deeper than before, but (and this is a characteristic feature in the tracing of the respiration in morphinised dogs), every now and then, in some cases frequently, the regular tracing is interrupted by a quick and very deep inspiration, followed by a slow and prolonged — a suppressed — expiration (Fig. 1).

This long and deep sigh which we called the morphia-sigh, we always met with in the tracing of the respiration of morphinised dogs.

Sounds in the neighbourhood, e. g. the clapping of hands, the rattling of the hammer of the induction- apparatus, may call it forth. Often, in the majority of cases even, it seems to come, without any exterior influence.

Another irregularity, which is closely connected with the typical one we described, is the following:

Short inspirations, not very deep, but quick, are noticed in the tracing, during the prolonged expirations, characteristic to the sigh. They change the aspect of the sigh. It seems, as if a secundary type of respiration with a quick rhythm, is superimposed upon the ordinary type. This superposition may be seen at the very first in the expiration-phase of the morphia-sigh, but often the quick respirations appear in the expiration-phase of several respirations, sometimes even in every one of them. (Fig. 2.)

Both changes, the deep inspiration with lengthened expiration, and the quickened respiration, at first only noticeable during the period of expiration, speak in favour of increased inspiratory functions.

They are of the same nature as the inspiratory effects, observed during the excitation of our point 16 of the frontal part of the cortex¹⁾. As we mentioned before, it is known that morphia increases

¹⁾ C. WINKLER l. c. p. 158 and fig. XXV, XXVII and XXVIII.

somewhat the excitability of the psycho-motor centres ²⁾). We stated, that ether, in diminishing the irritability of those centres ³⁾, causes all those irregularities to disappear.

We also observed that acoustic and optic impressions were in many cases the perceptible causes of these sighs, and it is known in human pathology, that a general stimulation of the human cortex (meningitis, tumour of the frontal lobe) may occasion similar sighs.

So there is room for the opinion, that morphia exercises the same influence on the point 16 of the frontal cortex as on the other psycho-motor centres, and that our experiments were made on dogs, whose inspiratory functions were increased by a tonus, proceeding from the cortex of the brain. We notice at the same time two different effects upon those inspiratory functions, 1^o the deep inspiration, 2^o the quickening of rhythm.

The influence of the operation.

Of still greater importance is the effect of the operation on the animal, after the increasing (by morphia) of its sensibility to influences which promote inspiratory effects.

Nerves (the optic Nerve and two branches of the trifacial Nerve) are exposed and cut through, and so are stimulated mechanically.

This stimulation changes the respiration.

Effect of the faradic excitation of the optic Nerve upon the Respiration.

The tracing of the respiration registered during the faradic excitation of the optic Nerve (fig. 3 N. opt.) is a very characteristic one.

When stimulating with a very weak current (12 cm. distance between the two coils) a quick and very deep inspiration is produced, immediately followed by a lengthened, suppressed expiration. The stimulus not being of too long duration, the suppression of the expiration lasts as long as the stimulation, and sometimes even longer.

After stimulating with moderate currents (10—6 cm. distance between the two coils) the tracing of respiration is that which is reproduced in fig. 3. A series of quick respirations is superimposed

¹⁾ HITZIG I. c. p. 38.

²⁾ HITZIG I. c. p. 39.

upon the tracing of the deep, sigh-like respiration, mentioned above. Always the deep inspiration sets in at the beginning, the suppression of the expiration lasts till the stimulus ceases. Usually the stimulation causes also stretching of the neck.

It proves that it is necessary to control the tracing of the thoracal movements (registered with the pneumograph of MAREY) by the tracing of the changes which the pressure of the air in the trachea¹⁾ undergoes. The quick respirations then appear as violent gasps, causing very extensive movements of the lever, inspiratory as well as expiratory.

The stimulation ceasing, the inspiratory effect still continues. After a strong stimulus a few quick respirations take place followed by deep inspiration.

*The effect of the excitation of the N. trigeminus upon
the Respiration.*

Equally characteristic is the effect of the excitation of the first branch of the N. trigeminus upon the respiration.

We examined the N. frontalis and the N. lacrymalis of the ramus ophthalmicus Nervi trigemini (Fig. 3 r. laer. N. V.). As yet we will not speak of the N. ethmoidalis and the N. naso-ciliaris.

Even the weakest currents (12 cm. distance between the coils) cause quick, violent gasps, inspiratory as well as expiratory, such as we mentioned previously, in speaking of the optic Nerve. The first of the quick respirations may be an expiration, in general this is not the case. The tracing now obtained, forms however a perfect contrast to that of the respiration produced by the excitement of the optic Nerve; the effect is not characterised by the sigh, the maximal inspiration followed by suppressed expiration does not exist. On the contrary, the inspiratory position of the thorax slowly increases. The stimulus having ceased, the slowly extended, but now distinctly inspiratory position of the thorax changes suddenly and makes place for a forced expiratory position, though the quick respiration continues

¹⁾ For this purpose the vertical tube of a hollow T-canule is introduced into the trachea. The air enters freely through one horizontal arm, the other is connected with MAREY's drum. The tracings show only the quick changes of pressure of the air in the trachea. Continuous conditions, such as a permanent inspiratory or expiratory condition of the thorax are registered as zeroline, and slow variations of pressure, even if they are noticeable, are registered (one of the legs of the T-canule being open) in an incomplete manner.

for a time. Then generally one or more sighs follow, after which normal respiration is restored.

The N. infra-orbitalis rami supra-maxillaris N. trigemini answers to the excitation by a change of respiration different from that produced by stimulation of the N. opticus, but also differing at least in some measure, from the effect of the excitation on the first branch of this Nerve. The effect, even when but very weak currents are used, is always the same, a strong permanent inspiratory situation of thorax, accompanied by very quick respiration (Fig. 4). The tracing of the respiration differs, in this case, from the maximal sigh-like inspiration produced by stimulating the optic nerve, by its permanence, and by the after-effect, which is a forced expiration and long duration of the quickened rhythm. It differs also from the tracing produced by excitation of the first branch, the quickening here taking place in a more marked manner and during permanent deep inspiratory position of the thorax, and the after-effect also being more marked and lasting longer.

These facts prove, that the effect of excitation of the N. opticus and of the N. trigeminus on respiration cannot be so simply formulated as CHRISTIANI¹⁾ did by saying that excitation of the optic Nerve produced an inspiratory, that of the N. trigeminus an expiratory effect.

Faradic excitation of the three nerves mentioned, always accelerates the respiration, that is to say, each of them exercises the same influence on the second of the inspiratory effects above mentioned, but each of them has a somewhat-different effect on the first of the so called inspiratory effects i. e. on the deepening of the inspiration.

The faradic excitation of the optic Nerve calls forth the maximal deep inspiration, directly followed by lengthened expiration, suppressed as long as the stimulus lasts — what we have called sigh —, that of the first branch of the N. trigeminus produces none or very little effect, it increases slowly the inspiratory position of the thorax, that of the second branch of this Nerve maintains the thorax in a permanent and maximally extended state.

The different tracings of respiration, obtained during excitation of the nerves, make it advisable to suppose that two different inspiratory effects, acceleration of rhythm and deepening of amplitude, exist.

The deepening of the amplitude however may appear in two forms¹⁰, as the quick, maximally deep inspiration followed by a suppression

¹⁾ ARTHUR CHRISTIANI. Ueber Atmungscentra und centripetale Atmungsnerven. Monatsberichte der königlich preussischen Akademie der Wissenschaften zu Berlin. 1882. (Sitz. 17 Febr. 1881). S. 213.

of the expiration — our sigh —, ^{2o} as a permanent and maximal inspiratory position of the thorax (SPENCER's over-inspiratory tonus).

The excitation of these nerves makes it possible to distinguish and separate the two inspiratory effects. The optic Nerve, when excited with a very weak current only produces the rise of the sigh, the excitation of the first branch of the fifth Nerve in that case quickens respiration. Somewhat stronger currents generally call forth both effects (SPENCER's over-inspiratory clonus) together. A very strong current may give in both cases a permanent inspiratory effect. At the same time the tracing of respiration during the excitation of the optic Nerve, can in no case be confounded with one, written during the excitation of the first branch of the fifth Nerve.

If the cortex of the brain is excited at the point 16 of our diagram the same order of effects is produced. Weak currents cause acceleration of rhythm,¹⁾ moderate currents deep inspiration, lengthened and suppressed expiration, often with a quick rhythm of respiration superimposed in the tracing,²⁾ strong currents may produce an over-inspiratory tonus.³⁾

The same effect is produced as soon as the side wall of the third ventricle is stimulated. We fully confirmed CHRISTIANI's results on this subject, as opposed to those of KNOLL⁴⁾. The excitation of a spot in the side wall of the ventricle, situated under the commissura posterior, produces inspiratory effects. Again weak currents cause acceleration of rhythm, moderate currents cause deepening of inspiration combined with acceleration, strong currents produce a forced, permanent inspiration (Fig. 5). It seems probable too, that in this case, no centres are excited, but nerve tracts originated from the cortex cerebri in the region of point 16 of our diagram.

Our operation often causes the accelerated type of respiration, by mechanical excitation of the Nerves. This is nearly always the case, if it is necessary to remove the contents of the orbita, in order to make room for the exposure of the frontal and basal parts of the brain.

The ophthalmic artery and the ophthalmic vein must be tied and often it is impossible to tie the ligature, without taking the optic

¹⁾ WINKLER l. c. fig. XXIV.

²⁾ WINKLER l. c. fig. XXV and fig. XXVIII,

³⁾ WINKLER l. c. fig. XXVI and fig. XXVII.

⁴⁾ Prof. Ph. KNOLL. Beiträge zur Lehre von der Athmungs-Innervation. 6te Mittb. Zur Lehre vom Einfluss des centralen Nervensystems auf die Athmung. Sitzungsber. d. K. Ak. d. Wissensch. in Wien. Bd. XII. Abth. III. S. 283.

Nerve and the first branch of the N. trigeminus into it. If this happens, one may suddenly see the regular rhythm which existed before quickened not only, but the acceleration of rhythm may be superimposed upon the existing rhythm (Fig. 6).

Then a very typical tracing of respiration is obtained. Not always the superposition of the quick rhythm takes place so completely, often the acceleration covers the former type partially, and if the sighs, mentioned before, interfere with the two rhythms of respiration described, the tracing may become of an extreme irregularity. In such cases it is impossible to experiment on the changes, which respiration undergoes during the stimulation of the cortex.

The effect of the excitation of bulbus, tractus and lobus olfactorius upon respiration.

If the consequences of the narcosis and the operation are known, and the tracings obtained in such abnormal conditions understood, it may be possible to study the strange effects, which the excitation of the bulbus, the tractus and the lobus olfactorius produces upon respiration.

MUNK, DANILEWSKY,¹⁾ UNVERRICHT²⁾ and others obtained expiratory effects on stimulating different parts, principally the basal parts of the brain. SPENCER mentions the effects after stimulating the lobus olfactorius in a most accurate manner.

Our results differ in many points from the results reported by these authors. In a single point our experiments agree with, and confirm the view of SPENCER viz. with regard to the existence of a well defined spot on the lobus olfactorius, stimulation of which produces slowing of respiration and arrest of the respiration in expiration, even if weak currents are used.

In our diagram (Fig. 9) this point is indicated by the letters *f* and *g*. It is found on the lobus olfactorius, on the border of the fissura rhinica, exactly behind the place where the fissura praesylvia touches the fissura rhinica.

But our experiments teach us more. The same effect may be obtained from the tractus and also from the caudal end of the bulbus olfactorius. However, we can state, that it can only be produced from the basal surface of the olfactory bulb, and from the basal

¹⁾ DANILEWSKY. Experimentelle Beitrage zur Physiologie des Gehirns PRUGER's Arch. f. Phys. Bd. XI. 1875. p. 128.

²⁾ UNVERRICHT. Experimentelle Untersuchungen über die Innervation der Atmungsbewegungen. Verhandlungen des Congresses für Innere Medicin. Wiesbaden 1888.

parts of the tractus. Therefore it is easier to find it at the points *f* and *g* of our diagram, where the tractus and lobus olfactorius reach each other, instead of exposing the basal surface of the brain, which is much more difficult.

Perhaps this is the reason why SPENCER does not state, that the expiratory effect, we now shall describe, may be produced by excitation of the olfactory bulb.

There is still another point, wherein our results differ from those of SPENCER. This author obtains "arrest" (and it sometimes takes place in inspiration, sometimes in expiration) at the beginning of the excitation with rather strong currents, and as soon as the stimulation stops, the "arrest" ends.

We observed, that even weak currents (10 c.m. distance between the two coils) and even better currents of moderate strength (8—6 c.m.) exciting the points *f*, *g*, or the basal middle part of the tractus and the bulbus olfactorius, produce the following effects.

The respiration may become slower (Fig. 7). In general however, after a few slow respirations, the thorax gains the position of expiration, and at last "*arrest in expiration*" takes place. This arrest may occur before the stimulus (during 10 seconds generally) ceases, but more often this arrest begins only a few seconds later. It may then last half a minute and longer. The current not being stopped, the arrest sets in, and may last for 1 or 2 minutes; ultimately deep inspirations set in.

If the currents last 10 sec. only, the *arrest* often begins after they have ceased, is prolonged 15 to 45 seconds, and then a few superficial respirations appear (often only visible in the tracing of the trachea-respiration), the thorax meanwhile taking an inspiratory position. This period may be indicated by a few inspirations only, (Fig. 9) at other times the thorax abruptly takes an inspiratory position and remains a long time in it, some irregular and superficial respirations coming between (Fig. 10), and often the thorax reaches its inspiratory position only very slowly after a greater number of irregular, superficial, gradually deepening respirations (Fig. 11 upper tracing). At the end one or two deep sighs appear, followed by very deep and slow respirations. 15 minutes and more may elapse before all has returned to the normal condition.

The arrest and the described mechanism of compensation of the arrest — as we have called this after-effect, because it may be supposed that deepening of the inspiration, compensates the arrest in expiration — may be called forth, by exciting the points *f* and *g* with the utmost constancy. With no greater constancy are the movements in

the opposite fore-leg caused by faradisation of the psycho-motor centre of the foreleg.

Occasionally the compensation-movements may be seen, even without the pre-existence of the arrest itself (Fig. 11 lower tracing).

Exceptionally, the mechanism of compensation is not brought forth, and in two cases the dog died from the shock, respiration being arrested in expiration. In the cases, in which this event took place, the inspiratory functions had been injured by preceding experiments of long duration.

It is clear, that these expiratory effects are due to the stimulation of the olfactory bulbus, tractus and lobus, consequently of a nerve-tract, not of a centre. It may be conceived, that the dog, as soon as strong olfactory impulses reach him, arrests the respiration in expiration, to prevent his smelling more. Then prudently, with irregular, superficial respirations, the thorax is brought into an inspiratory position and as soon as the danger seems past, he restores his respiration with deep sighs and long, deep inspirations. The tractus olfactorius answers faradic excitation by expiratory effects.

The difference between our results and those of SPENCER, which will make further experiments necessary, may perhaps be due to the fact, that SPENCER worked with ether-narcosis.

DESCRIPTION OF FIGURES

Fig. 1. Morphia-sigh of a dog.

1 is the zeroline.

2 is the tracing of a tuning-fork of 2 vibrations per second

3 is the tracing of the movements of the thorax registered with the pneumograph of MAREY.

4 is the tracing of the respiration registered from the trachea.

Fig. 2. Morphia-sigh with quickened respiration superimposed upon the phase of expiration.

1 is the zeroline.

2 is the tracing of a tuning-fork of 5 vibrations per second.

3 is the tracing of the movements of the thorax registered with the pneumograph of MAREY.

4 is the tracing of the pulsation of the arteria femoralis.

Fig. 3. Effect upon respiration during faradic excitation of the optic Nerve and during that of the first branch of the fifth Nerve.

1 is the tracing written by the electric signal, noticing the moment of excitation.

2 is the tracing of the tuning-fork of 2 vibrations per second.

3 is the tracing of the movements of the thorax registered with the pneumograph of MAREY.

4 is the tracing of respiration registered from the trachea (in all the following figures, the ciphers 1—4 have the signification here mentioned).

Fig. 1.

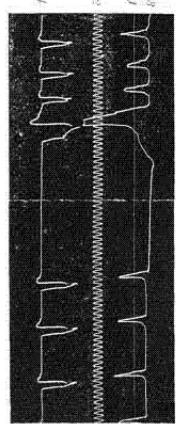


Fig. 2.

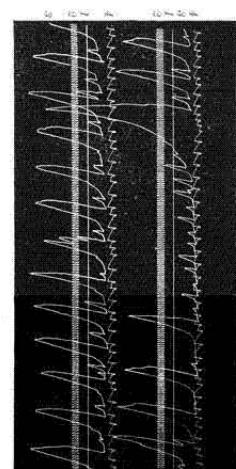


Fig. 3.

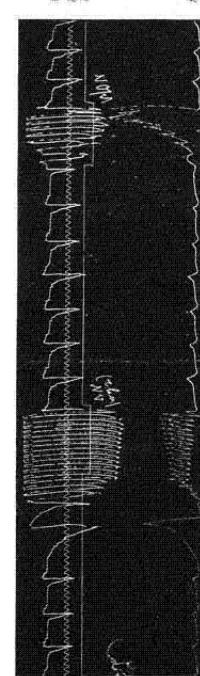


Fig. 5.

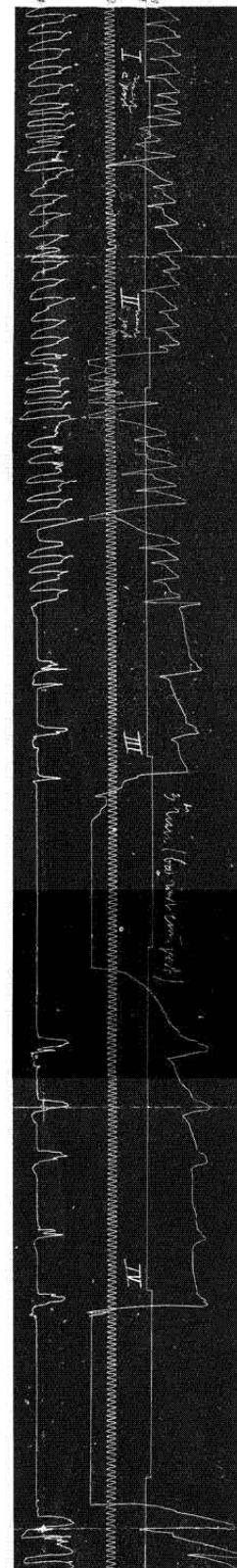


Fig. 9.

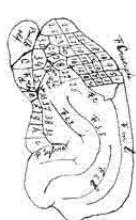


Fig. 4.

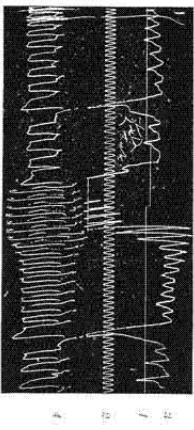


Fig. 6.

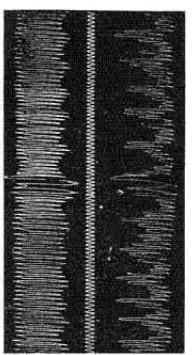


Fig. 7.



Fig. 8.

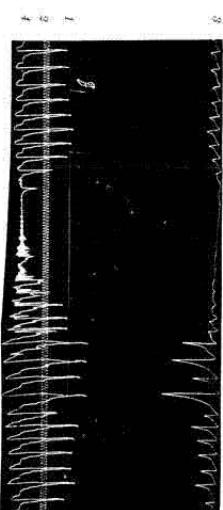


Fig. 11.

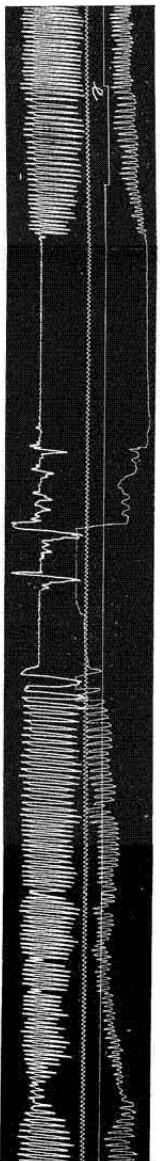
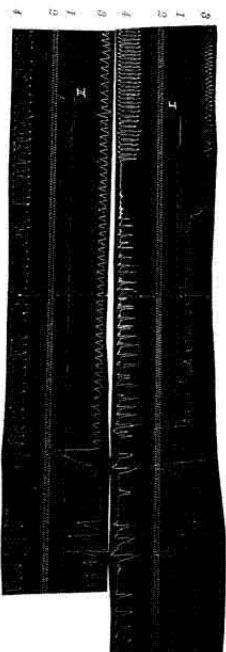


Fig. 10.



At N. opt. the optic Nerve is stimulated, At *r. lacr.* N. V. the first branch of the fifth Nerve. The signal causes the rise of the zero-line as long as the faradisation lasts. The distance of the coils in exciting the optic Nerve was 8 cm., in exciting the *r. lacrymalis* it was 12 cm.

- Fig. 4. Effect upon respiration during faradisation of the second branch of the fifth Nerve.

The tracings 1 and 2, does not exactly part from the same point as the tracings 3—4. The rise of the signal takes place at a moment that the tracing 3 and 4 have already passed. Tracing 1 and 2 must be read, placed somewhat more to the right side.

- Fig. 5. Effect upon respiration during faradisation of the wall of the posterior end of the 3rd ventricle.

In I, II, III and IV the stimulation takes places with increasing currents (Distance between the coils of 9, 7, 5 and (4 cm.).

- Fig. 6. Effect upon respiration of a mechanical stimulation (ligature) of the Nerves, which reach the orbita.

The slow respiration existing before, immediately varics. There exists a type of respiration, wherein two different rhythms can be recognised. The one is slow and the other, quick one, is superimposed upon it.

- Fig. 7-11. Effect upon respiration during and after the stimulation of the olfactory tract and of the lobus olfactorius.

- Fig. 7. During the faradisation of the olfactory tract respiration becomes slower. (Distance between the coils 6 cm.).

- Fig. 8. Faradisation of the lobus olfactorius (Distance between the coils 8 cm.) in *g*. Arrest in expiration, follows the stimulus. A few irregular respirations bring the thorax in an inspiratory position. Deep and slow inspirations follow.

- Fig. 10. Idem in point *e*. Slowing of respiration. increasing of the expiratory state, arrest in expiration. Afterwards arrest in inspiration, deep sighs and finally deep and slow respiration, lasting a very long time.

- Fig. 11. Upper tracing. Idem in point *g* (7 cm. distance between the coils). Slowing of respiration, arrest in expiration, afterwards irregular and superficial respirations extending the thorax, sighs, and finally deep and slow respiration.

Lower tracing. Idem in point *g* (10 cm. distance between the coils). Without a precedent arrest in expiration, the described movements of inspiration appear nearly at the same time after the stimulus, as in the upper tracing.

- Fig. 9. Diagram of the brain of a dog, showing the lateral parts of the rhinencephalon which were stimulated. After faradisation of the point *f* and *g* the expiratory effects are noticed.

Physiology. — "The influence of salt-solutions on the volume of animal cells". (2nd Communication). By Dr. H. J. HAMBURGER.

At the May-meeting of last year was discussed the influence of salt-solutions of the volume of red blood-corpuscles, white blood-corpuscles and spermatozoa. We can now communicate the result of similar experiments made with epithelium-cells, derived from four different places: the intestines, windpipe, urine-bladder and the oesophagus.

The epithelium was invariably obtained by carefully scraping it from the organ of an animal just killed. The scrapings were distributed in a little fresh blood-serum or in a Na Cl-solution of 0.9% and afterwards filtered through a small filter of unprepared gauze, in order to have the cells as far as possible quite free or in small aggregates. Equal quantities ($\pm \frac{1}{4}$ c.c) of the fluid thus obtained, were measured off and inserted in test tubes in which were equal quantities of the salt-solutions, to the influence of which the epithelium cells were to be exposed. After exposure for half an hour equal quantities of the mixtures were put into funnel-shaped tubes to be subjected to the influence of centrifugal force. When the level of the sediment had become fixed, the volume of the epithelium-column was considered to be determined. If after being centrifugalized for a short time, the small column of cells did not appear to be quite homogeneous, by far the greater part of the clear fluid was pipetted off, the small column with the remainder of the fluid stirred by means of a platina wire into a homogeneous mixture and again centrifugalized. If this was necessary with one of the tubes, the same was done with the other three.

I. INTESTINE-EPITHELIUM.

Experiment I.

Epithelium of the small intestine of a horse, at about one meter's distance from the pylorus.

The epithelium is distributed in a little blood-serum of the same animal.

Salt-solutions employed	Volume of the epithelium
Na Cl -sol. of 0.7 %	35 5
" " " 0 9 %	34
" " " 1 %	33
" " " 1 5 %	29

It will be seen that the volume of epithelium regularly decreases with the rise in the concentration of the salt-solution.

¹⁾ Cf. Verslag der Vergad. Kon. Akademie, April 21 1897 and May 28, 1898.

A priori, another result might have been expected. The intestinal mucosa may be considered as a resorbing organ *par excellence*, and, as experience teaches us, diluted Na Cl-solutions pass through it with the greatest facility. Hence it seemed obvious that when scraped - off intestinal epithelium would be distributed in large quantities of NaCl-solutions of different concentrations, the cells would be saturated with these solutions, so that there would soon be no more question of a difference in osmotic pressure between the contents of the cell and its surroundings; in other words, it was to be foreseen that the volume of the intestinal epithelium would not be affected by the influence of salt-solutions of different concentration.

The following experiment, made in the same manner and with the same sort of epithelium, was to give results quite different from those of the first experiment.

Experiment II.

Epithelium of the small intestine of a horse, at about half a meter's distance from the pylorus.

The epithelium is distributed in a little blood-serum of the same animal.

Saltsolutions employed		Volume of epithelium.
NaCl-solution	of 0 5%	106
" "	" 0.7%	103
" "	" 0 9%	103
" "	" 1.5	105

In this experiment the volume of the epithelium remained unchanged.

The difference in the results of the two experiments made a number of similar experiments necessary.¹⁾ They confirmed and enlarged the knowledge we had obtained from the two first: occasionally it appeared that, as in experiment I, the volume of epithelium was greatly under the influence of the concentration; in by far the most cases, however, the concentration exercised no per-

¹⁾ A full and detailed description of the experiments, hereby made, would require more space than we can here afford; that is also the case with other particulars. They will, however, be inserted in the Archiv f. Anat. u. Physiol. Physiol. Abth,

ceptible influence. This result was obtained not only with NaCl-solution, but also with grape-sugar and with mixtures of serum and water.

Seeing this difference in the results, obtained with the same sort of epithelium, only two possibilities suggested themselves: either the method of investigation was not to be relied on, or the epithelium, though drawn in the different cases from the same spot, was not always in the same condition.

The first possibility had to be rejected; 1. because in one and the same experiment the results with salt-solutions of different concentration corresponded, not only two to two, but also mutually, and that, indifferently whether the result did, or did not, indicate an influence on the volume; 2. because the method with the red and the white corpuscles, the spermatozoa, and also with the bladder- and oesophagus-epithelium obtained by scraping, had yielded uniform results.

There is nothing left but to assume that the epithelium is not always in the same state of permeability.

With respect to this we may form two conjectures: we may suppose that it is merely a question of time, in other words, that in those cases in which the volume of epithelium appears to be influenced by the concentration of the salt-solutions, the cells after half an hour's immersion are not yet, or but very imperfectly saturated with the solution, whereas the osmotic equilibrium has already been established. Under these circumstances more salt-solution might, after half an hour's action, have penetrated into the cell, and the concentration of the salt-solution would in a less degree cause its influence to be felt on the volume of the cells.

1. This, however, proved not to be the case.

Consequently we must consider the second possibility, viz., that we have to do with a modification in the state of the epithelium which continues till it is removed by some agent, a chemical one, for instance. And indeed, we sometimes succeeded in bringing epithelium from one state into the other; and that, by means of $Mg\text{SO}_4$, which is known to be able to bring about a modification in the resorbing power of the intestine, and also by the addition of traces of an acid. I say purposely „sometimes”, for now and then we failed.

For the rest, the conditions under which the intestinal epithelium passes from one state to another and what changes, the cell thereby undergoes, have hitherto remained unknown to us¹⁾.

1) I may here observe that this result is by no means a contradiction of the views I formerly expressed, viz., that the resorption phenomena, hitherto known, may be explained physically. At the time I stated emphatically: "I do not think of asserting

Involuntarily we are reminded here of what HEIDENHAIN observed in the live animal with respect to the power of the interstitial substance between the epithelium cells to allow methyleneblue to pass through it: in the same field he saw amidst cells whose interstitial substance was coloured blue, others whose interstitial substance had remained absolutely colourless.¹⁾.

In the most recent times they have tried in the Zürich laboratory to answer the question, whether in the intestine, besides the interstitial substance situated between the epithelium cells, the cells themselves allow dissolved substances to pass through them.²⁾

The investigations described above have, in my opinion, given an answer to this question. For, save in those exceptional cases, in which the scraped-off epithelium appears to be in a peculiar condition, the volumina of cells, influenced by salt-solutions of different concentration for half an hour, exhibit no mutual differences.

Now this phenomena might be explained by the supposition, that the epithelium cells are quite indifferent to the salt-solutions, that is to say, that they allow neither water nor salt to pass through them; but the supposition that they would not allow water to pass through them is immediately disproved by the fact that when, *immediately* after mixing them with salt-solutions, we centrifugalize them, and thus do not wait half an hour before doing so, a difference in volume is distinctly observed, viz., the volume increases as the concentration rises.

The cells then, must be permeable to water. Yet the fact, that the volumina of the epithelium cells after a longer immersion in salt-solutions approach nearer and nearer to each other, forces us no less to conclude that the salt-solution penetrates, as such, into the cell.

Our experiments teach us something, also with respect to the permeability of the *cell-nucleus*. They teach us that it is most probable that, beside the body of the cell, the cell-nucleus of the intestinal epithelium also allows salt solutions to pass through it. For, just

"that life neither can, nor will exercise an influence on the resorption process." "Under physiological and pathological conditions finely-shaded changes may unquestionably manifest themselves in living membranes, exercising no small influence on the physical process taking place in them, but by which these processes do not cease to be purely physical" (On "The significance of respiration and peristaltic for the resorption in the intestine." Verslag der Verg. Jan. 2th, 1896, note).

¹⁾ HEIDENHAIN. Beiträge zur Anatomie und Physiologie der Dünndarmschleimhaut. Pfluger's Archiv, Supplementheft. 1888, S. 49.

²⁾ R. HÖBER. Ueber Resorption im Dünndarm. Zweite Mittheilung. Pfluger's Archiv, B 74, S. 269, 1899.

suppose that the nucleus of the intestinal epithelium was permeable *exclusively* to water, its volume ought to be modified by the concentration of the surrounding salt solution, and the cell in its entirety would experience the influence. What do we observe, however? That the total volume of the cells is independent of the concentration of the surrounding salt-solution. Hence, it follows that, considering the relatively great share the nucleus has in the volume of the cell, the nucleus has also allowed the surrounding salt-solution to pass through it.

However, if the nucleus were quite impermeable, even to water, the total volume of the cell might still have been independent of the concentration of the surrounding salt-solutions. Considering however all we have seen concerning the influence of salt-solutions on the volume of spermatozoa, red and white corpuscles, bladder- and oesophagus-epithelium, the impermeability to water is highly improbable, and can hardly be supposed, if we consider the part, played by water in the vital processes of the cell.

II. CILIARY-EPITHELIUM.

The second sort of epithelium we have investigated is ciliary-epithelium from the trachea. The advantage of this is, that during the whole experiment, we can ascertain whether the object is yet in a live state. After the cells had been, four hours and even longer, in contact with the salt-solutions, cilia were or could be set in movement. Also after centrifugalizing ciliary movements could be produced.

Since the experiments were performed in the same manner as with the intestinal-epithelium, it will suffice to record the results.

Experiment XXI.

Ciliary-epithelium from the trachea of a horse just killed.

The epithelium is distributed in a little Na Cl-solution of 0.9%⁰. Immediately afterwards equal quantities of it are brought into contact with 15 c.c. of a solution of Na Cl 0.7%⁰ and 1.2%⁰. After half an hour's immersion, 8 c.c. are centrifugalized.

Salt-solutions employed.	Volume of the epithelium.
NaCl 0.7% ⁰ .	98
" 0.9% ⁰ .	98
" 1 2% ⁰	85

If we compare the numbers, obtained after an immersion in NaCl 0.7% and 0.9%, it appears, that the difference in concentration has exercised no influence on the volume of the cells, but it has with respect to the Na Cl-solutions 0.9% and 1.2%.

It was possible that the Na Cl-solution 0.9% and 1.2% had therefore caused some contraction, as half an hour's immersion had been insufficient to effect a complete exchange between the contents of the cell and its surroundings.

Under this supposition it seemed to us recommendable to centrifugalize the now remaining, but already two-hours old mixtures.

Salt-solutions employed.	Volume of the epithelium.
Na Cl-sol. of 0.7%	86
" " " 0.9%	86
" " " 1.2%	83

While thus after half an hour's immersion the difference in volume of the epithelium, lying in 0.9% and 1.2% Na Cl solution amounted to $\frac{98-85}{98} \times 100 = 13\%$, this difference, after two hours amounts only to $\frac{86-83}{86} \times 100 = \pm 4\%$.

It appears thus, that after a sufficiently long immersion of the epithelium in salt-solution, the difference in concentration exercises no influence on the volumina.

Of the many experiments, made in this direction, I will add two.

Experiment XXII.

Epithelium from the trachea of a horse just killed, distributed in serum and afterwards in salt-solutions.

Salt solutions employed.	Volume of the epithelium after an immersion in salt-solutions for		
	1/2 hour	2 hours	4 hours
NaCl-sol. of 0.7%	68	66	62
" " " 0.9%	63	64	61
" " " 1.2%	57	59	57
" " " 1.5%	54	57	57

It will be seen that the longer the epithelium is left in contact with the salt-solution, the less is the difference in volume. In this case a contact even of two hours is not sufficient to effect a complete exchange between the contents of the cell and its surroundings. At the end of the experiment, we can, by putting a preparation in a gas-camera and passing CO₂ through it, still set the cilia in movement.

Experiment XXIII.

Salt-solutions employed.	Volume of the epithelium after an immersion in salt-solutions for		
	1/2 hour	2 hours	3 hours
NaCl-sol. of 0.7%	90	83	80
" " " 0.9%	86	79	81
" " " 1.2%	78	76	78
" " " 1.5%	74	75	78

From this experiment we learn that, after half an hour's immersion, the volumina still differ, but after two hours only very little, and as little after three. After two hours thus the exchange between the contents of the cell and its surroundings is almost completely accomplished here.

At the end of the experiment the epithelium was still alive.

From these experiments, then, we may conclude that the ciliary-epithelium in a live state allows Na Cl-solutions to pass through it, though not so rapidly as does the intestinal epithelium.

This permeability of the isolated epithelium, which, although considerable, will of course be more intense *in situ*, is quite in accordance with the clinical experience, that remedies injected intratracheally are very rapidly resorbed. About the permeability of the nucleus, can be said the same what has been observed with respect to the intestinal epithelium.

III. BLADDER-EPITHELIUM.

If with the intestinal- and tracheal-epithelium we had to do with cells which might be expected to be easily permeable to salts, the reverse was to be expected of bladder-epithelium. The bladder, indeed, would very badly answer its purpose, if it possessed a great permeability for dissolved substances.

As is well known, the urine-bladder is lined inside with a fourfold layer of epithelium cells. In a pig-bladder these may be easily eliminated. It proved however to be necessary to wash the bladder out well, because not infrequently in the pig, and commonly in the horse, sediment is found on the mucous membrane.

Experiment XXXI.

Bladder epithelium of a pig just killed; distributed in a little Na Cl-solution of 0.9%.

Equal quantities of this are left for half an hour in contact with Na Cl-solutions of 0.7%, 0.9%, 1.2% and 1.5%. After being centrifugalized for about two hours, the contents of the tube are well stirred up by means of a platina wire, left an hour to themselves and again centrifugalized.

Salt-solutions employed.	Volume of epithelium after injection of	
	1/2 hour	1½ hour
NaCl-sol. of 0.7%	78	78
" " " 0.9%	70	70.5
" " " 1.2%	62.5	62
" " " 1.5%	58	58

From this experiment it appears, that the volume of epithelium decreases regularly and also considerably, with the rise of the concentration of the salt-solution, and that the figures obtained after an hour and a half's immersion are found to be the same as after an immersion of only half an hour.

On the ground of similar observations, made on red and white corpuscles and spermatozoa¹), we came at the time to the conclusion, that the cells consist of two substances, which behave differently with respect to hygroscopic power; one, the protoplasmatic, which has no share in the hygroscopic power of the cell, and the other, the intracellular fluid, representing the whole hygroscopic power of the cell. It was then possible by a simple calculation to determine the proportion of the volumina of protoplasmatic to intracellular

¹⁾ Proceedings Royal Academy Amsterdam, May 1898.

substance. And the figures obtained with the same species of cell by means of different salt-solutions agreed well with each other.

If we calculate this proportion also here (1st column), considering the Na Cl-solution of 0.9% as the physiological one, we find for the volume of the protoplasmatic substance with respect to that of the whole cell

from *a* and *d* 57.8%

from *a* and *c* 58.2%

from *b* and *d* 57.1%.

These figures correspond well to each other and, joined to other experiments yielding the same results, which we must omit however to describe here, justify the same conclusion which, at the time, we arrived at with respect to blood corpuscles and spermatozoa.

But not only is there considerable agreement among the figures obtained for the volume of protoplasmatic substance of the bladder epithelium of different pigs and horses, but it struck us that these figures deviated so little from those, which we had found with the red and the white corpuscles of the latter animal (Proceedings Royal Academy Amsterdam, May, 1898).

Hence the idea occurred to us that we should once more try simultaneously with *the same animal* to determine the volume of protoplasma-substance in bladder-epithelium and blood-corpuscles.

As will appear from the following experiment, the agreement was striking, indeed.

Experiment XLI.

Bladder-epithelium of a pig and defibrinated blood of the same animal are mixed with Na Cl-solutions of 0.7% and 1.5%, and half an hour afterwards both pairs are simultaneously centrifugalized.

	Volume of sediment	Contraction by NaCl 1.5%
<i>a.</i> NaCl-sol. of 0.7% } Bladder-epithelium	96	$\frac{96-69}{96} \times 100 = 28.1\%$
<i>b.</i> " " 1.5% }	69	
<i>c.</i> NaCl sol. of 0.7% } Red corpuscles.	134	$\frac{134-96}{134} \times 100 = 28.4\%$
" " 1.5% }	96	

Consequently there is a very striking agreement between the contraction of the bladder epithelium (containing the nucleus) and the red blood-corpuses (void of nucleus), effected by the *same* salt-solution.

Influence of Urea.

Considering the above-described behaviour of bladder-epithelium with respect to Na Cl, it appeared further of importance to me to ascertain whether this applied to urea, a substance which is present in large quantities in the urine of omnivora, and which according to determinations of the freezing-point made by me, constitutes in man more than a third part of the hygroscopic power of the whole of the urine.

The question interested me, especially from a general physiological point of view. As is well known, GRIJNS and KÖPPE observed that red blood-corpuses let urea easily pass through them. And I should not be surprised if that were the case with most of the other cells too. It could not but appear fitting to us, that the cells should be able to get rid of urea, the principal final product of albumen analysis. But it would certainly appear unfitting, if the urea should be able as easily to penetrate through the wall of the bladder. In that case the bladder would answer very badly to its purpose as a reservoir for the refuse products, of which the urea constitutes an important part, and one not without danger to the organism.

The question then arose: does the bladder-epithelium, in contrast with the red-blood corpuses possess the power of refusing a passage to the urea? In order to answer this question we prepared urea-solutions, isotomic with Na Cl 0.7, 0.9, 1.2 and 1.5%, caused them to act on the bladder-epithelium, but obtained only a comparatively slight influence on the volume. On a longer action the difference between the volumina was 0; in other words, in four different solutions the epithelium exhibited the same volume; the urea-solutions had thus penetrated as such into the cells.

Formerly we had observed that red-blood corpuses are destroyed in pure urea-solutions¹⁾; perhaps the epithelium was injured too by such solutions. We, therefore, resolved, instead of pure urea-solutions to take mixtures of Na Cl and urea-solutions, which were, two and two, isotonic with each other, the more so, because in the urine there is much Na Cl besides urea.

Here is an experiment that speaks distinctly.

¹⁾ Archiv. f. Anat. u. Physiol. Physiol. Abth. 1896. S. 481.

Solutions employed.	Volume of the epithelium
a NaCl-solution of 0.7%	85.5
b. 75 cc NaCl-sol. of 0.7% + 25 cc ur. sol isot. with NaCl 0.7%	102
c. NaCl-solution of 1.5%	61
d 75 cc NaCl-sol 1.5% + 25 cc ur. sol. isot. with NaCl. 1.5%	70

Had the Na Cl-urea-solution *b* acted like the Na Cl-solution *a*, with which it was isotonic, the volume would then have become, not 102, but 85.5. And had the Na Cl-urea-solution *d* acted like the Na Cl-solution *c*, with which it was isotonic, the volume would have become, not 70, but 61.

But the volume of *b* corresponds exactly to what would have been found, if 75 c.c. Na Cl-solution 0.7% had been diluted with 25 c.c. of water, instead of with 25 c.c. of urea-solution.

The same applies to *d* with respect to *c*.

The urea has thus distributed itself equally over the cell and its surroundings, without curtailing the power of the epithelium to resist Na Cl. For the calculation of the volume of protoplasma-substance from *d* and *b* (supposing the urea-solution added, be considered as water) and from *c* and *a* give the same figures.

We are, therefore placed before this question: What is the reason the urea so easily enters the scraped-off epithelium, whereas the intact wall of the bladder appears to exhibit if not an absolute, yet but a very slight permeability to urea.

With the answering of this question I am now engaged.

IV. · OESOPHAGUS-EPITHELIUM.

We now wish to examine another species of epithelium of which, as was the case with bladder-epithelium, an impermeability to salt was to be foreseen.

For this purpose we tried the epithelium of the oesophagus, which for resorption has no more significance than has the bladder. What we scrape off here consists of great flat cells in which are found no inconsiderable number of round and oval grains which make the impression of being bacteria, and which may also be deeply stained with alkaline coloring matter.

The experiments performed in precisely the same way as those

with the other species of epithelium, taught us that there is, as is the case with bladder-epithelium, a regular connection between the volume of the cells and the concentration of the salt-solution used. There was, however, considerable difference in the amount of the alterations of volume under the influence of salt-solutions in different oesophagi of the same species of animal.

On studying our notes, however, it was soon obvious that a slight dilation or contraction was always accompanied by the presence of many grains in the cell.

And so the idea occurred to us that these grains were answerable for the relatively slight changes in volume.

With this hypothesis in view, we made some comparative determinations with oesophagus-epithelium and blood-corpuscles of the same animal, and we found a striking correspondence between the magnitude of the changes in volume of the two cells produced by the same salt-solutions; but only in the case that the microscope showed a small quantity of grains in the epithelium.

As an instance of this we communicate the following experiment:

Experiment LII.

Oesophagus-epithelium of a pig and defibrinated blood of the same animal are mixed with NaCl-solutions of 0.7% and 1.5%, and both pairs are simultaneously centrifugalized half an hour afterwards.

		Volume of epithelium.	Contraction of NaCl of 1.5%
a	NaCl-sol. of 0.7% } b " " 1.5% } oesophagus	82.5	$\frac{82.5 - 59}{82.5} \times 100 = 28.8\%$
c	NaCl sol. of 0.7% } d. " " 1.5% } red blood- corpuscles.	59	
c.	NaCl sol. of 0.7% }	90	$\frac{90 - 64.5}{90} \times 100 = 28.3\%$
d.	" " 1.5% }	64.5	

The change of volume of the oesophagus-epithelium corresponds in a striking manner to that of the blood-corpuscles.

The experiments, then, have successively taught us that in white blood-corpuscles, spermatozoa and oesophagus-epithelium the proportion of the volume of the protoplasmatic substance (including the chromatinic-threads of the nucleus) to the volume of the intracellular

(including the intranuclear) substances, is the same as in the corresponding red blood corpuscles.

We may ask ourselves whether we have not here to do with a general phenomena which, consequently, also applies to the other cells.

Before we are justified to answer in the affirmative, we should of course have to investigate a number of the other cells. Most of them however don't admit of this. Cells such as the intestinal and the tracheal epithelium, which allow salt-solutions of all concentrations to pass through them and thus neither contract nor dilate, are, as is obvious, excluded.

Summary.

The principal results to which the preceding investigation has led, are as follow.

I. Of the four kinds of epithelium examined, the intestinal-epithelium and the ciliary epithelium of the trachea undergo no modification of volume under the influence of Na Cl-solutions of different concentration; they allow the Na Cl-solutions easily to pass through them. And this is the case, not only with the body of the cell, but probably also with the nucleus.

II. The fact, that the ciliary-epithelium, at the end of the experiments, is found to be still alive, increase the value of the results, not only with respect to the ciliary-epithelium itself, but also with respect to the intestinal-epithelium.

III. In contrast with the intestinal and the ciliary-epithelium, the bladder- and oesophagus-epithelium exhibit a contraction by hyperisotonic and a dilation by hypotonic Na Cl-solutions.

IV. From the amount of this contraction and dilation may be calculated the proportion between the volume of protoplasma substance and the intracellular contents.

This proportion, for the oesophagus-epithelium as well as for the bladder epithelium, appears to correspond to that in the red blood-corpuscles of the same animal.

V. Urea-solutions, in contrast with Na Cl-solutions, leave the volume of the isolated bladder-epithelium unaltered. From mixtures of urea- and Na Cl-solutions indeed only the urea makes its way through

the cells, whereas exclusively the Na Cl-solution exercises an influence on the volume in consequence of its concentration.

This does not seem to be in accordance with the function of the bladder as a reservoir for the refuse products, of which the urea constitutes an important, and for the organisation a very injurious part.

There must be factors then, which counteract the resorption of urea in the wall of the bladder.

Physics. — *"An anomaly in the course of the plaitpointcurve in a mixture of anomalous substances."* By Prof. J. D. VAN DER WAALS.

In the „Zeitschrift für physikalische Chemie XXVIII Heft 2” KUENEN and ROBSON have communicated observations on the mutual solubility of liquids, which give occasion for a short remark. In mixtures of ethane and ethylalcohol or one of the following alcohols they found, that the plaitpointcurve consists of two isolated branches, which intersect, and end on the curve which indicates the pressure of the three phases. In the following figure their result has been represented schemetically.

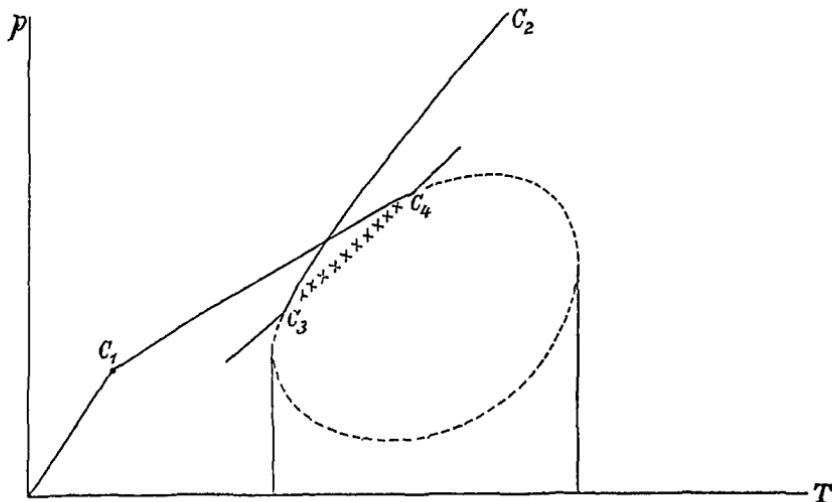


Fig. 1.

Let C_1 be the critical point of ethane, C_2 that of alcohol, then C_4 indicates the point, in which the plaitpoint, originating in C_1 can be observed no longer and it seems to serve as terminating point of the pressure of the three phases. Then C_3 indicates the point, in which the plaitpoint, originating in C_2 cannot be observed any

longer, because it disappears there above the triangle of the three phases-pressure. It appears then as the beginning point of the three phases-pressure.

Led by the thought that if the temperature rises a plaitpoint can only appear on the surface ψ , if a plait splits into two parts or can disappear, if two plaitpoints fall together, and that therefore a branch of a plaitpointcurve can never end in any other point than in the critical points of the components or in a point at an infinite distance. I have occupied myself with the question how the two separate branches can be joined into one curve. The simplest way of doing this is by joining the two branches, as is indicated by the dotted line in the figure.

The vertical lines, between which the closed part of the curve lies, represent then a maximum and a minimum temperature. The minimum temperature is the temperature at which the transverse plait splits into two parts and from which a plaitpoint starts to the right and to the left towards the critical point of the components. The greatest irregularity of the point, going towards C_2 , the critical point of alcohol, is that in its course it approaches a little nearer to the side of the small volumes. But the point, describing its way to C_3 has greater irregularities. At the maximum temperature it meets another, which has come from the opposite direction, starting from C_1 and at their meeting its movement ends. The three branches mentioned however, form a continuous curve on the $\nu\tau$ -plane.

At a given temperature, between the minimum and the maximum temperature, the projection of the spinodal curve consists of two separate parts. One part forms a small closed curve round the point where the meeting will take place; the other part has an almost regular form.

But the two plait-points, which belong to the first part, are quite covered by the ruled surface of the connodal curve of the second part. Moreover the peculiarity presents itself that there is a great distance between the spinodal and the connodal curve- a distance great enough to contain the closed branch of the spinodal curve. After I had convinced myself that an exact description of the phenomenon has been given in what I have said before, the question was to be solved, how this would agree with the fact, which I had remarked before. I have remarked (Molekulartheorie V 2) that for a mixture of two substances there can only be question of a maximum or of a minimumtemperature, but that the existence of the two at once is excluded. I think that we must look for the explanation of this in the fact that in the experiments of KUENEN and

ROBSON one of the components is an anomalous substance. It is almost generally accepted, especially because they do not follow the law of the correspondent states, that alcohols when in the liquid state possess complex molecules. At $x = 1$, i. e. for pure alcohol, the association into complex molecules is perfect or maximum. But when x is very small the dissociation into simple molecules is nearly perfect. Under these circumstances the critical temperature is not determined by the course of $\frac{a_x}{b_x}$ alone.

The critical temperature would be proportional to this quantity, if the molecules of alcohol also in a rarefied solution continued to be, what they were in the dense state when the substance is taken pure.

Now in general the critical temperature of smaller molecules will be lower than that of more complex molecules. Starting from ethane the temperature would have to rise considerably till it reached that of alcohol according to the course of $\frac{a_x}{b_x}$. In consequence of the dissociation of the alcohol molecules the course will be determined by $f \frac{a_x}{b_x}$, where f is smaller than unity. If x is near unity this factor too will not differ much from unity. For values of x which do not exceed $\frac{1}{2}$, an idea of the value of this factor is obtained by equating it to $\frac{1}{1+x}$. Now it is easy to see that even if $\frac{a_x}{b_x}$ had no minimum or maximum, there would be numerous cases, in which the factor f causes such a maximum, and this would involve that there was also a minimum.

If the true interpretation of the phenomenon is to be found in this, a course such as KUENEN and ROBSON have found for mixtures of alcohol, can never be found if the substances do not associate. But the reverse may not be stated. What appeared as a longitudinal plait in KUENEN and ROBSON's experiments, was in reality nothing but a somewhat modified transverse plait. The following phrase already refers to such a modification : (Mol. Theorie p. 172) „Es zeigt sich dann aber, dass in diesem Falle die beiden Falten auf ihre Bildungsweise anders angesehen werden müssen u. s. w.” In the figure on p. 173 the spinodal curve has, however, not been traced correctly. The bulging to the right of P should not be there. It should be replaced by a small isolated closed curve, which has separated from the other parts at the minimum temperature.

It is not without interest to remark that the mixture of ethane and methylalcohol behaves so differently. To find the probable cause

of this deviation I have construed the locus of the point for which $\frac{dp}{dv}$ and $\frac{d^2p}{dv^2} = 0$, as has been done in "Een benaderde loop voor de plooipuntslijn van een mengsel" (Verslag der vergadering Kon. Akad., 27 Nov. 1897). Now we had also to take into consideration, that one of the substances is abnormal. The influence of this property can only be calculated in a very incomplete manner, because the way in which association of the molecules takes place in such an abnormal substance, is not known. In consequence the result which I have obtained by this wholly approximative calculation may only be accepted with great reserve.

The locus mentioned may occur in two different forms, which depend on the fact whether in the gaseous state the molecules of the abnormal substance are larger or smaller than the molecules of the substance with which they are mixed. If they are larger the curve has a loop which hangs down as in the figure (1). That the molecules of C_2H_6O , C_3H_8O , etc. are greater than that of C_2H_6 will not be doubted. If on the other hand the molecules of the abnormal substance in the gaseous state are smaller, the loop is turned upwards. To all probability, however, the molecules of CH_4O will be smaller than those of C_2H_6 . In this case the following schematical figure may make clear the probable course.

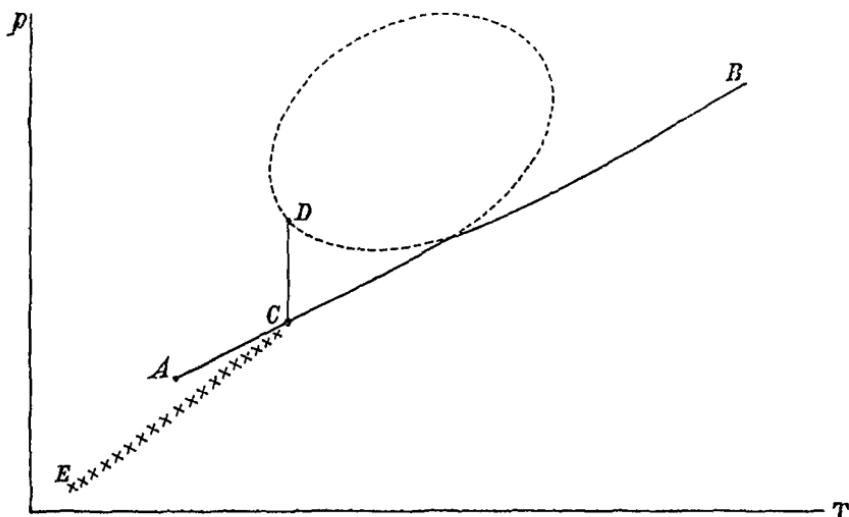


Fig. 2.

Let A be the critical point of ethane, B that of methylalcohol. Let EC be the pressure-curve of the three phases and C the point, in which the plaitpoint-curve, originating in A , meets the three-phase-

pressure-curve. In this case point D , which lies on that branch of the plait-curve that originates in B above C , is the last of the plait-points that may be realized if the phenomena of retardation are excluded. From this follows, that however much we may lower the temperature, the pressure of the three phases will continue to exist. The points between C and D , marked on the curve, indicate plait-points of such values of x , at which they lie above the derived surface, resting on the connodal line which encloses the two separate branches of the spinodal line. From the phrase of KUENEN and ROBSON, p. 357 : "Oberhalb 35,87° ist nur eine Falte vorhanden, deren Faltenpunkt wir bei Methylalcohol nicht erreichen konnten, weil Druck und Temperatur zu hoch waren" seems to follow, that point D of our figure 2 must not lie above C , but that it would have to be taken more to the side of B . As there remain so many uncertainties, it is to be regretted, that in the case of methylalcohol they have not been able to carry out their investigation. We must not attach greater value to fig. 2 than that it has led to the supposition that the cause of the difference in behaviour for mixtures of ethane and methylalcohol must be found in the smallness of the methyl molecules¹⁾.

I shall avail myself of this opportunity to make the following remark about what is really properly to be called "longitudinal plait."

In a mixture of normal substances a relation may exist between the quantities a_1 , a_{12} , a_2 , b_1 , b_{12} and b_2 , of such a nature that there actually exist two plaits, each of which possesses a connodal curve of its own. At a same temperature both may be proved experimentally. Then the principal direction of the transverse plait is // v -axis and the principal direction of the longitudinal plait is // x -axis. But in this case the plaitpoint of the longitudinal plait lies on the side of the large volumes. This has been proved theoretically by Prof. KORTEWEG in the special case of symmetry and a similar case has been worked out experimentally by Mr. VAN DER LEE. (Proc. Royal Acad. 1898). Theory has not yet been able to decide whether this plait may be again closed if the volumes are much smaller still, or whether it continues to diverge, even when the limiting volumes are reached. If the latter should

¹⁾ The geometrical locus of the points, for which $\frac{dp}{dv}$ and $\frac{dp^2}{dv^2}$ is equal to 0, cannot

teach anything about the plaitpoints of the real longitudinal plait. Moreover I do not share Mr. KUENEN's expectation that also for methylalcohol the plait is the same as for the following alcohols. I expect that the course of the plaitpoint-curve of a mixture of methylalcohol and ethane shows great resemblance to that of a mixture of water and ether.

be the case, a plait, which appears as a longitudinal plait, but which has its plaitpoint on the side of the small volumes, as in the observations of KUENEN and ROBSON, is not a longitudinal plait, but a somewhat modified transverse plait. If then in the liquid state complete mixture does not take place under all circumstances, the phenomenon is to be ascribed to other causes than in case of a real longitudinal plait.

If further investigations confirm what has been said, we should have to find the cause in this case in the fact that at least one of the substances is abnormal.

Physics. — "On variation of volume and of pressure". III. By Prof. J. D. VAN DER WAALS.

In order to be able to judge about the extent of the variation of volume and of pressure in mixtures of two substances, it will be necessary, to find for the different proportions of mixing the pressures at which the number of the molecules is the same for the two substances, if the volume and the temperature are also the same. If there were no deviations from the law of BOYLE, the pressures required would be equal. Assume that for a mixture the characteristic equation is :

$$\left(p + \frac{a_x}{v^2} \right) (v - b_x) = MRT ,$$

at a given temperature the 2nd member of this equation has a constant value for mixtures with the same number of molecules, and so we may find the pressure which satisfies the preceding condition in solving the value of p from this equation, if the value of v is constant. Take as unity of pressure 1 atmosphere and as unity of volume, the volume occupied by a molecular quantity of the mixture under that pressure. The value of the second member is then

$$(1 + a_x) (1 - b_x) (1 + \alpha t) ,$$

for which we may write with a sufficient degree of approximation

$$(1 + a_x - b_x) (1 + \alpha t) .$$

If we write p_0 for the value of the pressure taken as unity, and

$(v_0)_x$, for the volume occupied by a molecular quantity at 0°, the characteristic equation is

$$\left[p + a_x p_0 \frac{(v_0)_x^2}{v^2} \right] \left[1 - b_x (v_0)_x \right] = p_0 (v_0)_x (1 + a_x) (1 - b_x) (1 + \alpha t),$$

from which we deduce that the conditions that in 1 cM³. the same number of molecules be found are, that we have different mixtures with the same value for v , the same value for t and that we have that value of p , for which the first member and therefore also the second, is the same. The simple result is that the quantity of the different mixtures is to be chosen in such a way that $(v_0)_x (1 + a_x) (1 - b_x)$ has the same value for them all.

As $p_0 (v_0)_x (1 + a_x) (1 - b_x) (1 + \alpha t)$ is the limiting value of the product pv , the preceding condition is also fulfilled, when this limiting value is the same for the different mixtures.

In the two preceding proceedings of the Academy Mr. J. VERSCHAFFELT has published observations about mixtures of carbonic acid and hydrogen. In order to determine the volumes, which contain an equal number of molecules, the observer has followed a course which agrees with the determination of $(v_0)_x (1 + a_x) (1 - b_x)$. He is of opinion that the investigation of the limiting value of pv would not serve the purpose. I think that this conclusion must be drawn from his remark p. 332. „From the point of view” etc.

Be this as it may, I shall prove that both ways may be followed. At the same time I shall investigate, in how far his observations agree with my characteristic equation. But first a remark about the accurate form of the characteristic equation, or rather about the value of the quantities a_x and b_x .

I have always taken for them the following expressions:

$$a_x = a_1 (1 - x)^2 + 2 a_{12} x (1 - x) + a_2 x^2$$

$$\text{and } b_x = b_1 (1 - x)^2 + 2 b_{12} x (1 - x) + b_2 x^2.$$

It is easy to see that if we want to be perfectly accurate we must put:

$$a_x (v_0)_x^2 = a_1 (v_0)_1^2 (1 - x)^2 + 2 a_{12} (v_0)_1 (v_0)_2 x (1 - x) + a_2 (v_0)_2^2 x^2$$

$$\text{and } b_1 (v_0)_x = b_1 (v_0)_1 (1 - x)^2 + 2 b_{12} \sqrt{(v_0)_1 (v_0)_2} x (1 - x) + b_2 (v_0)_2 x^2.$$

If we were allowed to equate $(v_0)_1$, $(v_0)_2$ and $(v_0)_x$, there would

be no difference between the two values of a_x and b_x ; and at any rate the difference will be scarcely noticeable, if the observations on the mixtures do not reach the highest degree of accuracy. But to know the accurate form is necessary in the first place for the sake of perfect accuracy of the theoretical considerations, and in the second place in his observations Mr. VERSCHAFFELT seems to have aimed at and perhaps reached that high degree of accuracy, at which differences between the two sets of values of a_x and b_x might be of influence.

Let us now proceed to investigate in how far Mr. VERSCHAFFELT has succeeded in determining the volumes which contain an equal number of molecules under a pressure of 1 atmosphere.

He puts for this :

$$y = 0,99931 + 0,006 (1-x)^2 .$$

From this formula we obtain the value 0,99931 for hydrogen ($x = 1$), the value 1,0053 for carbonic acid ($x = 0$) and the value 1,00081 for $x = \frac{1}{2}$. By means of these three values, taken from observations, he has calculated his formula, assuming that y might be put under the form $a + bx + cx^2$.

According to the theory the factor, with which $(v_0)_x$ is to be multiplied in order to obtain the volume, which contains the same number of molecules, must be equal to:

$$(1 + a_x) (1 - b_x) = 1 + a_x - b_x .$$

If we take the latter form, viz $(1 + a_x - b_x)$, we make already use of an approximation. But even with this approximation we find:

$$y = 1 + (1 + a_x - b_x)^2 \left[\frac{a_1 (1-x)^2}{(1+a_1-b_1)^2} + 2a_{12} \frac{x(1-x)}{(1+a_1-b_1)(1+a_2-b_2)} + a_2 \frac{x^2}{(1+a_2-b_2)^2} \right] - \\ - (1 + a_x - b_x) \left[\frac{b_1 (1-x)^2}{1+a_1-b_1} + 2b_{12} \frac{x(1-x)}{\sqrt{(1+a_1-b_1)(1+a_2-b_2)}} + b_2 \frac{x^2}{1+a_2-b_2} \right] ,$$

so that we must make use of another approximation to get a form like $y = a + bx + cx^2$. All this may cause deviations, but the error which is committed by assuming this form, will remain but insignificant. My objection to the formula:

$$y = 0,99931 + 0,006 (1-x)^2$$

has another ground. Mr. VERSCHAFFELT himself remarks, that if he had made use of other experimental data to determine the values of a , b and c , he would have found the following formula:

$$y = 0,9995 + 0,00136(1-x) + 0,0056(1-x)^2.$$

But he uses the former — and now I shall show that the latter agrees to a great extent with what his own experiments teach, and that the former certainly cannot be true.

When two gases are mixed in different proportions — and one of the gases (carbonic acid) deviates in one direction from the law of BOYLE, whereas the other gas (hydrogen) does so in the other direction, we may expect the existence of a mixture, which follows the law of BOYLE. What may be brought about by change of temperature in case of a simple gas, occurs here by change of the mixing proportion. For such a mixture $y = 1$. From the formula

$$1 = 0,99931 + 0,006(1-x)^2$$

follows $x = \pm \frac{2}{3}$. From the formula

$$1 = 0,9995 + 0,00136(1-x) + 0,0056(1-x)^2$$

follows $x = 0,8$.

Mr. VERSCHAFFELT has made observations for $x = 0,7963$ and $x = 0,6445$. The products pv for $x = 0,7963$ are respectively

$$1,0740, \quad 1,0756, \quad 1,0764, \quad 1,0749, \quad 1,0748.$$

At $v = 0,02$ pv has still the value of 1,0750, and not before $v = 0,01$ it has reached the value of 1,0960.

From these values of pv we may conclude that the mixture has nearly that composition, in which it would follow the law of BOYLE in great volumes. From the long series of larger volumes, in which actually constancy of this product has been found, we might deduce, that the mixture deviates still in the direction of carbonic acid — and that therefore x should have a somewhat greater value in order to form a mixture, which follows the law of BOYLE only in very large volumes, but yet shows an increasing product from the beginning.

If we take the value of pv that belongs to the mixture, in which $x = 0,6445$, we find:

$$1,0431, \quad 1,0425, \quad 1,0413, \quad 1,0411, \quad 1,0413, \quad 1,041,$$

while at $v=0,002$ the product is diminished to 1,036 and at $v=0,01$ it descends still more down to 1,021. From the former series of values we might still be in doubt whether this mixture also possibly follows the law of BOYLE, but the value at $v=0,01$, which has perceptibly decreased, decides, and shows convincingly, that this mixture still deviates in the direction of carbonic acid. My conclusion is therefore that for the mixture for which $y=1$ the value of x cannot descend below 0,8.

In such a mixture the product $pv=(1+\alpha t)$, or ($t=18^\circ$) $pv=1,06606$. For this value we found above 1,074. This would lead to the conclusion that if Mr. VERSCHAFFELT equates the volume to 0,03, it is in reality no more than 0,02983. As a similar difference is also found for the following volumes, it would point to the fact that he has taken the unity of volume in this mixture $\pm \frac{1}{2}$ pCt. too large, an error which surpasses the amount of the above discussed corrections. If we would not acknowledge this error, we should have to take as the mixture for which $pv=1+\alpha t$ one for which x is smaller, which is contradicted by the experiments on compression, as has been shown before.

Put 35,80 as the value of the pressure, which a gas would exercise, if it should follow the law of BOYLE. For every series of observations if the volume $x=0,02983$, we find in Table XII, p. 334 indicated the pressure which is to be subtracted from 35,8, to find the influence of the deviations.

If we put

$$(35,8 - p) = \frac{ax - bx(1 + \alpha t)}{v^2},$$

the value of $ax - bx(1 + \alpha t)$ may be calculated for every mixture from this approximated equation. If we calculate $a_1 - b_1(1 + \alpha t)$ (for carbonic acid) we find 0,00614 and $-0,000454$ for $a_2 - b_2(1 + \alpha t)$ (hydrogen). By means of these values we may determine the constants for $y=a+bx+cx^2$, if we make also use of the circumstance that the value of $y=1$ is to be found for $x=0,8$. Then we find:

$$y = 0,999546 + 0,001189(1-x) + 0,005405(1-x)^2,$$

an equation which closely resembles that one, which Mr. VERSCHAFFELT thought, that he ought not to make use of, and which yet has been deduced from his own observations only. In the following table we find the value of y for values of $x=0,1, 0,2$ etc.

$x = 0$	$y = 1,00614$
0,1	1,00499
0,2	1,00396
0,3	1,00303
0,4	1,00220
0,5	1,00149
0,6	1,00089
0,7	1,00039
0,8	1
0,9	0,99972
1	0,99954

By application of the approximated formula

$$(35,8 - p) v^2 = a_x - b_x (1 + \alpha t)$$

we find from the series of observations

$x = 0,0995$	$y = 1,00483$
$x = 0,1990$	1,00398
$x = 0,3528$	1,00276
$x = 0,4993$	1,00177
$x = 0,6445$	1,00093
$x = 0,7963$	1
$x = 0,8972$	0,99965

Only at $x = 0,5$ a deviation of importance is found.

For $t = 18^\circ$ we may calculate from

$$a_x - b_x (1 + \alpha t) = - 0,000454 + 0,001189 (1-x) + 0,005405 (1-x)^2$$

the value $a_{12} - b_{12} (1 + \alpha t) = 0,0001375$ and

$$a_1 + a_2 - 2 a_{12} - (b_1 + b_2 - 2 b_{12}) (1 + \alpha t) = 0,005404 .$$

According to this value of $a_{12} - b_{12} (1 + \alpha t)$ the deviation ¹⁾ from the law of DALTON would be in the usual direction, i. e. in such a way that the pressure of the mixture at larger volumes is smaller than the sum of the separate pressures, whereas at smaller volumes the sign of the deviation is reversed. In consequence of the small value of $a_{12} - b_{12} (1 + \alpha t) = 0,0001375$, the deviation will be but slight. We may examine whether this is confirmed by the observations of Mr. VERSCHAFFELT (table XII), if $x = 1/2$. So for $v = 0,03$

1) See Proceedings of Nov. 1898.

$p_1 = 28,9$ and $p_2 = 36,31$. If an equal number of molecules of carbonic acid and hydrogen so that $x = \frac{1}{2}$, are mixed in the same volume $v = 0,03$, the volume contains twice as many molecules, so the same number in $v = 0,015$. For this volume Mr. VERSCHAFFELT finds $p = 65$. The value of $p_1 + p_2 = 65,21$.

In the same way

at $v = 0,028$	$p_1 = 30,58$	$p_2 = 38,97$	$p_1 + p_2 = 69,45$	and at $v = 0,014$	$p = 69,30$
" $v = 0,026$	$p_1 = 32,40$	$p_2 = 42,04$	$p_1 + p_2 = 74,41$	and at $v = 0,013$	$p = 74,20$
" $v = 0,024$	$p_1 = 34,38$	$p_2 = 45,65$	$p_1 + p_2 = 80,03$	and at $v = 0,012$	$p = 80,00$
" $v = 0,022$	$p_1 = 36,55$	$p_2 = 49,94$	$p_1 + p_2 = 86,49$	and at $v = 0,011$	$p = 86,75$
" $v = 0,020$	$p_1 = 39,08$	$p_2 = 55,10$	$p_1 + p_2 = 94,18$	and at $v = 0,010$	$p = 94,40$

So in the case of carbonic acid and hydrogen, the quantity $a_{12} - b_{12}(1+\alpha t)$ is not large and $(a_1 + a_2 - 2a_{12}) - (b_1 + b_2 - 2b_{12})(1+\alpha t)$ small, but the contrary. The latter may be expected for substances which differ much in physical properties.

In my communications under the same title, in the proceedings of November and December 1898, I have discussed two rules of approximation for mixtures, viz. the law of DALTON and that of AMAGAT. As a third rule of approximation the following rule might be given: In a mixture a substance exercises the pressure that it would exercise if the other molecules were substituted by molecules of its own kind. Let us call the pressure which the first mentioned gas would exercise, if all the molecules were of the same kind p_1 , and that of the second gas p_2 , then this rule of approximation comes to the same as putting

$$p = p_1(1-x) + p_2x .$$

From the graphical representation of Mr. VERSCHAFFELT p. 329 it appears that for carbonic acid and hydrogen $p - [p_1(1-x) + p_2x]$ is positive.

From the characteristic equation we may deduce for this difference:

$$p - p_1(1-x) - p_2x = x(1-x) \frac{(a_1 + a_2 - 2a_{12}) - (b_1 + b_2 - 2b_{12})(1+\alpha t)}{v^2}$$

for all volumes large enough that we may put $1 + \frac{b}{v}$ for $\frac{v}{v-b}$.

So we see that for large volumes this third rule of approximation is exactly the same as that of AMAGAT.

At a given volume p is a function of x of the second degree and the maximum deviation will be found at $x = \frac{1}{2}$.

For $v = 0,024$ the following table gives the calculated and the observed values of the pressure.

In the formula $p = p_1(1-x) + p_2x + A x(1-x)$ we have taken A as equal to 8.

x	0	0,05	0,1	0,2	0,3528	0,5	0,6445	0,8	0,9	1
p calculated	34,38	35,32	36,22	37,91	40,18	42,01	43,47	44,67	45,24	45,65
p observed	34,38	35,53	36,54	38,04	40,12	41,80	43,33	44,80	45,48	45,65

For the value of A which might have been calculated by means of the relation

$$A = \frac{(a_1 + a_2 - 2 a_{12}) - (b_1 + b_2 - 2 b_{12})(1 + \alpha t)}{v^2},$$

we should have found about 9,5, if $\Delta_a - \Delta_b(1 + \alpha t) = 0,005405$ (see p. 395).

The value of p , putting $A = 9,5$, agrees nearly perfectly for small values of x and $1-x$, but in this case the differences are larger again for values of x near $1/2$. From $A = 8$ would follow $\Delta_a - \Delta_b(1 + \alpha t) = 0,00461^1$.

From all this follows that absolute agreement between the theory and the observations of Mr. VERSCHAFFELT does not yet exist. But the differences remain below 1 pCt. It would be premature to try to decide as yet whether the differences are to be ascribed to the theory or to the experiments. Yet we may say that in these observations the differences are smaller than in those that I have tested before. And the fact that up to now the differences grow less as the observations grow in accuracy, seems to plead in favour of the theory.

¹⁾ In calculating the coefficient in the equation

$$y = 0,999546 + 0,001189(1-x) + 0,005405(1-x)^2$$

we have made use of the data for carbonic acid and hydrogen, and of the supposition that $y = 1$, if $x = 0,8$.

If we take $y = 1$ for $x = 0,82$, the coefficients become

$$y = 0,999546 + 0,001618(1-x) + 0,00497(1-x)^2.$$

These values of the coefficients seem more probable to me; moreover in this case the values of the two tables on p. 395 agree still better.

With these coefficients is

$$a_{12} - b_{12}(1 + \alpha t) = 0,000855$$

and

$$(a_1 + a_2 - 2 a_{12}) - (b_1 + b_2 - 2 b_{12})(1 + \alpha t) = 0,00497.$$

Physics. — Prof. VAN DER WAALS presents for the Proceedings of the meeting a paper of Prof. L. BOLTZMANN, foreign member of the Academy,: “*On the characteristic equation of v. d. WAALS,*” with an accompanying letter from which the following extract has been taken.

VIENNA, March 2nd 1899.

Dear Sir,

Mr. VAN LAAR has sent me from Utrecht a calculation which he undertook at your demand. From this—I have calculated the next correction term of your formula in the same way as I have followed in the second part of my gas-theory, and I take the liberty of sending the MS. to you. The result will differ in some respects from the one you obtained, but I should consider a discussion about it very interesting on mathematical grounds; not so much on physical grounds, as the further correction terms are certainly not calculable. Under slight pressures the observations are too inaccurate for this term to be useful, while for high pressures more approximation terms would be required. Moreover no observations have been made on *Hg*-vapour, argon, helium, where spherical molecules may be presumed. I should be pleased if you would lay my MS. before the Amsterdam Academy of Sciences and I should like it to be printed in its Proceedings, because I think this the best way to attain my purpose, viz. to incite those who are interested in this question to a discussion which might be useful to science.

.

Yours truly
LUDWIG BOLTZMANN.

At the demand of Prof. VAN DER WAALS, Mr. VAN LAAR has calculated a formula which may be used for the calculation of a further approximation term in the former's formula. I shall show how Mr. VAN LAAR's formula may be used for the further development of my calculation relating to this subject, and make use of the same notations as in the second part of my “*Lectures on Gas-theory*”, which I shall always briefly quote as l. c.

I. *Calculation of the space left for the centre of a new molecule to be introduced into the gas.*

Let V be the volume of a vessel, in which there are found n very

small, rigid spheres (molecules), all of them having the same properties and of the mass m and the diameter σ .

We represent by D_1 the first approximated value of the space D left for the centre of a molecule new added into the gas. This molecule has the same properties as the others. Then $D_1 = V$. As a second approximation we must subtract from this the volume of the distance-spheres of all the n molecules. The term distance-sphere stands here for a sphere concentric to the molecule and with the radius σ , so having a volume of $\frac{4}{3} \pi \sigma^3$. If we put $\frac{2 \pi \sigma^3}{3m} = b$ and the total mass of the gas $m n = G$, then the sum of all the volumes of all the n distance-spheres is $2 G b$ and the second approximated value of D is therefore

$$D_2 = V - 2 G b \quad (1)$$

For the third approximation we must take into account that not the sum of the volumes of all the distance-spheres is to be subtracted from V , as here and there two distance spheres cover each other partially, and that then the space which is common to them, is to be subtracted but once. So we must add to D_2 the sum Z of all spaces which two distance-spheres have anywhere in common.

We find the volume Z in the following way: We construe round the centre of every one of the n molecules a spherical shell concentrical with the molecule, with an inner radius x and the very small thickness dx , which spherical shell we shall call S . The volume R , which is the sum of all the spherical shells S construed in such a way, is $4 \pi n x^2 dx$. The number dn of the centres of the molecules which are found in any of these spherical shells is to the total number of the molecules in the first approximation as R is to the volume of the vessel V , so that we get

$$dn : n = 4 \pi n x^2 dx : V \quad (2)$$

or

$$dn = \frac{4 \pi n^2 x^2 dx}{V} \quad (3)$$

The last expression gives the number of molecules whose centres lie at a distance between x and $x + dx$ from the centre of any other molecule. The number of the pairs of molecules, whose centra have a distance between these limits, is $\frac{1}{2} dn$. As soon as x is between σ and 2σ , the distance-spheres of the two molecules of

the pairs in question will have a lens-shaped space of the volume

$$2K = \frac{\pi}{12} (16\sigma^3 - 12\sigma^2 x + x^3) \dots \dots \dots \quad (4)$$

in common (l.c. p. 166)¹). The sum of all these lens-shaped spaces which occur with all possible pairs of molecules, is represented by Z . Therefore

$$Z = \int K dn = \frac{\pi^2 n^2}{6 V} \int_{\sigma}^{2\sigma} x^2 dx (16\sigma^3 - 12\sigma^2 x + x^3) = \frac{17}{16} \frac{G^2 b^2}{V} \dots \quad (5)$$

and

$$D_3 = V - 2Gb + Z \dots \dots \dots \quad (6)$$

The value of D , in which the approximation has been worked out one term further, is called D_4 . To find it, we must first subtract from D_3 the sums of all volumes which belong to the distance-spheres of three molecules at the same time, and which is according to Mr. VAN LAAR

$$\frac{2\beta G^3 b^3}{V^2}$$

in which β is the quantity which he has calculated and which he has also represented by β on the last page of his discussion. Secondly however we have also to add a correction term to Z , which we shall represent by ζ , so that

$$D_4 = V - 2Gb + \frac{17}{16} \frac{G^2 b^2}{V} - 2\beta \frac{G^3 b^3}{V^2} + \zeta \dots \dots \quad (7)$$

We get the correction term ζ by the following consideration. The proportion (2) is only right as a first approximation. If we try to obtain to a greater accuracy, the last term of the proportion should not be represented simply by V , as the whole volume of the vessel is not at the disposal of all the n molecules. In the same way a correction term is to be inserted in the last member but one of the

¹) Compare also. VAN DER WAALS, Amst. Acad. 31 Oct. 1896 and 29 Oct. 1898.

proportion, while also a part of the volume, represented above by R , will fall in the distance-sphere of other molecules.

Next we may state the following rule. According to probability the relation between the number represented above by d_n and the total number n of the molecules will be the same as between that part R_1 of the space R which is left for the centre of a new-added molecule and the whole space which is left inside the vessel for the centre of a new-added molecule.¹⁾

According to formula (1) the last mentioned space is $V - 2 Gb$. The last term of the proportion (2) should therefore be represented by $V - 2 Gb$ instead of by V . R_1 is still to be found. For this purpose we construe inside each of the above considered n spherical shells with a radius x and a thickness dx , which we have called the spherical shells S , a concentrical spherical shell with an inner radius y and a thickness dy . The latter spherical shells we shall call the spherical shells T . The sum of the number of the centra of molecules, which lie in any of the spherical shells T is, analogous to the equation (3)

$$d\nu = \frac{4\pi n^2 y^2 dy}{V} \dots \dots \dots \dots \quad (8)$$

The part

$$\pi x \frac{\sigma^2 - (x - y)^2}{y}$$

of the surface lies within the distance spheres of every separate one of the molecules ; thence the part

$$\omega = \pi x dx \frac{\sigma^2 - (x - y)^2}{y} \dots \dots \dots \quad (9)$$

of the inner space of the spherical shells in question with a radius x and a thickness dx , as an easy calculation shows. That part of the volumes of all the spherical shells S which is covered by the distance spheres of all the $d\nu$ molecules together is therefore $\omega d\nu$.

If no molecules were to be found inside the spherical shells S , we might integrate this expression with respect to y from $x - \sigma$ to $x + \sigma$ and

$$\int \omega d\nu = \frac{16\pi^2 n^2 x^2 \sigma^3}{3V} dx = \frac{2bG}{V} \cdot 4\pi n x^2 dx \dots \dots \quad (10)$$

would follow.

¹⁾ For a fuller exposition of this rule comp. l. c. § 51.

Then the space of the sum of the volumes $R = 4 \pi n x^2 dx$ of all the spherical shells S together, which is enclosed in the distance spheres, would be to this total space R , as the whole space $2 G b$ occupied by the distance spheres of all the molecules of the gas to the whole volume of the vessel V , which was to be expected a priori.

In every spherical shell S , however, a molecule is found, so that the centre of another molecule cannot come closer than at a distance σ from the centre of the spherical shell. Therefore we have to subtract from the value (10).

$$\int_{y=x-\sigma}^{y=\sigma} \omega d\nu = \frac{4 \pi^2 n^2 x^2 dx}{V} \left(\frac{x^3}{12} - x \sigma^2 + \frac{4\sigma^3}{3} \right) = 4 \pi n x^2 dx \gamma,$$

in which

$$\gamma = \frac{\pi n}{12 V} (x^3 - 12 x \sigma^2 + 16 \sigma^3).$$

Instead of the last term but one in the proportion (2) we have therefore to put

$$4 \pi n x^2 dx \left(1 - \frac{2 G b}{V} + \gamma \right).$$

As we ought to substitute $V \left(1 - \frac{2 G b}{V} \right)$ for the last term, it comes to the same thing as if the last term were left as it is, and as if

$$4 \pi n x^2 dx (1 + \gamma)$$

were substituted for the last term but one, at least if terms of still higher order are neglected.

We obtain therefore for $d\eta$ the correction term

$$d\nu = \gamma d\eta$$

and for Z the correction term

$$\begin{aligned} \zeta &= \int K \gamma d\nu = \frac{\pi^3 n^3}{72 V^2} \int_{\sigma}^{2\sigma} x^2 dx (x^3 - 12 x \sigma^2 + 16 \sigma^3)^2 = \\ &= \frac{2357}{22680} \cdot \frac{\pi^3 n^3 \sigma^9}{V^2} = \frac{2357}{6720} \frac{G^3 b^3}{V^2}; \end{aligned}$$

By the substitution of these values, formula (7) becomes:

$$D_4 = V - 2 G b + \frac{17}{16} \frac{G^2 b^2}{V} + \left(\frac{2357}{6720} - 2\beta \right) \frac{G^3 b^3}{V^2} = \\ = V - 2 G b \left[1 - \frac{17}{32} \frac{G b}{V} + \left(\beta - \frac{2357}{13440} \right) \frac{G^2 b^2}{V^2} \right] \quad (11)$$

This is therefore the space left in the vessel for the centre of a molecule, when terms of four different orders with respect to $\frac{G b}{V}$ are considered.

II. Correction of the equation of VAN DER WAALS.

The shortest way to calculate this correction by means of formula 11 is that which I followed l. c. § 61. If we substitute the expression D_4 , which has now been found for the expression D , which has been used l. c. § 61 and has been represented by equation 173 there, we obtain

$$l \left[v - 2 \nu m b + \frac{17}{16} \frac{\nu^2 m^2 b^2}{v} + \left(\frac{2357}{6720} - 2\beta \right) \frac{\nu^3 m^3 b^3}{v^2} \right] = \\ = l v - \frac{2 \nu m b}{v} - \frac{15}{16} \frac{\nu^2 m^2 b^2}{v^2} - \left(\frac{1283}{6720} + 2\beta \right) \frac{\nu^3 m^3 b^3}{v^3},$$

instead of the first formula on p. 174, and so instead of the formula which is found for S six lines lower

$$S = \frac{3 n r}{2} \int (1 + \beta) \frac{d T}{T} + r \left[l v - \frac{b}{v} - \frac{5}{16} \frac{b^2}{v^2} - \left(\frac{1283}{26880} + \frac{\beta}{2} \right) \frac{b^3}{v^3} \right].$$

The formula for

$$\frac{\partial (T S)}{\partial v}$$

which follows on this one in l. c. is changed to

$$\frac{\partial (T S)}{\partial v} = r T \left[\frac{1}{v} + \frac{b}{v^2} + \frac{5}{8} \frac{b^2}{v^3} + \left(\frac{1283}{8960} + \frac{3\beta}{2} \right) \frac{b^3}{v^4} \right]$$

so that the corrected equation of VAN DER WAALS would have the following form

$$p + \frac{a}{v^2} = r T \left[\frac{1}{v} + \frac{b}{v^2} + \frac{5}{8} \frac{b^2}{v^3} + \left(\frac{1283}{8960} + \frac{3\beta}{2} \right) \frac{b^3}{v^4} \right]$$

$$= \frac{r T}{v - b + \frac{3}{8} \frac{b^2}{v} + \left(\frac{957}{8960} - \frac{3\beta}{2} \right) \frac{b^3}{v^2}} \quad (12)$$

In this formula is according to Mr. VAN LAAR

$$\beta = \frac{73\sqrt{2} + 81.17 \left(\operatorname{arctg} \sqrt{2} - \frac{\pi}{4} \right)}{32.35 \pi} = 0.0958 .$$

Physics. — “*The galvano-magnetic and thermo-magnetic phenomena in bismuth*”. (Second Communication.) By Dr. E. VAN EVERDINGEN JR. (Communication №. 48 from the Physical Laboratory at Leiden, by Prof. H. KAMERLINGH ONNES.)

1. In the Proceedings of the meeting of June 25th 1898 we have communicated the results of the observations of the four transverse galvano-magnetic and thermo-magnetic phenomena, all of them made in one and the same magnetic field with one and the same electrolytically prepared plate of bismuth. By means of the same plate of bismuth we have now observed the decrease of conductivity for electricity and heat and also the longitudinal thermo-magnetic phenomenon. It has given us much trouble to measure the two last phenomena with sufficient accuracy, and the variation of the conductivity for heat can only approximately be deduced from the measurements. Yet I communicate the results for two reasons: in the first place, for a preliminary theory it is sufficient that the order of magnitude of the phenomena is known, and secondly in consequence of the small dimensions of the plate it is not probable that further measurements with the same plate would yield a much more exact result for the absolute value. Moreover the plate during one of the last experiments has developed a crack and this has put an end to all further observations under the same circumstances, even if we had wished to continue them.

These measurements having been finished we have obtained for

the first time results in one and the same plate, in the same magnetic field and at the same temperature, for all the phenomena relating to the theory of the conduction of electricity and heat in metals. For a complete theory it would also be necessary to investigate the variation with temperature of the different phenomena; at the same time a question would arise about a possible change of the THOMSON-effect in the magnetic field. But at this stage I wished to leave variation with temperature out of consideration. It is true, in measurements for which a current of heat is required, different temperatures occur, but we will suppose this variation with temperature small enough to be neglected in a preliminary theory of the phenomena. So far as we can judge from the results obtained, they are not contradictory to this supposition.

2. I have succeeded in representing variation of resistance and longitudinal-effect by means of an empirical formula of the form

$$E = \frac{C_2 M^2}{1 + C_1 \sqrt{M^2}} {}^1)$$

in which E represents the phenomenon observed and M the magnetic force (in these calculations expressed in the unit 1000 C. G. S.).

Together with the quantities observed we also give the values obtained by means of this empirical formula. It will be shown that it represents the observations very satisfactorily; applied to those made by others, it offers an easy way of comparing their results with mine. Moreover the formula is of great use in deciding that variation of resistance and longitudinal-effect are proportional for all magnetisations, a result which is very important for the theory of the phenomenon. If for instance, we consider the longitudinal-effect as a variation of the thermo-electric power, it follows that we might deduce the change of the THOMSON-effect in the magnetic field from the variation of the increase of resistance with change of temperature.

3. Variation of the electric resistance.

a. Measurements. The observations of this variation are made

¹⁾ In the denominator $\sqrt{M^2}$, where the root has to be taken with a positive value, was written in view of GOLDHAMMER's remark (Wied. Ann. 36, p. 824, 1889) that the phenomena which do not change sign on reversing the field should be functions of M^2 .

following the method, described in the "Verslag der Vergad." of April 21st 1897, p. 493¹⁾.

As resistance electrodes we used the same thermo-elements which were used in the measurements of the transverse phenomena, the copper wires of which were again connected with one of the coils of a differential-galvanometer. Fig. 1 shows the principle of this method of observation.

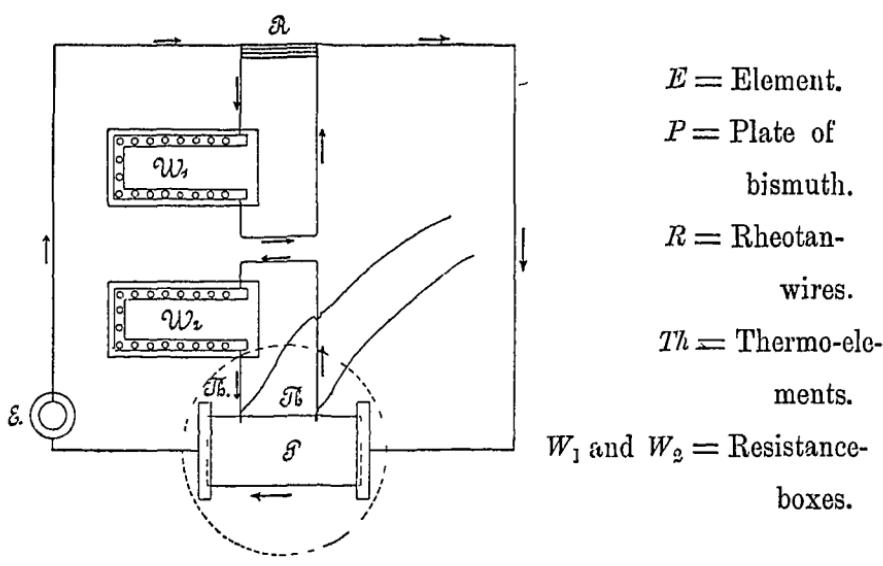


Fig. 1.

$$C_1 = 0,258$$

$$C_2 = 1,000$$

Magnetic field	Percentage Observed	increase Calculated	Difference.
4650	9,7	9,8	+ 0,1
6100	14,5	14,5	0,0
9200	25,1	25,1	0,0

From these observations we could also deduce an approximate value for the specific resistance of this bismuth at about 20°, for which 182,000 C. G. S was found.

We need not be astonished at this number being much larger than that of FLEMING and DEWAR, 116,000, which we used in the calculations of the previous communication. For, our method, although

¹⁾ Comm. Phys. Lab. Leiden, N°. 37, p. 5. Compare also my thesis for the doctorate.

it shows us very precisely the proportion of the resistances in the different magnetic fields, was neither devised nor fit for precise measurements of the specific resistance.

First, the distance of the resistance-electrodes cannot be accurately determined, and second, it is not certain that the current-lines are entirely parallel with the sides of the plate, a supposition which must be made in the calculation. In order to convince myself that these circumstances did not influence the measuring of the increase of resistance I have moreover repeated that measurement with another plate of a very regular shape, electrolytically prepared in the same way. The specific resistance being found to be here 121,000, the percentage increase of resistance was almost the same as in the case of the other plate.

b. Results of other observers.

HENDERSON¹⁾

Temp. 18°.	$C_1 = 0,2847$	$C_2 = 1,798$	
Magnetic field.	Percentage observed	increase calculated.	Difference.
960	1,5	1,3	- 0,2
1740	3,4	3,6	+ 0,2
2860	8,2	8,1	- 0,1
4160	14,8	14,2	- 0,6
6260	25,9	25,3	- 0,6
7190	30,7	30,5	- 0,2
8740	39,7	39,4	- 0,3
9650	45,2	44,7	- 0,5
10950	53,0	52,4	- 0,6
12750	62,3	63,1	+ 0,8

In order to investigate how far the empirical formula retains its meaning outside the range of the observations which were used in calculating the constants, I substituted the value $M = 38,900$, with which HENDERSON found 233,4%; this gave 245,7% which is in sufficient agreement considering the extent of the extrapolation.

¹⁾ Wied. Ann. 53 p. 912, 1894. The numbers here mentioned concern the measurements with the large spiral of bismuth, which are graphically represented in fig. 4 of Table X of that Volume.

VON ETTINGSHAUSEN and NERNST¹⁾

$C_1 = 0,1341$

$C_2 = 0,8882$

Magnetic field	Percentage observed	increase calculated	Difference.
1600	2,53	1,87	- 0,7
3160	7,87	6,24	- 1,6
5880	19,7	17,2	- 2,5
8410	30,8	29,6	- 1,2
11200	43,6	14,6	+ 1,0

FLEMING and DEWAR²⁾

Temp. 19°.

$C_1 = - 0,03084$

$C_2 = 0,1805$

Magnetic field.	Percentage observed	increase calculated	Difference
2450	6,3	1,2	- 5,1
5500	13,6	6,6	- 7,0
11200	60,9	64,9	+ 4,0

As appears from the negative sign of C_1 and the large deviations, the observations in the last table cannot be represented by our formula. But from this discussion it would appear that the accurate representation of my three results by a formula with two constants signifies more than would at first be expected.

4. Variation of conductivity for heat.

a. *Measurements.* It being our object to observe all the phenomena as far as possible under the same circumstances, we in this case had to determine the variation of temperature at different distances from the source of heat, in order to calculate from it the variation of the conductivity.

We therefore first observed the temperature in the stationary condition without magnetic field, then that temperature in the new stationary condition in the magnetic field. It appeared however after

¹⁾ Wied. Ann. 33 p 474, 1888.

²⁾ Proc. Roy. Soc. 60, p 425, 1896.

some time that if the plate were mounted between the poles as had been done until then, the condition was never steady enough to make the measurement in this way; the irregular variations of temperature as compared with the fall of temperature to be measured were too large. This difficulty was almost entirely removed by using a *U*-shaped water-jacket placed between the poles of the electromagnet, through which water was streaming under a constant difference of pressure, so that the temperature remained constant for several hours within a few tenths of a degree.

The new mounting of the plate may be seen from fig. 2.

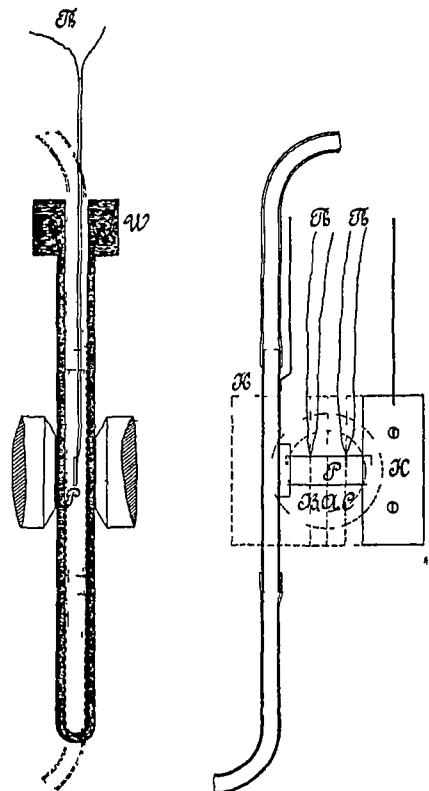


Fig. 2.

- P.* = Plate of bismuth.
- Th.* = Thermo-element.
- W.* = Water-jacket.
- K.* = Copper tube through which the steam is driven.
- A. B. C.* = Grooves for introducing the thermo-elements.
- H:* = Piece of wood for clamping the plate.

The plate having been put into this jacket, the remaining space was filled with cotton-wool. A test-experiment in which the jacket was closed on all sides with paper did not yield any different results, so that we may assume that no errors arise from currents of air.

In inserting the thermo-elements in their places we had to secure proper contact on the one hand, and on the other, we had to make

sure that the copper or the German-silver wires did not separately touch the bismuth.

The observations were made in the following way:

1. Observe for five minutes the deflection by the thermo-element without magnetic field.
 2. Observe for five minutes with the magnetic field in one direction.
 3. " " " " without " "
 4. " " " " with the " " " other "
 5. " " " " without " " "
- And so on.

During the five minutes after interrupting the magnetising current the temperature returns slowly to its original value.

Mainly by help of the deflections just before applying the magnetic field, the remaining variation of the deflection without magnetic field was represented graphically and from this was interpolated the value of that deflection at a moment five minutes after the closing of the current. The difference of this value and the deflection observed just before interrupting the current gives the fall of temperature we require.

By taking the mean for the two directions of magnetisation the error is avoided, which would otherwise be caused by the rotation of the isothermals.

Here follow some of the results obtained. *A*, *B* and *C* again indicate the three fixed places on the plate; the temperatures above each column give approximately the values as they occurred during the observations.

Magnetic field	A ± 24°	B ± 36°	C ± 19°
2800	—	0°,38	0°,07
4800	0°,48	0°,83	0°,18
6100	0°,67	1°,25	0°,28

The temperature of the water-jacket in all these observations was about 8°.

The following are the results from which the number 0°,28 for *C*. was deduced, given in order that we may judge of the accuracy of the separate determinations.

Deflection without magn (interpolated).	Deflection with magn A.	Deflection with magn B	Variation	Mean
37,90	36,80		1,10	
38,06		37,35	0,71	0,90
38,14	36,95		1,19	0,95
38,12		37,52	0,60	0,90
38,33	37,22		1,11	0,86
38,86		38,20	0,66	0,89
39,42	38,28		1,14	0,90
39,80		39,13	0,67	0,90
				—
				0,90

With several of the observations mentioned above the agreement of the separate values was still greater.

c. *Calculation of the variation of conductivity for heat.*

Let t be the difference between the temperature in the plate and that of the surrounding atmosphere, and let x be the distance from the heated end, then we can assume when using certain simplifying suppositions that

$$\frac{d^2t}{dx^2} - a^2t = 0$$

in which

$$a^2 = \frac{o s}{d g}$$

d = section of the plate.

o = circumference of the plate.

s = coefficient of the exterior conduction of heat.

$g = n n n$ interior $n n n$

We assume that s is independent of the magnetic force.
For t we find

$$t = p e^{-ax} + q e^{+ax}.$$

The great difficulty in the measurements with our plate of bismuth, is that, as would be the case for a longer plate, we cannot neglect the second term for moderate values of x and take the coefficient p constant (this could only be the case if the plate were soldered immediately to the heated tube). In the magnetic field p , q and a

vary; it is probable that the temperature at both ends does also not remain the same, and only with the aid of three observed temperatures we can calculate α . It is evident that in this way errors of observation should acquire a great influence, especially as the determination of the falls of temperature could not possibly be made at the three places at the same time; the stationary temperatures were about the same in each experiment, but small differences could not be avoided.

For the calculation, the temperatures, observed at the heated end and in the places A , B and C were united in a graphical representation with x as abscissa. From this the temperatures in the places $x = 0, 0,6, 1,2, 1,8$, and $2,4$ were deduced. These must satisfy the relation.

$$\frac{t_n + t_{n+2}}{t_{n+1}} = e^{-0,6\alpha} + e^{+0,6\alpha}.$$

Putting $e^{-0,6\alpha} = k$, we find k from the equation

$$k^2 - \frac{t_n + t_{n+2}}{t_{n+1}} k + 1 = 0.$$

Supposing the mean value of k to be correct, I then calculated from the set of 5 temperatures the coefficients p and q , and from these again the temperatures themselves. The result being:

$$k = 0,626$$

$$p = 43,16$$

$$q = 0,39$$

x	t (interpolated)	$\frac{t_n + t_{n+2}}{t_{n+1}}$	k	t (calculated)	Difference.
0	43,5			43,5	0,0
0,6	28,0	2,17	0,666	27,6	-0,4
1,2	17,5	2,29	0,584	17,9	+0,4
1,8	12,1	2,22	0,629	12,2	+0,1
2,4	9,4			9,2	-0,2

Fig. 3 shows the calculated curve. The signs \times show the temperatures and the places which where used in the calculation, the signs $+$ show the really observed temperature in the places A , B and C .

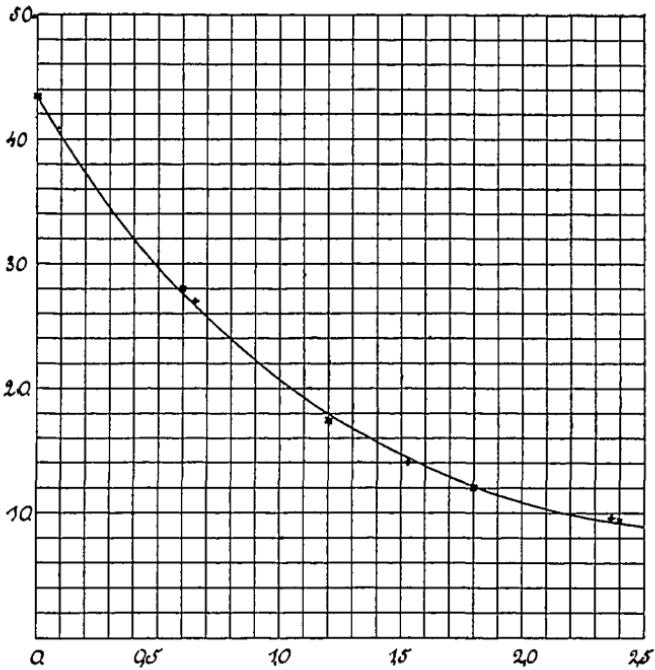


Fig. 3.

Next, we have read the falls of temperature also from a graphical representation; with a view to the small accuracy of the results I have confined myself to the observations in the strongest field and have omitted the value for $x = 0$ as being unreliable. I then found

$$M = \pm 6000$$

x	Δt	t (calculated)	$t - \Delta t$	$\frac{t_n + t_{n+2}}{t_{n+1}}$	k
0,6	1°,25	27°,6	26°,35		
1,2	0,90	17°,9	17°,00	2,235	0,6185
1,8	0,55	12°,2	11°,65	2,227	0,6237
2,4	0,26	9°,2	8°,94		

As the distance from C to the heated place was only 2,37 the value for 2,4 has been extrapolated already. Let us therefore take the value of k 0,6185 as a basis. From the value 0,626 follows a value of $a = 0,778$; 0,6185 gives $a' = 0,801$.

$$\frac{g}{g'} = \frac{a'^2}{a^2} = 1,058.$$

For an investigation of the variation in different magnetic fields the observations are not accurate enough; yet they already show that the increase is more rapid than proportional to the first power of the magnetic force, but not so rapid as proportional to the second power¹⁾.

b. Results of other observers.

As far as I know it is only LEDUC²⁾, the discoverer, and VON ETTINGSHAUSEN³⁾, who have observed this phenomenon.

From the former's observations follows for $M = 7800 \frac{g}{g'} = 1,057$.

VON ETTINGSHAUSEN finds numbers varying from 1,052 to 1,021 in a field of about 9000.

In both these cases the bismuth was directly soldered to the source of heat, and for the calculation use was made of the term $p e^{-ax}$ only. As my value, 1,058 in the field 6000 is rather too large than too small, we can deduce with certainty from the observations that this change is considerably smaller than of the electrical conductivity.

5. Longitudinal thermo-magnetic phenomenon.

a. *Measurements.* This phenomenon may be considered as an increase of the thermo-electric power of bismuth in the magnetic field. I therefore wished to measure the variation of the deflection, observed with the galvanometer, when the copper wires of the thermo-electric couples were connected to it, in percentage values for different magnetic fields, always with both directions of the magnetic field, in order to avoid the disturbances caused by transverse phenomena; afterwards it would only be necessary to determine the thermo-electric power outside the magnetic field once for all, in order to be able to express the longitudinal effect also in absolute measure. Now it was also desirable to make these measurements with the thermo-elements at the places *B* and *C*; but as the temperatures of these places vary on applying the magnetic field, a disturbance arises which reduces the change in the deflection, because the fall of tem-

¹⁾ A variation of g with temperature, outside or in the magnetic field, ought to appear from a similar change in k . It is evident that the accuracy of the observations is not sufficient to draw from them conclusions as to such a variation; however the variation cannot be very large.

²⁾ C. R. 104, p. 1783, 1887.

³⁾ Wied. Ann. 33, p. 129, 1888.

perature is greatest at the highest temperature. At first I intended to calculate this disturbance from the measurements of the fall of temperature mentioned in the preceding part. But I have changed my mind because it appeared that in spite of accurate placing of the thermo-elements, deviating falls of temperature occurred with different measurements. For the most part, these deviations must be ascribed to the existence of an other slope of temperature. Therefore of late I have determined the fall of temperature on both these places before and after the measurement of the longitudinal effect. If we knew exactly the difference of temperature between the two places of contact we might have expressed the correction in percentages. Although the temperature of the places of contact between German-silver and copper is exactly known, there is however no sufficient certainty that this is at the same time the temperature of the place of contact of bismuth and copper. The same uncertainty is the reason that the thermo-electric power of bismuth can also not be exactly deduced from these measurements; the values of the difference bismuth-copper, calculated from the results of different experiments, range from 8400 to 10500 per 1° for temperatures ranging from 20° to 50° . It should be remembered that in the place *B* we find a slope of temperature from $2,5^{\circ}$ to 5° per mm., while the whole difference of temperature was 20° to 30° . For these reasons I have for the time being given up a calculation of the longitudinal effect in absolute measure and have calculated the correction by taking for the thermo-electric difference bismuth-copper 10000 per 1° ; for the thermo-element German-silver and copper was found 1590. In the correction there remains an uncertainty of probably not more than 10%; the correction itself amounted at most to 25% of the total effect, so that for this there remains an uncertainty of 2,5%; as moreover the fall of temperature, as afterwards appeared, shows almost the same variation with the magnetic force as the longitudinal-effect, the proportion of the phenomenon in different magnetic fields will not be changed by the error.

A great number of measurements had been made already before the method of observation described here was applied. In order to use these in the calculation, I interpolated from all the sets of observations the percentage variation for the fields 2000 to 6000; then I always assumed 10 as the value for the field 6000 which was the result according to the last method, and changed the other figures proportionally. The following results were obtained.

Magnet. field.	21 July.	9 Sept.	16 Sept.	18 Febr.	6-9 March	Mean.
2000	1,77	1,60	1,94	2,27	1,60	1,84
3000	3,66	3,21	3,69	3,86	3,35	3,55
4000	5,79	5,02	5,60	5,70	5,50	5,52
5000	7,83	7,21	7,69	7,67	7,75	7,63
6000	10,0	10,0	10,0	10,0	10,0	10,0

The empirical formula was applied to these mean results.

$$C_1 = 0,2341$$

$$C_2 = 0,6663$$

Magnet. field.	Percentage observed.	increase calculated.	Difference.
2000	1,84	1,81	-0,03
3000	3,55	3,52	-0,03
4000	5,52	5,51	-0,01
5000	7,63	7,67	+0,04
6000	10,00	9,98	-0,02

As we see, the agreement is very satisfactory. The coefficient C_1 is almost the same as for the increase of the resistance, whence we conclude: — *the two phenomena differ in different magnetic fields only by an almost constant factor*¹⁾.

b. Results of other observers.

It appears that the observations of VON ETTINGSHAUSEN and NERNST cannot be represented by our formula. Therefore I shall only recalculate their results for the longitudinal effect and compare them with the increase of resistance in their bismuth.

¹⁾ In my thesis for the doctorate (p. 111—114) I pointed out that the phenomena mentioned might have a common cause in a variation of the number of free charged particles due to the magnetic field. The found proportionality renders a common cause very probable.

Magnet. field.	Percentage value of long. effect.	Percentage increase of resistance.	Ratio.
2800	1,01	7,0	0,14
4720	2,57	14,7	0,18
9480	11,00	3,55	0,31

That nothing can be seen here of the proportionality found by me, may probably be ascribed to some extent to errors of observation; the more so, as the object of the measurements was not to determine the relation between the strength of the magnetic field and the two phenomena, but to compare the behaviour of bismuth with that of bismuth-tin alloys.

Moreover their bismuth was cast. Therefore disturbances, caused by irregular crystallisation were probably present.

Chemistry. — Professor BAKHUIS ROOZEBOOM presents a communication by Dr. ERNST COHEN, "On the velocity of electrical Reaction. II."

The velocity of the reaction $Zn + Hg_2 SO_4 \rightleftharpoons Hg_2 + Zn SO_4$.

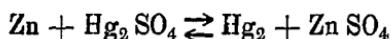
I. An open galvanic element is not a system in equilibrium; this is obvious from the fact that if the circuit of such an element is closed by means of a wire, the reaction which can take place in the cell at once occurs.

We must assume, that, owing to the resistance of an open cell being very (infinitely) great, the velocity of the reaction is practically reduced to zero¹⁾.

2. If, on the other hand, it is, for example, desired to study the course of a chemical reaction at different temperatures as a function of the temperature, a galvanic element may be constructed in which the reaction will take place as soon as the circuit is closed.

In what follows I shall describe briefly the results obtained in the study of the reaction

¹⁾ Compare NERNST, Theor. Chemie. 2e Aufl. 1898, 658.



at different temperatures.

The reaction represented by the above equation takes place in a CLARK-cell when its circuit is closed.

When the cell is short-circuited by a short, thick wire (whose resistance is practically zero) the constant of the electrical velocity of reaction (see my first communication) at t° is

$$K_1 = \frac{E}{\Omega},$$

where E is the electromotive force of the cell at t° and Ω the specific resistance of the saturated solution of zinc sulphate in the cell at the same temperature.

According as the solid substance present is $\text{Zn SO}_4 \cdot 7 \text{ H}_2\text{O}$ or $\text{Zn SO}_4 \cdot 6 \text{ H}_2\text{O}$, we shall find at one and the same temperature, two values for K_1 since both E and Ω depend on the nature of the solid phase.

3. JAEGER¹⁾ has found that for an element in which the stable solid phase is $\text{Zn SO}_4 \cdot 7 \text{ H}_2\text{O}$, the electromotive force at the temperature t° may be represented by:

$$E_t = 1.400 - 0,00152(t-39) - 0,000007(t-39)^2 \text{ Volts.}$$

If, however, the solid phase is $\text{Zn SO}_4 \cdot 6 \text{ H}_2\text{O}$, we have

$$E'_t = 1.400 - 0,00102(t-39) - 0,000004(t-39)^2 \text{ Volts}$$

I have determined the resistance of the different solutions by means of a KOHLRAUSCH dipping-electrode²⁾ (*Tauchelectrode*) the resistance capacity of which, at 18° , was determined by a 0.5 N. KCl solution, and controlled by a 0.5 N. NaCl solution at the same temperature.

Table I contains the values of E , Ω , and K_1 for the case that $\text{Zn SO}_4 \cdot 7 \text{ H}_2\text{O}$ is present in the element and Table II, the same quantities for $\text{Zn SO}_4 \cdot 6 \text{ H}_2\text{O}$.

¹⁾ WIED. Ann. Bd. 63 (1897) 354.

²⁾ KOHLRAUSCH a. HOLBORN, Das Leitvermögen der Electrolyte. (Leipzig 1898). p. 18 u. 19.

T A B L E I.

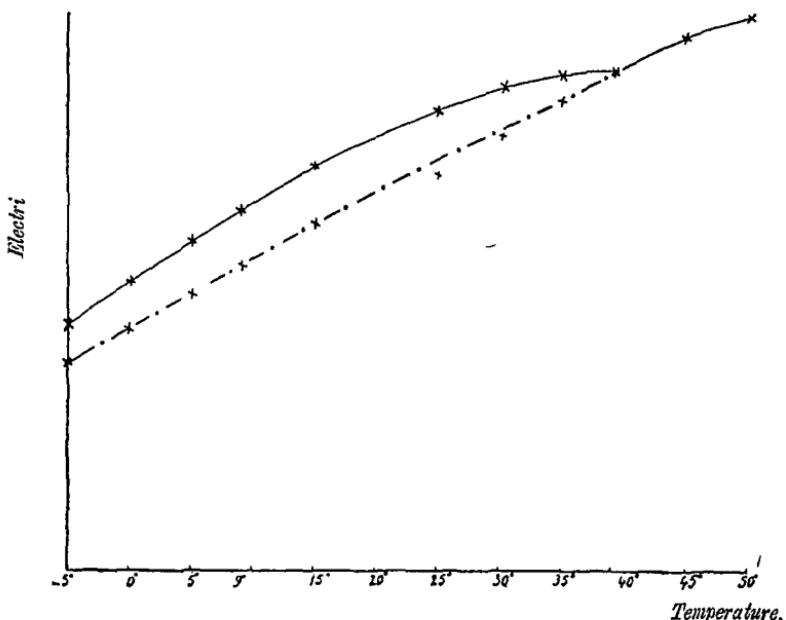
Temperature	E (Millivolts).	\mathcal{Q}	$K_1 \times 10^{-2}$
— 5°.0	1453.3	0.0491	296.0
0°.0	1448.6	0.0423	342.0
5°.0	1443.6	0.0371	388.5
9°.0	1439.3	0.0337	426.6
15°.0	1432.5	0.0299	478.3
25°.0	1419.9	0.0260	544.6
30°.0	1413.1	0.0248	574.2
35°.0	1406.0	0.0241	583.6
39°.0	1400.0	0.0237	590.9

T A B L E II.

Temperature.	E (Millivolts).	\mathcal{Q}	$K_1 \times 10^{-2}$
— 5°.0	1437.1	0.0577	248.7
0°.0	1433.7	0.0498	287.8
5°.0	1430.1	0.0437	326.8
9°.0	1427.0	0.0397	359.5
15°.0	1422.2	0.0348	408.5
25°.0	1413.5	0.0302	467.5
30°.0	1408.9	0.0274	513.8
35°.0	1404.0	0.0252	557.3
39°.0	1400.0	0.0237	590.9
45°.0	1393.7	0.0220	631.8
50°.0	1388.3	0.0211	659.0

4. Representing the electrical velocity of reaction graphically as a function of the temperature, the two curves shown in the follow-

wing figure are obtained according as we are dealing with the modification which is stable, or with that which is metastable below 39°.



As to be foreseen, the velocity of reaction at 39° is the same in both cases; below this temperature the reaction in the element containing the stable phase is always the faster (at the same temperature).

Phys. — “*Diffraction of RÖNTGEN Rays*”. By Prof. H. HAGA and Dr. C. H. WIND.

Investigations¹⁾ formerly undertaken in the Groningen Laboratory rendered it already clear that, if X rays are due to vibrations of the ether, their wavelength can be but a few ANGSTROM units. In the course of the further enquiry a phenomenon was observed, indicating traces of real diffraction and tending to give a wavelength of one or two ANGSTROM units. Now it became possible to perform a new series of experiments under circumstances more qualified to exhibit diffraction. A simple consideration makes it clear that in order to obtain great intensity it is better to use narrow slits than to make the distances great.

¹⁾ C. H. WIND, “On the influence of the dimensions of the source of light in diffraction phenomena of FRESNEL and on the diffraction of X rays”. Kon. Akad. van Wetenschappen. Proceedings, June 1898.

All the apparatus could now be mounted on one plate of free-stone, whereby moreover greater steadiness was secured.

The first slit (X slit), the second (diffraction slit) and the photographic plate were mounted on heavy metallic stands, resting on a free-stone plate ($200 \times 40 \times 30$ cm.) This plate was supported by three columns of the same material, resting on the great pillar of the edifice; stands, plate, column and pillar were united by plaster of Paris.

Behind the first slit was the RÖNTGEN tube, one of the excellent tubes made by MÜLLER (Hamburg) with automatic vacuum regulator.

The induction coil was an excellent piece of apparatus made by SIEMENS and HALSKE, with a maximum spark-length of 30 cm. and a DEPREZ interruptor with two contacts.

The current for the induction coil was given by 6 accumulators. The tube and the first slit were surrounded at all sides — except at the backside, where space was kept for the connecting wires between the coil and the tube — with thick leaden plates; in the direction of the diffraction slit a small aperture was spared in order to enable the X rays to reach the photographic plate. Much pain has been taken to obtain the slits as excellent as possible; the small platinum plates, thick $\frac{1}{2}$ mm., which formed the edges of the slit, had been very carefully flattened and ground and were screwed on flattened plates of brass. The width of the X slit was 14, 18 or 25 microns; by means of a leaden plate the height was limited to 1 cm.

The diffraction slit (height 3 cm.) was 14 microns at the upper end and gradually narrowing to a width of a few microns.

On one of the sides of the diffraction slit and very near to it there had been drilled small round holes in the platinum, near to the upper and lower end and to the centre of the slit, in order to enable us, in a way afterwards to be mentioned, to know the effective width of each part of the diffraction slit. The diffraction slit ended at the top and at the bottom in a slit of considerable width (3 m.m.), the axis of which was the prolongation of the axis of the diffraction slit; the RÖNTGEN tube was placed behind the first slit in such a position that these wider slits cut from the pencil of X rays the most intensive, middle part, as could easily be controlled, from time to time, by means of a fluorescent screen; of course the rays then passed also in the desired manner by the diffraction slit.

With this arrangement were taken the experiments of which the results are given in the following table, where signifies

σ : width of the first slit in microns.

s : " " " diffraction slit " "

a : distance between first and diffraction slit in cm.

b : " " " diffraction slit and photographic plate in cm.

$a = 75$ cm. $s : 2 \text{ à } 14 \mu$.

σ	b	Time of exposition Hours.	No. of experiment.
25	1	29	5
25	20	.57	3
25	45	66	4
14	75	60	1
14	75	100	2
25	105,5	150	6
18	1	30	8
18	75	180	7
18	75	200	9

In the experiment Nr. 1 a plate sensitive on both sides was used — mark B II of the Actien-Gesellschaft für Anilin-Fabrikation. The image on the second side of the plate however was many times fainter than that on the first side; moreover on account of the rather strong fog it seemed very desirable to destroy the image on the second side and to remove the gelatine film; in the other experiments LOMBERG-plates were used. The plates were developed a very long time (at least half an hour) using rodinal (1 at 35), potassium bromide being added.

In experiments 5 and 8, the photographic plate was pressed against the brass plate, on which the diffraction slit was mounted. Measurements, in order to give the width of the slit, were made on the images obtained by these experiments, greater accuracy being attainable in this manner than by direct measurement of the slit; on account of the small distance between the plate and the slit, the image could not be sensibly broadened nor by diffraction nor by the X slit being

H. HAGA and C. H. WIND.
THE DIFFRACTION OF THE RÖNTGEN-RAYS.

FIG. 1.

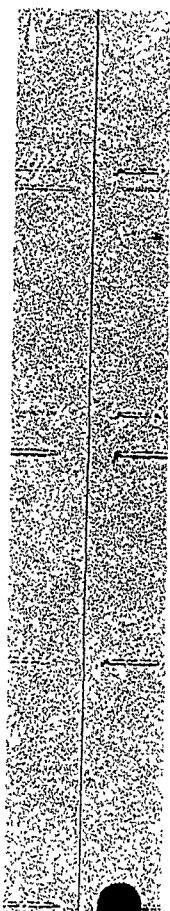
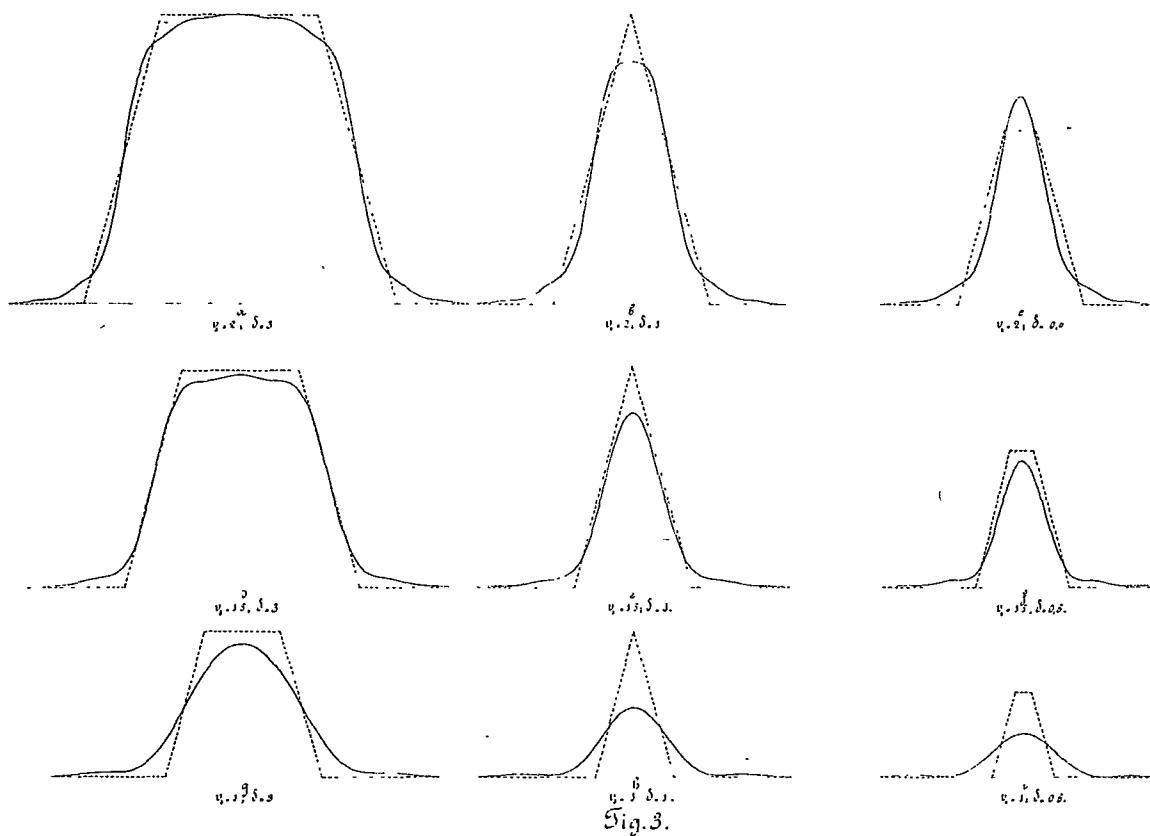


FIG. 2.



H. HAGA and C. H. WIND, Diffraction of Röntgen Rays.



not infinitely narrow. The measurements were made by means of a microscope made by ZEISS, object-glass D, compensation eye-piece 6 with micrometer; also low powers were used.

The above mentioned small holes in one of the slit sides gave on 5 and 8 circular and on the other plates — on account of the height of the X ray source — elongated dark images (cf. fig. 1 and 2). On all plates the distances between the centres of these images were divided with the dividing engine in the same number of equal parts. Whenever for a definite division in one of the diffraction images the corresponding width of the diffraction slit was wanted, we could by this way immediately take it from the measurements of 5 and 8.

If, using small power — object-glass a*, index at 10, compensation eye-glass 6 — plate N°. 2 is gradually displaced, one sees that the image of the slit appears in the broader part as a black line, dark in the centre and with hazy edges, that however in the narrower part the darker part — nucleus — ceases and the image of the slit broadens out somewhat like a plume, showing but little difference of intensity in a direction perpendicular to the length of the image. In accordance with the gradual narrowing of the diffraction slit, the image becomes of course weaker, but narrower alone at a few places, in such a manner that from the point, where the nucleus seems to disappear, maxima and minima begin to appear in the width of the image.

The same thing occurs also in the other negatives, most clearly in N°. 2, 6 and 9.

In order to give some idea of the character of these faint broadenings, which are observed at their best with the microscope, we have made of the narrowest part of 5 and 2 enlarged photographs by means of a so called mikroplanar N°. I_a made by ZEISS. Figures 1 and 2 are reproductions of these enlargements by means of heliography.

Fig. 1 is enlarged 16 times, fig. 2 14 times.

Identical numbers at the divisions indicate corresponding places. In fig. 1 is given also the width of the diffraction slit.

In order to be sure that the real cause of the described phenomena is to be attributed to diffraction, we have carefully considered other different causes which might cause a broadening of the image of the slit. It appeared however, that though photographic irradiation, local differences in the sensibility of the photographic film, secondary rays (SAGNAC), small motions of the stands during the experiments, often extending over more than 10 days, may have changed in different degrees the image of the slit, these causes

cannot explain the broadenings of the image, which as we observed occur with the narrowing of the diffraction slit.

It might however be argued that if the influence of the width of the first slit is taken into account, also with linear propagation of the rays, the image of the slit *might* make upon us the impression of a broadening with the gradually narrowing of the second slit. In this case however the apparent limits of the image should *at the utmost* diverge at the same rate as the edges of the second slit approach each other. In our experiments however the diffraction slit narrows in so slow a manner that a broadening from *this* cause would be totally imperceptible.

Hence we can only draw the conclusion, that the broadenings of the slit image we have observed must be attributed to a **Diffractiōn of the Röntgen rays**. With this hypothesis the different broadenings of the slit image, which are to be interpreted as manifold formations of plumes, are easily explained, because it is only necessary to suppose that there are rays of considerably different wavelengths, rays of some definite wavelengths possessing greater energy than others; the rays with great energy cause the separate broadenings resembling plumes, the height of the ray source favouring haziness.

The small intensity of the broadenings resembling plumes, the haziness of their edges make accurate measurements and hence a precise determination of the wavelength impossible. We were obliged to make only an estimation of the wavelength of the more prominent rays.

In FRESNEL's diffraction theory is introduced the quantity v ; in conformity therewith we now introduce the quantity v_s , as given by the relation

$$v_s = s \sqrt{\frac{2(a+b)}{ab\lambda}}.$$

This quantity determines, s , a , b and λ being given, the kind of the primary diffraction image to be expected, so that by means of CORNU's spiral the intensity curve of the diffraction image can be constructed. We have done so for the values 2, 1.5 and 1 of the quantity v_s . The width of the first slit, σ , must however be taken into account, or which comes to the same thing, from the primary the secondary diffraction images must be deduced.¹⁾

¹⁾ C. H. WIND, "Over den invloed van de afmetingen der lichtbron bij FRESNEL'sche buigingsverschijnselen". Verslag der vergad. April 1897.

In order to obtain a somewhat general survey of the different possible cases, which were for us of importance, we have made concerning the ratio

$$\delta = \frac{\frac{b}{a} \sigma}{\frac{a+b}{a} s},$$

the significance of which will be clear, three hypotheses, viz. $\delta=3$, $\delta=1$ and $\delta=0,6$.

The first slit is with the first hypothesis rather wide, with the third rather narrow as compared with the second slit. The intensity curves for the diffraction images in the 9 cases are plotted down in fig. 3 a—i.

In the same plate are represented also by dotted lines the intensity curves with the same slits and with rectilinear propagation, using the circumstance that the areas enclosed between the intensity curves and the axis must be the same with and without diffraction.

From these figures it appears that already with great values of v_s broadening will be visible, provided that the least perceptible intensity, is *very* small as compared with the greatest intensity, which with given width occurs in the image of the slit.

However in our case of the broadenings resembling plumes the last condition is certainly not fulfilled; in the centre of the slit image the intensity is already small, and a high ratio at the edges would be necessary in order to be perceptible.

In this case we see at once from the curves, that with a value $v_s=1$ there must be in *all* cases a distinct broadening, with $v_s=2$ probably never, with $v_s=1.5$ only under favourable circumstances as to the width of the first slit, and then only in a small degree.

It is also clear that, though the influence of the value of δ on the visibility of a broadening exists in general, yet this influence is not great for values of v_s between 1 and 1.5. Taking this into account we think to be justified to take $v_s=1,3$ in every case the slit image is sensibly broadened, that is in the case of every plume.

Hence we find by means of the relation

$$\lambda = \frac{1}{1.3^2} \frac{2(a+b)}{ab} s_1^2,$$

where s_1 is the width of the diffraction slit corresponding with a broadening, the values in the following table :

Nº. of experiment.	s_1 in micra.	λ in ANGSTROM units.
2	7	1,5
	6	1,1
	5	0,8
	4	0,5
4	8	2,7
	(5)	(1,0)
	3	0,4
	2,5	0,25
6	8	1,7
	7	1,3
	4,5	0,5
	3	0,25
9	4	0,5
	2	0,12

If, and this is not impossible, these values of s_1 , are enlarged by photographic irradiation, the wave lengths would still become smaller. Though very desirous to be enabled to give the limits of the RÖNTGEN rays with more certainty, we think this an impossibility with the means at our disposal. Not till RÖNTGEN tubes are produced, remaining in good working condition as long as those we have made use of, but giving out rays of much greater energy, one may succeed to make measurements instead of estimations.

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Physics. — “*Simplified Theory of Electrical and Optical Phenomena in Moving Systems*”. By Prof. H. A. LORENTZ.

§ 1. In former investigations I have assumed that, in all electrical and optical phenomena, taking place in ponderable matter, we have to do with small charged particles or ions, having determinate positions of equilibrium in dielectrics, but free to move in conductors except in so far as there is a resistance, depending on their velocities. According to these views an electric current in a conductor is to be considered as a progressive motion of the ions, and a dielectric polarization in a non-conductor as a displacement of the ions from their positions of equilibrium. The ions were supposed to be perfectly permeable to the aether, so that they can move while the aether remains at rest. I applied to the aether the ordinary electromagnetic equations, and to the ions certain other equations which seemed to present themselves rather naturally. In this way I arrived at a system of formulae which were found sufficient to account for a number of phenomena.

In the course of the investigation some artifices served to shorten the mathematical treatment. I shall now show that the theory may be still further simplified if the fundamental equations are immediately transformed in an appropriate manner.

§ 2. I shall start from the same hypotheses and introduce the same notations as in my „Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpern”. Thus, \mathfrak{d} and \mathfrak{H} will represent the dielectric displacement and the magnetic force, ϱ the density to which the ponderable matter is charged, v the velocity of this matter, and \mathfrak{E} the force acting on it per unit charge (electric force). It is only in the interior of the ions that the density ϱ differs from 0; for simplicity's sake I shall take it to be a continuous function of the coordinates, even at the surface of the ions. Finally, I suppose that each element of an ion retains its charge while it moves.

If, now, V be the velocity of light in the aether, the fundamental equations will be

$$\text{Div } \mathfrak{d} = \varrho, \quad \dots \dots \dots \dots \quad (\text{I}_a)$$

$$\text{Div } \mathfrak{H} = 0, \quad \dots \dots \dots \dots \quad (\text{II}_a)$$

$$\text{Rot } \mathfrak{H} = 4\pi\varrho v + 4\pi \dot{\mathfrak{d}}, \quad \dots \dots \dots \dots \quad (\text{III}_a)$$

$$4\pi V^2 \text{Rot } \mathfrak{d} = -\ddot{\mathfrak{H}}, \quad \dots \dots \dots \dots \quad (\text{IV}_a)$$

$$\mathfrak{E} = 4\pi V^2 \mathfrak{d} + [v \cdot \mathfrak{H}] \dots \dots \dots \dots \quad (\text{V}_a)$$

We shall apply these equations to a system of bodies, a common velocity of translation \mathfrak{p} , of constant direction and magnitude, the aether remaining at rest, and we shall hence denote by v , not the whole velocity of a material element, the velocity it may have in addition to \mathfrak{p} .

It is natural to use a system of axes of coordinates, which are parallel to the axes of the translation \mathfrak{p} . If we give to the axis of x the direction of the translation, so that \mathfrak{p}_y and \mathfrak{p}_z are 0, the equations (I) will have to be replaced by

$$\operatorname{Div} \mathfrak{v} = \varrho, \dots \quad (I_b)$$

$$\operatorname{Div} \mathfrak{H} = 0, \dots \quad (II_b)$$

$$\left. \begin{aligned} \frac{\partial \mathfrak{H}_z}{\partial y} - \frac{\partial \mathfrak{H}_y}{\partial z} &= 4\pi \varrho (\mathfrak{p}_x + v_x) + 4\pi \left(\frac{\partial}{\partial t} - \mathfrak{p}_x \frac{\partial}{\partial x} \right) \mathfrak{v}_x, \\ \frac{\partial \mathfrak{H}_x}{\partial z} - \frac{\partial \mathfrak{H}_z}{\partial x} &= 4\pi \varrho v_y + 4\pi \left(\frac{\partial}{\partial t} - \mathfrak{p}_x \frac{\partial}{\partial x} \right) \mathfrak{v}_y, \\ \frac{\partial \mathfrak{H}_y}{\partial x} - \frac{\partial \mathfrak{H}_x}{\partial y} &= 4\pi \varrho v_z + 4\pi \left(\frac{\partial}{\partial t} - \mathfrak{p}_x \frac{\partial}{\partial x} \right) \mathfrak{v}_z, \end{aligned} \right\} \quad (III_b)$$

$$\left. \begin{aligned} 4\pi V^2 \left(\frac{\partial \mathfrak{v}_z}{\partial y} - \frac{\partial \mathfrak{v}_y}{\partial z} \right) &= - \left(\frac{\partial}{\partial t} - \mathfrak{p}_x \frac{\partial}{\partial x} \right) \mathfrak{H}_x, \\ 4\pi V^2 \left(\frac{\partial \mathfrak{v}_x}{\partial z} - \frac{\partial \mathfrak{v}_z}{\partial x} \right) &= - \left(\frac{\partial}{\partial t} - \mathfrak{p}_x \frac{\partial}{\partial x} \right) \mathfrak{H}_y, \\ 4\pi V^2 \left(\frac{\partial \mathfrak{v}_y}{\partial x} - \frac{\partial \mathfrak{v}_x}{\partial y} \right) &= - \left(\frac{\partial}{\partial t} - \mathfrak{p}_x \frac{\partial}{\partial x} \right) \mathfrak{H}_z, \end{aligned} \right\} \quad (IV_b)$$

$$\mathfrak{E} = 4\pi V^2 \mathfrak{v} + [\mathfrak{p} \cdot \mathfrak{H}] + [v \cdot \mathfrak{H}] \dots \quad (V_b)$$

In these formulae the sign Div , applied to a vector \mathfrak{A} , has still the meaning defined by

$$\operatorname{Div} \mathfrak{A} = \frac{\partial \mathfrak{A}_x}{\partial x} + \frac{\partial \mathfrak{A}_y}{\partial y} + \frac{\partial \mathfrak{A}_z}{\partial z}.$$

As already been said, v is the relative velocity with regard to the moving axes of coordinates. If $v = 0$, we shall speak of a body at rest; this expression therefore means relative rest with regard to the moving axes.

In most applications \mathfrak{p} would be the velocity of the earth in its yearly motion.

§ 4. Now, in order to simplify the equations, the following quantities may be taken as independent variables

$$x' = \frac{V}{\sqrt{V^2 - p_x^2}} x, \quad y' = y, \quad z' = z, \quad t' = t - \frac{p_x}{V^2 - p_x^2} x \dots \quad (1)$$

The last of these is the time, reckoned from an instant that is not the same for all points of space, but depends on the place we wish to consider. We may call it the *local time*, to distinguish it from the *universal time* t .

If we put

$$\frac{V}{\sqrt{V^2 - p_x^2}} = k,$$

we shall have

$$-\frac{\partial}{\partial x} = k \frac{\partial}{\partial x'} - k^2 \frac{p_x}{V^2} \frac{\partial}{\partial t'}, \quad \frac{\partial}{\partial y} = \frac{\partial}{\partial y'}, \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial z'}, \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial t'}.$$

The expression

$$\frac{\partial \mathfrak{U}_x}{\partial x'} + \frac{\partial \mathfrak{U}_y}{\partial y'} + \frac{\partial \mathfrak{U}_z}{\partial z'}$$

will be denoted by

$$Div' \mathfrak{U}.$$

We shall also introduce, as new dependent variables instead of the components of \mathfrak{d} and \mathfrak{h} , those of two other vectors \mathfrak{F}' and \mathfrak{H} , which we define as follows

$$\mathfrak{F}'_x = 4\pi V^2 \mathfrak{d}_x, \quad \mathfrak{F}'_y = 4\pi k V^2 \mathfrak{d}_y - k p_x \mathfrak{h}_z, \quad \mathfrak{F}'_z = 4\pi k V^2 \mathfrak{d}_z + k p_x \mathfrak{h}_y,$$

$$\mathfrak{H}'_x = k \mathfrak{h}_x, \quad \mathfrak{H}'_y = k^2 \mathfrak{h}_y + 4\pi k^2 p_x \mathfrak{d}_z, \quad \mathfrak{H}'_z = k^2 \mathfrak{h}_z - 4\pi k^2 p_x \mathfrak{d}_y.$$

In this way I find by transformation and mutual combination of the equations (I_b)—(V_b):

(430)

$$Div' \mathfrak{F}' = \frac{4\pi}{k} V^2 \varrho - 4\pi k p_x \varrho v_x, \quad \dots \quad (I_c)$$

$$Div' \mathfrak{H}' = 0, \quad \dots \quad (II_c)$$

$$\left. \begin{aligned} \frac{\partial \mathfrak{H}'_z}{\partial y'} - \frac{\partial \mathfrak{H}'_y}{\partial z'} &= 4\pi k^2 \varrho v_x + \frac{k^2}{V^2} \frac{\partial \mathfrak{F}'_x}{\partial t'} \\ \frac{\partial \mathfrak{H}'_x}{\partial z'} - \frac{\partial \mathfrak{H}'_z}{\partial x'} &= 4\pi k \varrho v_y + \frac{k^2}{V^2} \frac{\partial \mathfrak{F}'_y}{\partial t'} \\ \frac{\partial \mathfrak{H}'_y}{\partial x'} - \frac{\partial \mathfrak{H}'_x}{\partial y'} &= 4\pi k \varrho v_z + \frac{k^2}{V^2} \frac{\partial \mathfrak{F}'_z}{\partial t'} \end{aligned} \right\}, \quad \dots \quad (III_c)$$

$$\left. \begin{aligned} \frac{\partial \mathfrak{F}'_z}{\partial y'} - \frac{\partial \mathfrak{F}'_y}{\partial z'} &= - \frac{\partial \mathfrak{H}'_x}{\partial t'} \\ \frac{\partial \mathfrak{F}'_x}{\partial z'} - \frac{\partial \mathfrak{F}'_z}{\partial x'} &= - \frac{\partial \mathfrak{H}'_y}{\partial t'} \\ \frac{\partial \mathfrak{F}'_y}{\partial x'} - \frac{\partial \mathfrak{F}'_x}{\partial y'} &= - \frac{\partial \mathfrak{H}'_z}{\partial t'} \end{aligned} \right\}, \quad \dots \quad (IV_c)$$

$$\left. \begin{aligned} \mathfrak{E}_x &= \mathfrak{F}'_x + k \frac{p_x}{V^2} (v_y \mathfrak{F}'_y + v_z \mathfrak{F}'_z) + (v_y \mathfrak{H}'_z - v_z \mathfrak{H}'_y) \\ \mathfrak{E}_y &= \frac{1}{k} \mathfrak{F}'_y - k \frac{p_x}{V^2} v_x \mathfrak{F}'_y + \left(\frac{1}{k} v_z \mathfrak{H}'_x - v_x \mathfrak{H}'_z \right) \\ \mathfrak{E}_z &= \frac{1}{k} \mathfrak{F}'_z - k \frac{p_x}{V^2} v_x \mathfrak{F}'_z + \left(v_x \mathfrak{H}'_y - \frac{1}{k} v_y \mathfrak{H}'_x \right) \end{aligned} \right\}. \quad (V_c)$$

Putting $v=0$ in the three last equations we see that

$$\mathfrak{F}'_x, \frac{1}{k} \mathfrak{F}'_y, \frac{1}{k} \mathfrak{F}'_z$$

are the components of the electric force that would act on a particle at rest.

§ 5. We shall begin with an application of the equations to

electrostatic phenomena. In these we have $v = 0$ and \mathfrak{F}' independent of the time. Hence, by (II_c) and (III_c)

$$\mathfrak{H}' = 0,$$

and by (IV_c) and (I_c)

$$\frac{\partial \mathfrak{F}'_z}{\partial y'} - \frac{\partial \mathfrak{F}'_y}{\partial z'} = 0, \quad \frac{\partial \mathfrak{F}'_x}{\partial z'} - \frac{\partial \mathfrak{F}'_z}{\partial x'} = 0, \quad \frac{\partial \mathfrak{F}'_y}{\partial x'} - \frac{\partial \mathfrak{F}'_x}{\partial y'} = 0,$$

$$Div' \mathfrak{F}' = \frac{4\pi}{k} V^2 \varrho.$$

These equations show that \mathfrak{F}' depends on a potential ω , so that

$$\mathfrak{F}'_x = -\frac{\partial \omega}{\partial x'}, \quad \mathfrak{F}'_y = -\frac{\partial \omega}{\partial y'}, \quad \mathfrak{F}'_z = -\frac{\partial \omega}{\partial z'}$$

and

$$\frac{\partial^2 \omega}{\partial x'^2} + \frac{\partial^2 \omega}{\partial y'^2} + \frac{\partial^2 \omega}{\partial z'^2} = -\frac{4\pi}{k} V^2 \varrho \quad \quad (2)$$

Let S be the system of ions with the translation p_x , to which the above formulae are applied. We can conceive a second system S_0 with no translation and consequently no motion at all; we shall suppose that S is changed into S_0 by a dilatation in which the dimensions parallel to OX are changed in ratio of 1 to k , the dimensions perpendicular to OX remaining what they were. Moreover we shall attribute equal charges to corresponding volume-elements in S and S_0 ; if then ϱ_0 be the density in a point P of S , the density in the corresponding point P_0 of S_0 will be

$$\varrho_0 = \frac{1}{k} \varrho.$$

If x, y, z are the coordinates of P , the quantities x', y', z' , determined by (1), may be considered as the coordinates of P_0 .

In the system S_0 , the electric force, which we shall call \mathfrak{E}_0 may evidently be derived from a potential ω_0 , by means of the equations

$$\mathfrak{E}_{0x} = -\frac{\partial \omega_0}{\partial x'}, \quad \mathfrak{E}_{0y} = -\frac{\partial \omega_0}{\partial y'}, \quad \mathfrak{E}_{0z} = -\frac{\partial \omega_0}{\partial z'},$$

and the function ω_0 itself will satisfy the condition

$$\frac{\partial^2 \omega_0}{\partial x'^2} + \frac{\partial^2 \omega_0}{\partial y'^2} + \frac{\partial^2 \omega_0}{\partial z'^2} = -4\pi V^2 \varrho_0 = -\frac{4\pi}{k} V^2 \varrho.$$

Comparing this with (2), we see that in corresponding points

$$\omega = \omega_0,$$

and consequently

$$\mathfrak{F}'_x = \mathfrak{E}_{0x}, \mathfrak{F}'_y = \mathfrak{E}_{0y}, \mathfrak{F}'_z = \mathfrak{E}_{0z}.$$

In virtue of what has been remarked at the end of § 4, the components of the electric force in the system S will therefore be

$$\mathfrak{E}_{0x}, \frac{1}{k} \mathfrak{E}_{0y}, \frac{1}{k} \mathfrak{E}_{0z}.$$

Parallel to OX we have the same electric force in S and S_0 , but in a direction perpendicular to OX the electric force in S will be $\frac{1}{k}$ times the electric force in S_0 .

By means of this result every electrostatic problem for a moving system may be reduced to a similar problem for a system at rest; only the dimensions in the direction of translation must be slightly different in the two systems. If, e.g., we wish to determine in what way innumerable ions will distribute themselves over a moving conductor C , we have to solve the same problem for a conductor C_0 , having no translation. It is easy to show that if the dimensions of C_0 and C differ from each other in the way that has been indicated, the electric force in one case will be perpendicular to the surface of C , as soon as, in the other case, the force \mathfrak{E}_0 is normal to the surface of C_0 .

Since

$$k = \left(1 - \frac{p_x^2}{V^2}\right)^{-\frac{1}{2}}$$

exceeds unity only by a quantity of the second order — if we call $\frac{p_x}{V}$ of the first order — the influence of the earth's yearly motion on electrostatic phenomena will likewise be of the second order.

§ 6. We shall now shew how our general equations (I_c)—(V_c) may be applied to optical phenomena. For this purpose we consider a system of ponderable bodies, the ions in which are capable of vibrating about determinate positions of equilibrium. If the system be traversed by waves of light, there will be oscillations of the ions, accompanied by electric vibrations in the aether. For convenience of treatment we shall suppose that, in the absence of light-waves, there is no motion at all; this amounts to ignoring all molecular motion.

Our first step will be to omit all terms of the second order. Thus, we shall put $k=1$, and the electric force acting on ions at rest will become \mathbf{F}' itself.

We shall further introduce certain restrictions, by means of which we get rid of the last term in (I_c) and of the terms containing v_x , v_y , v_z in (V_c).

The first of these restrictions relates to the magnitude of the displacements a from the positions of equilibrium. We shall suppose them to be exceedingly small; even relatively to the dimensions of the ions and we shall on this ground neglect all quantities which are of the second order with respect to a .

It is easily seen that, in consequence of the displacements, the electric density in a fixed point will no longer have its original value ϱ_0 , but will have become

$$\varrho = \varrho_0 - \frac{\partial}{\partial x} (\varrho_0 a_x) - \frac{\partial}{\partial y} (\varrho_0 a_y) - \frac{\partial}{\partial z} (\varrho_0 a_z).$$

Here, the last terms, which evidently must be taken into account, have the order of magnitude $\frac{c \varrho_0}{a}$, if c denotes the amplitude of the vibrations; consequently, the first term of the right-hand member of (I_c) will contain quantities of the order

$$\frac{V^2 c \varrho_0}{a} \dots \dots \dots \dots \quad (3)$$

On the other hand, if T is the time of vibration, the last term in (I_c) will be of the order

$$\frac{p_x \varrho_0 c}{T}, \dots \dots \dots \dots \quad (4)$$

Dividing this by (3), we get

$$\frac{p_x}{V} \cdot \frac{a}{VT},$$

an extremely small quantity, because the diameter of the ions is a very small fraction of the wave-length. This is the reason why we may omit the last term in (I_e) .

As to the equations (V_e) , it must be remarked that, if the displacements are infinitely small, the same will be true of the velocities and, in general, of all quantities which do not exist as long as the system is at rest and are entirely produced by the motion. Such are \mathfrak{H}'_x , \mathfrak{H}'_y , \mathfrak{H}'_z . We may therefore omit the last terms in (V_e) , as being of the second order.

The same reasoning would apply to the terms containing $\frac{p_x}{V^2}$, if we could be sure that in the state of equilibrium there are no electric forces at all. If, however, in the absence of any vibrations, the vector \mathfrak{F}' has already a certain value \mathfrak{F}'_0 , it will only be the difference $\mathfrak{F}' - \mathfrak{F}'_0$, that may be called infinitely small; it will then be permitted to replace \mathfrak{F}'_y and \mathfrak{F}'_z by \mathfrak{F}'_{0y} and \mathfrak{F}'_{0z} .

Another restriction consists in supposing that an ion is incapable of any motion but a translation as a whole, and that, in the position of equilibrium, though its parts may be acted on by electric forces, as has just been said, yet the whole ion does not experience a resultant electric force. Then, if $d\tau$ is an element of volume, and the integrations are extended all over the ion,

$$\int \varrho_0 \mathfrak{F}'_{0y} d\tau = \int \varrho_0 \mathfrak{F}'_{0z} d\tau = 0. \dots \dots \quad (5)$$

Again, in the case of vibrations, the equations (V_e) will only serve to calculate the resultant force acting on an ion. In the direction of the axis of y e.g. this force will be

$$\int \varrho \mathfrak{E}_y d\tau.$$

Its value may be found, if we begin by applying the second of the three equations to each point of the ion, always for the same universal time t , and then integrate. From the second term on the right-hand side we find

$$-\frac{p_x}{V^2} \int \rho v_x \mathfrak{F}'_y d\tau,$$

or, since we may replace \mathfrak{F}'_y by \mathfrak{F}'_{0y} and ρ by ρ_0 ,

$$-\frac{p_x}{V^2} v_x \int \rho_0 \mathfrak{F}'_{0y} d\tau,$$

which vanishes on account of (5).

Hence, as far as regards the resultant force, we may put $\mathfrak{E} = \mathfrak{F}'$, that is to say, we may take \mathfrak{F}' as the electric force, acting not only on ions at rest, but also on moving ions.

The equations will be somewhat simplified, if, instead of \mathfrak{F}' , we introduce the already mentioned difference $\mathfrak{F}' - \mathfrak{F}'_0$. In order to do this, we have only twice to write down the equations (I_c)—(IV_c), once for the vibrating system and a second time for the same system in a state of rest; and then to subtract the equations of the second system from those of the first. In the resulting equations, I shall, for the sake of brevity, write \mathfrak{F}' instead of $\mathfrak{F}' - \mathfrak{F}'_0$, so that henceforth \mathfrak{F}' will denote not the total electric force, but only the part of it that is due to the vibrations. At the same time we shall replace the value of ρ , given above, by

$$\rho_0 - a_x \frac{\partial \rho_0}{\partial x'} - a_y \frac{\partial \rho_0}{\partial y'} - a_z \frac{\partial \rho_0}{\partial z'}.$$

We may do so, because we have supposed a_x , a_y , a_z to have the same values all over an ion, and because ρ_0 is independent of the time, so that

$$\frac{\partial \rho_0}{\partial x} = \frac{\partial \rho_0}{\partial x'}.$$

Finally we have

$$Div' \mathfrak{F}' = -4\pi V^2 \left(a_x \frac{\partial \rho_0}{\partial x'} + a_y \frac{\partial \rho_0}{\partial y'} + a_z \frac{\partial \rho_0}{\partial z'} \right), \dots \quad (I_d)$$

$$Div' \mathfrak{H}' = 0, \dots \dots \dots \quad (II_d)$$

$$\frac{\partial \mathfrak{H}'_z}{\partial y'} - \frac{\partial \mathfrak{H}'_y}{\partial z'} = 4\pi \rho_0 \frac{\partial a_x}{\partial t'} + \frac{1}{V^2} \frac{\partial \mathfrak{F}'_x}{\partial t'}, \text{ etc.} \dots \quad (III_d)$$

$$\frac{\partial \mathfrak{F}'_z}{\partial y'} - \frac{\partial \mathfrak{F}'_y}{\partial z'} = -\frac{\partial \mathfrak{H}'_x}{\partial t'}, \text{ etc.} \dots \dots \quad (IV_d)$$

Since these equations do no longer explicitly contain the velocity p_x , they will hold, without any change of form, for a system that has no translation, in which case, of course, t' would be the same thing as the universal time t .

Yet, strictly speaking, there would be a slight difference in the formulae, when applied to the two cases. In the system without a translation a_x , a_y , a_z would be, in all points of an ion, the same functions of t' , i. e. of the universal time, whereas, in the moving system, these components would not depend in the same way on t' in different parts of the ion, just because they must everywhere be the same functions of t .

However, we may ignore this difference, of the ions are so small, that we may assign to each of them a single local time, applicable to all its parts.

The equality of form of the electromagnetic equations for the two cases of which we have spoken will serve to simplify to a large extent our investigation. However, it should be kept in mind, that, to the equations (I_d)—(IV_d), we must add the equations of motion for the ions themselves. In establishing these, we have to take into account, not only the electric forces, but also all other forces acting on the ions. We shall call these latter the molecular forces and we shall begin by supposing them to be sensible only at such small distances, that two particles of matter, acting on each other, may be said to have the same local time.

§ 7. Let us now imagine two systems of ponderable bodies, the one S with a translation, and the other one S_0 without such a motion, but equal to each other in all other respects. Since we neglect quantities of the order p_x^2/V^2 , the electric force will, by § 5 be the same in both systems, as long as there are no vibrations.

After these have been excited, we shall have for both systems the equations (I_d)—(IV_d).

Further we shall imagine motions of such a kind, that, if in a point (x', y', z') of S_0 we find a certain quantity of matter or a certain electric charge at the universal time t' , an equal quantity of matter or an equal charge will be found in the corresponding point of S at the local time t' . Of course, this involves that at these corresponding times we shall have, in the point (x', y', z') of both systems, the same electric density, the same displacement a , and equal velocities and accelerations.

Thus, some of the dependent variables in our equations (I_d)—(IV_d) will be represented in S_0 and S by the same functions of x' , y' , z' , t' ,

whence we conclude that the equations will be satisfied by values of \mathfrak{H}'_x , \mathfrak{H}'_y , \mathfrak{H}'_z , \mathfrak{F}'_x , \mathfrak{F}'_y , \mathfrak{F}'_z , which are likewise in both cases the same functions of x' , y' , z' , t' . By what has been said at the beginning of this §, not only \mathfrak{F}' , but also the total electric force will be the same in S_0 and S , always provided that corresponding ions at corresponding times (i. e. for equal values of t') be considered.

As to the molecular forces, acting on an ion, they are confined to a certain small space surrounding it, and by what has been said in § 6, the difference of local times within this space may be neglected. Moreover, if equal spaces of this kind are considered in S_0 and S , there will be, at corresponding times, in both the same distribution of matter. This is a consequence of what has been supposed concerning the two motions.

Now, the simplest assumption we can make on the molecular forces is this, that they are *not* changed by the translation of the system. If this be admitted, it appears from the above considerations that corresponding ions in S_0 and S will be acted on by the same molecular forces, as well as by the same electric forces. Therefore, since the masses and accelerations are the same, the supposed motion in S will be possible as soon as the corresponding motion in S_0 can really exist. In this way we are led to the following theorem.

If, in a body or a system of bodies, without a translation, a system of vibrations be given, in which the displacements of the ions and the components of \mathfrak{F}' and \mathfrak{H}' are certain functions of the coordinates and the time, then, if a translation be given to the system, there can exist vibrations, in which the displacements and the components of \mathfrak{F}' and \mathfrak{H}' are the same functions of the coordinates and the *local* time. This is the theorem, to which I have been led in a much more troublesome way in my „Versuch einer Theorie, etc.”, and by which most of the phenomena, belonging to the theory of aberration may be explained.

§ 8. In what precedes, the molecular forces have been supposed to be confined to excessively small distances. If two particles of matter were to act upon each other at such a distance that the difference of their local times might not be neglected, the theorem would no longer be true in the case of molecular forces that are not altered at all by the translation. However, one soon perceives that the theorem would again hold good, if these forces were changed by the translation in a definite way, in such a way namely that the action between two quantities of matter were determined, not by the *simultaneous* values of their coordinates, but by their values

at *equal local times*. If therefore, we should meet with phenomena, in which the difference of the local times for mutually acting particles might have a sensible influence, and in which yet observation showed the above theorem to be true, this would indicate a modification, like the one we have just specified, of the molecular forces by the influence of a translation. Of course, such a modification would only be possible, if the molecular forces were no direct actions at a distance, but were propagated by the aether in a similar way as the electromagnetic actions. Perhaps the rotation of the plane of polarization in the so-called active bodies will be found to be a phenomenon of the kind, just mentioned.

§ 9. Hitherto all quantities of the order p_x^2/V^2 have been neglected. As is well known, these must be taken into account in the discussion of MICHELSON's experiment, in which two rays of light interfered after having traversed rather long paths, the one parallel to the direction of the earth's motion, and the other perpendicular to it. In order to explain the negative result of this experiment FITZGERALD and myself have supposed that, in consequence of the translation, the dimensions of the solid bodies serving to support the optical apparatus, are altered in a certain ratio.

Some time ago, M. LIÉNARD¹⁾ has emitted the opinion that, according to my theory, the experiment should have a positive result, if it were modified in so far, that the rays had to pass through a solid or a liquid dielectric.

It is impossible to say with certainty what would be observed in such a case, for, if the explication of MICHELSON's result which I have proposed is accepted, we must also assume that the mutual distances of the molecules of transparent media are altered by the translation.

Besides, we must keep in view the possibility of an influence, be it of the second order, of the translation on the molecular forces.

In what follows I shall shew, not that the result of the experiment must necessarily be negative, but that this might very well be the case. At the same time it will appear what would be the theoretical meaning of such a result.

Let us return again to the equations $(I_c) - (V_c)$. This time we shall not put in them $k = 1$, but the other simplifications of which we have spoken in § 6 will again be introduced. We shall now have to distinguish between the vectors \mathfrak{E} and \mathfrak{F}' , the former alone

¹⁾ L'Éclairage Électrique, 20 et 27 août 1898.

being the electric force. By both signs I shall now denote, not the whole vector, but the part that is due to the vibrations.

The equations may again be written in a form in which the velocity of translation does not explicitly appear. For this purpose, it is necessary to replace the variables x' , y' , z' , t' , \mathfrak{F}' , \mathfrak{H}' , a and ϱ_0 by new ones, differing from the original quantities by certain constant factors.

For the sake of uniformity of notation all these new variables will be distinguished by double accents. Let ϵ be an indeterminate coefficient, differing from unity by a quantity of the order p_x^2/V^2 , and let us put

$$x = \frac{\epsilon}{k} x'', \quad y = \epsilon y'', \quad z = \epsilon z'', \quad \dots \quad (6)$$

$$a_x = \frac{\epsilon}{k} a''_x, \quad a_y = \epsilon a''_y, \quad a_z = \epsilon a''_z, \quad \dots \quad (7)$$

$$\varrho_0 = \frac{k}{\epsilon^3} \varrho_0'', \quad \dots \quad (8)$$

$$\mathfrak{F}'_x = \frac{1}{\epsilon^2} \mathfrak{F}''_x, \quad \mathfrak{F}'_y = \frac{1}{\epsilon^2} \mathfrak{F}''_y, \quad \mathfrak{F}'_z = \frac{1}{\epsilon^2} \mathfrak{F}''_z,$$

$$\mathfrak{H}'_x = \frac{k}{\epsilon^2} \mathfrak{H}''_x, \quad \mathfrak{H}'_y = \frac{k}{\epsilon^2} \mathfrak{H}''_y, \quad \mathfrak{H}'_z = \frac{k}{\epsilon^2} \mathfrak{H}''_z,$$

$$t' = k \epsilon t'', \quad \dots \quad (9)$$

so that t'' is a modified local time; then we find

$$Div'' \mathfrak{F}'' = 4 \pi V^2 \left(- a''_x \frac{\partial \varrho_0''}{\partial x''} - a''_y \frac{\partial \varrho_0''}{\partial y''} - a''_z \frac{\partial \varrho_0''}{\partial z''} \right), \quad (I_e)$$

$$Div'' \mathfrak{H}'' = 0, \quad \dots \quad (II_e)$$

$$\frac{\partial \mathfrak{H}''_z}{\partial y''} - \frac{\partial \mathfrak{H}''_y}{\partial z''} = 4 \pi \varrho_0'' \frac{\partial a''_x}{\partial t''} + \frac{1}{V^2} \frac{\partial \mathfrak{F}''_x}{\partial t''}, \text{ etc.} \quad (III_e)$$

$$\frac{\partial \mathfrak{F}''_z}{\partial y''} - \frac{\partial \mathfrak{F}''_y}{\partial z''} = - \frac{\partial \mathfrak{H}''_x}{\partial t''}, \text{ etc.} \quad (IV_e)$$

$$\mathfrak{E}_x = \frac{1}{\epsilon^2} \mathfrak{F}''_x, \quad \mathfrak{E}_y = \frac{1}{k \epsilon^2} \mathfrak{F}''_y, \quad \mathfrak{E}_z = \frac{1}{k \epsilon^2} \mathfrak{F}''_z \quad \dots \quad (V_e)$$

These formulae will also hold for a system without translation; only, in this case we must take $k = 1$, and we shall likewise take $\epsilon = 1$, though this is not necessary. Thus, x'', y'', z'' will then be the coordinates, t'' the same thing as t , i. e. the universal time, a'' the displacement, φ_0'' the electric density, \mathfrak{F}'' and \mathfrak{B}'' the magnetic and electric forces, the last in so far as it is due to the vibrations.

Our next object will be to ascertain under what conditions, now that we retain the terms with ρ_x^2/V^2 , two systems S and S_0 , the first having a translation, and the second having none, may be in vibratory states that are related to each other in some definite way. This investigation resembles much the one that has been given in § 7; it may therefore be expressed in somewhat shorter terms.

To begin with, we shall agree upon the degree of similarity there shall be between the two systems in their states of equilibrium. In this respect we define S by saying that the system S_0 may be changed into it by means of the dilatations indicated by (6); we shall suppose that, in undergoing these dilatations, each element of volume retains its ponderable matter, as well as its charge. It is easily seen that this agrees with the relation (8).

We shall not only suppose that the system S_0 *may* be changed in this way into an imaginary system S , but that, as soon as the translation is given to it, the transformation *really* takes place, of itself, i. e. by the action of the forces acting between the particles of the system, and the aether. Thus, after all, S will be the *same* material system as S_0 .

The transformation of which I have now spoken, is precisely such a one as is required in my explication of MICHELSON's experiment. In this explication the factor ϵ may be left indeterminate. We need hardly remark that for the real transformation produced by a translatory motion, the factor should have a definite value. I see, however, no means to determine it.

Before we proceed further, a word on the electric forces in S and S_0 in their states of equilibrium. If $\epsilon = 1$, the relation between these forces will be given by the equations of § 5. Now ϵ indicates an alteration of all dimensions in the same ratio, and it is very easy to see what influence this will have on the electric forces. Thus, it will be found that, in passing from S_0 to S , the electric force in the direction of OX will be changed in the ratio of 1 to $\frac{1}{\epsilon^2}$, and that the corresponding ratio for the other components will

be as 1 to $\frac{1}{k\epsilon^2}$.

As to the corresponding vibratory motions, we shall require that at corresponding times, i. e. for equal values of t'' , the configuration of S may always be got from that of S_0 by the above mentioned dilatations. Then, it appears from (7) that a''_x, a''_y, a''_z will be, in both systems, the same functions of x'', y'', z'', t'' , whence we conclude that the equations (I_e)—(IV_e) can be satisfied by values of $\xi''_x, \xi''_y, \xi''_z$, etc., which are likewise, in S_0 and in S , the same functions of x'', y'', z'', t'' .

Always provided that we start from a vibratory motion in S_0 that can really exist, we have now arrived at a motion in S , that is possible in so far as it satisfies the electromagnetic equations. The last stage of our reasoning will be to attend to the molecular forces. In S_0 we imagine again, around one of the ions, the same small space, we have considered in § 7 and to which the molecular forces acting on the ion are confined; in the other system we shall now conceive the corresponding small space, i. e. the space that may be derived from the first one by applying to it the dilatations (6). As before, we shall suppose these spaces to be so small that in the second of them there is no necessity to distinguish the local times in its different parts; then we may say that in the two spaces there will be, at corresponding times, corresponding distributions of matter.

We have already seen that, in the states of equilibrium, the electric forces parallel to OX, OY, OZ , existing in S differ from the corresponding forces in S_0 by the factors

$$\frac{1}{\varepsilon^2}, \frac{1}{k\varepsilon^2} \text{ and } \frac{1}{k\varepsilon^2}.$$

From (V_e) it appears that the same factors come into play when we consider the part of the electric forces that is due to the vibrations. If, now, we suppose that the molecular forces are modified in quite the same way in consequence of the translation, we may apply the just mentioned factors to the components of the *total* force acting on an ion. Then, the imagined motion in S will be a possible one, provided that these same factors to which we have been led in examining the forces present themselves again, when we treat of the product of the masses and the accelerations.

According to our suppositions, the accelerations in the directions of OX, OY, OZ in S are resp. $\frac{1}{k^3\varepsilon}, \frac{1}{k^3\varepsilon}$ and $\frac{1}{k^3\varepsilon}$ times what they

are in S_0 . If therefore the required agreement is to exist with regard to the vibrations parallel to OX , the ratio of the masses of the ions in S and S_0 should be $\frac{k^3}{\epsilon}$; on the contrary we find for this ratio $\frac{k}{\epsilon}$, if we consider in the same way the forces and the accelerations in the directions of OY and OZ .

Since k is different from unity, these values cannot both be 1; consequently, states of motion, related to each other in the way we have indicated, will only be possible, if in the transformation of S_0 into S the masses of the ions change; even, this must take place in such a way that the same ion will have different masses for vibrations parallel and perpendicular to the velocity of translation.

Such a hypothesis seems very startling at first sight. Nevertheless we need not wholly reject it. Indeed, as is well known, the effective mass of an ion depends on what goes on in the aether; it may therefore very well be altered by a translation and even to different degrees for vibrations of different directions.

If the hypothesis might be taken for granted, MICHELSON's experiment should always give a negative result, whatever transparent media were placed on the path of the rays of light, and even if one of these went through air, and the other, say through glass. This is seen by remarking that the correspondence between the two motions we have examined is such that, if in S_0 we had a certain distribution of light and dark (interference-bands) we should have in S a similar distribution, which might be got from that in S_0 by the dilatations (6), provided however that in S the time of vibration be $k\epsilon$ times as great as in S_0 . The necessity of this last difference follows from (9). Now the number $k\epsilon$ would be the same in all positions we can give to the apparatus; therefore, if we continue to use the *same* sort of light, while rotating the instruments, the interference-bands will never leave the parts of the ponderable system, e. g. the lines of a micrometer, with which they coincided at first.

We shall conclude by remarking that the alteration of the molecular forces that has been spoken of in this § would be one of the second order, so that we have not come into contradiction with what has been said in § 7.

Physics. — STOKES's *Theory of Aberration in the Supposition of a Variable Density of the Aether.* By Prof. H. A. LORENTZ.

In the theory of aberration that has been proposed by Prof. STOKES it must be assumed that the aether has an irrotational motion and that, all over the earth's surface, its velocity is equal to that of the planet itself, in its yearly motion. These two conditions are easily shown to contradict each other, if the aether is understood to have everywhere the same invariable density.

Prof. PLANCK of Berlin had the kindness to call my attention to the fact that both conditions might be satisfied at the same time, if the aether were compressible and subject to gravity, so that it could be condensed around the earth like a gas. It is true, a certain amount of sliding is not to be avoided, but the relative velocity of the aether with regard to the earth may be made as small as we like by supposing the condensation sufficiently large.

At my request Prof. PLANCK permitted me to communicate his treatment of the case; it is as follows.

Instead of considering the earth moving through the aether we shall suppose the planet to be at rest and the aether to flow along it; this comes to the same thing. Let this motion be steady and irrotational and let the velocity at infinite distance be c , constant in direction and magnitude. Let the aether obey BOYLE's law and be attracted by the earth according to the law of NEWTON.

We shall place the origin of coordinates in the centre of the planet and give to the axis of z the direction of the velocity c . Finally we shall call the distance to the centre r , the radius of the earth r_0 , the velocity-potential φ , the pressure p , the density k , and the potential of gravity per unit mass V . We shall denote by μ the constant ratio $\frac{k}{p}$, and by g the value

$$\frac{\partial V}{\partial r} (r = r_0)$$

of the acceleration at the surface of the earth.

The motion will be determined by the equations

$$\frac{\partial}{\partial x} \left(k \frac{\partial \varphi}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial \varphi}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial \varphi}{\partial z} \right) = 0 \dots \quad (1)$$

and

$$\int \frac{dp}{k} + V + \frac{1}{2} \left\{ \left(\frac{\partial \varphi}{\partial x} \right)^2 + \left(\frac{\partial \varphi}{\partial y} \right)^2 + \left(\frac{\partial \varphi}{\partial z} \right)^2 \right\} = \text{const.} \dots \quad (2)$$

The problem becomes much simpler if, in the second equation, we suppose the variations of the square of the velocity to be much smaller than those of either of the first terms. We may then write

$$\int \frac{dp}{k} + V = \text{const.},$$

or, since

$$V = -g \frac{r_0^2}{r},$$

$$\log k - \mu g \frac{r_0^2}{r} = \text{const.}$$

If k_0 be the density at the surface, and

$$\mu g r_0^2 = \alpha,$$

the last equation becomes

$$\log k - \log k_0 - \alpha \left(\frac{1}{r} - \frac{1}{r_0} \right) = 0 \quad (3)$$

As we see, our simplification consists in this, that the distribution of the aether is independent of its motion, that is to say that it is condensed to the same degree as if it were at rest.

Substituting the value of k from (3) in (1), we find a differential equation for the determination of φ . It can be satisfied by

$$\varphi = z \left[a \left(\frac{\alpha}{2r} - 1 \right) + b \left(\frac{\alpha}{2r} + 1 \right) e^{-\frac{\alpha}{r}} \right], \quad (4)$$

the form of the solution being chosen with a view to the remaining conditions of the problem. These are

1^o. for $r = \infty$

$$\frac{\partial \varphi}{\partial x} = \frac{\partial \varphi}{\partial y} = 0, \quad \frac{\partial \varphi}{\partial z} = c,$$

2^o. for $r = r_0$

$$\frac{\partial \varphi}{\partial r} = 0.$$

They give us the following relations between the constants of integration a and b :

$$-a + b = c \quad \dots \dots \dots \dots \dots \quad (5)$$

$$a = \left(\frac{\alpha^2}{2r_0^2} + \frac{\alpha}{r_0} + 1 \right) e^{-\frac{\alpha}{r_0}} b. \quad \dots \dots \dots \quad (6)$$

The velocity with which the aether slides along the earth is found to be

$$v = \frac{\alpha^3}{4r_0^3} \cdot e^{-\frac{\alpha}{r_0}} b \sin \vartheta, \quad \dots \dots \dots \quad (7)$$

where ϑ is the angle between the radius of the point considered and the direction of the velocity c . Now, Prof. PLANCK remarks that, by (6), if only $\frac{\alpha}{r_0}$ be large enough, a will be very small relatively to b , so that, as (5) shows, b is nearly equal to c . But then, the value of v given by (7) will be a very small fraction of c itself.

If the quotient of the pressure and the density had the same value for aether as for air of 0° , and if the force of gravity acted with the same intensity on the aether as on ponderable matter, we should have

$$\frac{\alpha}{r_0} = 800, \text{ approximately.}$$

The sliding would then be absolutely imperceptible, but it should be noticed that this would be due to an enormous condensation, the ratio n between the densities for $r = r_0$ and $r = \infty$ being by (3)

$$e^{-\frac{\alpha}{r_0}}.$$

In order that the aether may follow the earth in its motion in so far as is necessary for the explication of the phenomena, we need not require that the condensation should have such a high value. Of course, it would be less, if either $\frac{k}{p}$ or g were smaller than for air.

We can easily determine what degree of condensation must necessarily be admitted. Indeed, the constant of aberration may be reckoned to correspond to within $\frac{1}{2}$ pCt. to the value given by the elementary theory of the phenomenon; consequently, in the theory of STOKES, the velocity of sliding should be no more than about $\frac{1}{2}$ pCt. of the earth's velocity. Now, putting $\frac{\alpha}{r_0} = 10$, I find for the maximum value of the velocity of sliding $0,011 c$. If $\frac{\alpha}{r_0} = 11$,

this value would be $0,0055 c.$ Thus we are led to the conclusion that $\frac{\alpha}{r_0}$ cannot be much different from 11, so that the least admissible value of the condensation is nearly $n = e^{11}.$

Calculations which we shall omit here may serve to estimate the error that has been committed in simplifying the equation (2). It is found that far away from the earth the error may become rather large, but that nearer the surface, precisely on account of the smallness of the velocity in these parts, we need not trouble ourselves about it. Thus, what has been said about the condensation may be true, even though the state of motion in the rarefied aether, at great distances, depart widely from the equation (4).

Strictly speaking, the condensation must be still more considerable than the value we have found to be necessary. If the aether be attracted by the earth, it is natural to suppose that it is acted on likewise by the sun; thus, the earth will describe its orbit in a space in which the aether is already condensed. In this dense aether the earth must produce a new condensation.

Of course it is not necessary that the attraction follow precisely the law of inverse squares; any law which leads to a sufficient condensation will suffice for our purpose. To understand the connexion between the condensation and the velocity of sliding, we may consider a simple case. Let the aether have a constant small density k outside a certain sphere, concentric with the earth, and within this sphere a constant density $k' > k.$

If now the earth were at rest, and the aether flowed along it, a diametral plane of the sphere, perpendicular to the mean direction of flow, would be traversed by a quantity of aether, equal to that which enters the sphere on one side and leaves it on the other side. If this shall be the case, the velocities inside the sphere must be of the order $\frac{k}{k'} c$, if outside the surface they are of the order $c.$

If we wish to maintain the theory of Prof. STOKES by the superposition of a condensation in the neighbourhood of the earth, it will be necessary to add a second hypothesis, namely that the velocity of light be the same in the highly condensed and in the not condensed aether. This is the theory that may be opposed to that of FRESNEL, according to which the aether has no motion at all. In comparing the two we should, I believe, pay attention to the following points.

1. The latter theory can only serve its purpose if we introduce the well known coefficient of FRESNEL, concerning the propagation

of light in moving media. Now, this coefficient has been found to be true by direct measurements and may be calculated by means of well founded theoretical considerations. It might be deemed strange, if in these ways we arrived precisely at the value that is required by a wrong theory.

2. If we hope some time to account for the force of gravitation by means of actions going on in the aether, it is natural to suppose that the aether itself is not subject to this force.

On these and other grounds, I consider FRESNEL's theory as the more satisfactory of the two. Prof. PLANCK is of the same opinion. Nevertheless it will be of importance to consider the question from all sides, and it is for this reason that the following remarks may here be allowed.

1. If the large condensation that has been spoken of and the constancy of the velocity of propagation, whatever be the density, be taken for granted, one can indeed explain all observed phenomena. At least, I for one have been unable to find a contradiction. It is true, as has already been stated, that, far away from the earth, the equation (4) will no longer hold. In considering the motion in those distant regions, the square of the velocity in the equation (2) has to be taken into account, and the sun's attraction will have to be considered. But, after all, I find that there may always exist an irrotational motion, and this, in addition to a sufficient condensation near the earth, is all that is required.

2. If we apply to the moving aether the equations which HERTZ has proposed for moving dielectrics¹⁾ the propagation of light will obey very simple laws. Suppose the earth to be at rest, and the aether to flow, and let the axes of coordinates be fixed in space. Then, if \mathbf{d} be the dielectric displacement, \mathbf{H} the magnetic force, \mathbf{v} the velocity of the aether and V that of light, and if the electromagnetic properties of the aether be supposed to be wholly independent of its density, the equation may be put in the form

$$\text{Div } \mathbf{d} = 0,$$

$$\frac{\partial \mathfrak{H}_z}{\partial y} - \frac{\partial \mathfrak{H}_y}{\partial z} = 4\pi \left[\frac{\partial \mathbf{d}_x}{\partial t} + \frac{\partial}{\partial y} (\mathbf{v}_y \mathbf{d}_x - \mathbf{v}_x \mathbf{d}_y) - \frac{\partial}{\partial z} (\mathbf{v}_x \mathbf{d}_z - \mathbf{v}_z \mathbf{d}_x) \right], \text{ etc.}$$

$$\text{Div } \mathbf{H} = 0,$$

$$4\pi V^2 \left(\frac{\partial \mathbf{d}_z}{\partial y} - \frac{\partial \mathbf{d}_y}{\partial z} \right) = - \frac{\partial \mathfrak{H}_x}{\partial t} - \frac{\partial}{\partial y} (\mathbf{v}_y \mathfrak{H}_x - \mathbf{v}_x \mathfrak{H}_y) + \frac{\partial}{\partial z} (\mathbf{v}_x \mathfrak{H}_z - \mathbf{v}_z \mathfrak{H}_x), \text{ etc.}$$

¹⁾ Wied. Ann. Bd. 41, p. 369.

We shall apply these equations to a steady motion with velocity-potential φ , without supposing that $\text{Div } v$ vanishes. We shall however neglect quantities of the order v^2 .

Now if, instead of t , we introduce as a new independent variable

$$t' = t + \frac{\varphi}{V^2},$$

and instead of v and \mathfrak{h} the vectors \mathfrak{F}' and \mathfrak{H}' , defined by

$$\mathfrak{F}'_x = 4\pi V^2 \mathfrak{v}_x + (\mathfrak{v}_z \mathfrak{h}_y - \mathfrak{v}_y \mathfrak{h}_z), \text{ etc.,}$$

and

$$\mathfrak{H}'_x = \mathfrak{h}_x - 4\pi (\mathfrak{v}_z \mathfrak{v}_y - \mathfrak{v}_y \mathfrak{v}_z), \text{ etc.,}$$

the equations become

$$\text{Div } \mathfrak{F}' = 0,$$

$$\frac{\partial \mathfrak{H}'_z}{\partial y} - \frac{\partial \mathfrak{H}'_y}{\partial z} = \frac{1}{V^2} \frac{\partial \mathfrak{F}'_x}{\partial t'}, \text{ etc.}$$

$$\text{Div } \mathfrak{H}' = 0,$$

$$\frac{\partial \mathfrak{F}'_z}{\partial y} - \frac{\partial \mathfrak{F}'_y}{\partial z} = - \frac{\partial \mathfrak{H}'_x}{\partial t'}, \text{ etc.}$$

These formulae have the same form as those that would hold for an aether without motion, and this is sufficient to obtain in a moment the well known theorems concerning the rotation of the wave-fronts and the rectilinearity of the rays of light. At the same time we see that at the boundary of the different layers of the aether, which slide one over the other, there is never a reflection of light.

It is curious that in the two rival theories somewhat the same mathematical artifices may be used.

3. There seems to be nothing against the assumption that, while the aether may be condensed by gravitation, molecular forces are incapable of producing this effect. In this way it might be explained that small masses, e. g. the flowing water in FIZEAU's experiments, cannot drag the aether along with it. In these cases the coefficient of FRESNEL would remain of use.

4. A decision between the two theories would be soon obtained, if the phenomena of the daily aberration were sufficiently known. Unfortunately, this is by no means the case; even, as Prof. VAN DE SANDE BAKHUYZEN assures me, one has never purposely examined what the existing observations teach us concerning this aberration.

Mathematics. — “On reducible hyperelliptic Integrals.” By Prof. J. C. KLUYVER.

(Will be published in the Proceedings of the next meeting.)

(April 22nd 1899.)

KONINKLIJKE AKADEMIE VAN WETENSCHAPPEN
TE AMSTERDAM.

PROCEEDINGS OF THE MEETING
of Saturday April 22th 1899.

(Translated from: Verslag van de gewone vergadering der Wis- en Natuurkundige Afdeeling van Zaterdag 25 Maart 1899 Dl VII)

CONTENTS "On reducible hyperelliptic Integrals" By Prof J C KLUYVFR, p 449. — „Meltin points in systems of optic isomers" By Prof H. W. BALKHUIS ROOZEBOOM, p 466 — „On the deduction of the characteristic equation" By Prof J. D. VAN DER WAALS — discussion with Prof BOLTZMANN, p 468. — „The galvano-magnetic and thermo-magnetic phenomena in bismuth (2nd Communication continued)" By Dr. E VAN EVERDINGEN JR (Communicated by Prof. H. KAMERLINGH ONNES) p. 473.

The following papers were read:

Mathematics. — "On reducible hyperelliptic Integrals". By Prof. J. C. KLUYVER.

(Read in the Meeting of March 25th 1899)

In some cases an Abelian integral of the first kind and of deficiency p can be reduced to an elliptic integral. According to a theorem of WEIERSTRASS¹⁾ when such a reducible integral presents itself, it is possible to transform by a substitution of order r the ϑ -function of the first order into an other one of order r , so that the $p-1$ constituents $\tau'_{12}, \tau'_{13}, \dots, \tau'_{1p}$ in the first row of the period matrix all assume the value zero. If, conversely, it is possible by a substitution of order r to find a ϑ -function whose period matrix shows this peculiarity, at least one of the integral

¹⁾ KOWALEVSKI, *Acta Math.* IV, p 395.

of the first kind is reducible, and it is possible to construct rational functions on the RIEMANN surface T , which are doubly periodic functions of this integral.

Let us suppose that in the ϑ -function of p variables $\vartheta(u; \tau)$, where u denotes the p normal integrals and τ denotes the given period matrix, we make a substitution of order r associated with the Abelian matrix ¹⁾

$$\begin{pmatrix} \alpha & \beta \\ \alpha' & \beta' \end{pmatrix}$$

of $2p$ rows and $2p$ columns of integers, in such a way that the separate matrices $\alpha, \beta, \alpha', \beta'$ satisfy the equations

$$\alpha' = 0, \quad \bar{\alpha}\beta' = \bar{\beta}'\alpha = \alpha\bar{\beta}' = \beta'\bar{\alpha} = r, \quad \bar{\beta}\beta' - \bar{\beta}'\beta = 0.$$

According to this substitution the integrals u are replaced by other integrals w determined by the equation

$$u = \alpha w,$$

and $\vartheta(u; \tau)$ of the first order becomes a function $\vartheta(w; \tau')$ of order r with a period matrix τ' which can be derived from the equation

$$\alpha\tau' = \beta + \tau\beta'.$$

From the above relations we can immediately calculate the increments Ω , taken by the integrals w , when by describing some closed curve on T the normal integrals u are increased by

$$\omega = k + \tau k',$$

where k and k' denote two columnletters.

In the first place we find

$$\alpha\Omega = \omega = k + \tau k',$$

and also by multiplying by the matrix $\bar{\beta}'$

$$\bar{\beta}'\alpha\Omega = r\Omega = \bar{\beta}'k + \bar{\beta}'\tau k'.$$

¹⁾ For the notation compare: BAKER, *Abel's theorem and the allied theory*. Cambridge 1897.

From

$$\alpha \tau' = \beta + \tau \beta'$$

we have

$$\bar{\beta}' \tau = \tau' \bar{\alpha} - \bar{\beta},$$

so that we get for the system of the increments Ω of the integrals w

$$r\Omega = \bar{\beta}' k - \bar{\beta} k' + \tau' \alpha k'.$$

By supposing that in the first row of the matrix τ' the constituents $\tau'_{12}, \tau'_{13}, \dots, \tau'_{1p}$ are all equal to zero we shall find that every increment Ω_1 of the integral w_1 is expressed by

$$r\Omega_1 = \beta'_1 k - \beta_1 k' + \tau'_{11} \alpha_1 k',$$

where the first columns of the matrices β', β, α are denoted by $\beta'_1, \beta_1, \alpha_1$.

Hence the moduli of periodicity of the integral rw_1 corresponding to any closed circuit are always multiples of 1 and τ'_{11} and therefore this integral must be an elliptic integral.

It may be noticed that in the case $p = 2$, the same conclusion holds for the integral rw_2 , so that for $p = 2$ there exist two reducible integrals or there is none.

Assuming rw_1 to be an elliptic integral we can easily find how many zeros the function $\theta(rw_1; \tau'_{11})$, of the single variable rw_1 and the period τ'_{11} , possesses on the surface T . We have only to calculate the value of the integral

$$\frac{1}{2\pi i} \int_{T'} d \log \theta(rw_1; \tau'_{11}),$$

taken round the boundary of the simply connected surface T' , into which T is resolved by the customary p pairs of cross-cuts A_h and B_h . On opposite edges of a cross-cut A_h the variable rw_1 has values the difference of which amounts to β'_h , so that on both edges $d \log \theta(rw_1; \tau'_{11})$ has the same value and the integrals taken in opposite directions round these edges, destroy one another.

On the contrary by crossing B_h the integral rw_1 increases by

$$-\beta_{h1} + \tau'_{11} \alpha_{h1},$$

and as

$$d \log \bar{O}(rw_1 - \beta_{h1} + \tau'_{11} \alpha_{h1}; \tau'_{11}) = d \log O(rw_1; \tau'_{11}) - 2\pi i \alpha_{h1} drw_1,$$

we finally get

$$\frac{1}{2\pi i} \int_T d \log O(rw_1; \tau'_{11}) = - \sum_h a_{h1} \int_{B_h} drw_1 = \sum_h \alpha_{h1} \beta'_{h1} = r.$$

Consequently the function $O(rw_1; \tau'_{11})$ admits r zeros on T' , so that any quotient of the squares of two thetas must be a uniform function of position on the undissected surface T with r double zeros and r double poles. Hence as soon as one of the integrals of the first kind W is reducible, there exist four adjoint curves R_1, R_2, R_3 and R_4 , belonging to a pencil, which each separately, letting alone any possible common points of intersection with the fundamental curve f , touch — or at least intersect in two coincident points — the latter r times. The three quotients $R_1 : R_4, R_2 : R_4, R_3 : R_4$ being quotients of the squares of two thetas save as to some constant factor, may be taken equal to $\bar{p}' W - \varepsilon_1, \bar{p}' W - \varepsilon_2, \bar{p}' W - \varepsilon_3$, whence

$$\bar{p}' W = \frac{2}{R_4^2} \sqrt{R_1 R_2 R_3 R_4}.$$

The function $\bar{p}' W$ being however likewise uniform on T it must be possible to replace the product of the four functions R by the square of a rational function F , otherwise said: through the $4r$ points of contact of the curves R there can be made to pass an adjoint curve F , the order of which is the double of the order of the curves R , which touches the fundamental curve f in the common points of intersection with the curves R and which for the rest intersects f only in the double points. The elliptic integral itself is now given by the equation

$$dW = \frac{R_4 \frac{dR_1}{dx} - R_1 \frac{dR_4}{dr}}{\sqrt{4 R_1 R_2 R_3 R_4}} dx,$$

or, when homogeneous variables x, y, z are introduced, by

$$\varrho dw = \frac{J(x, y, z)}{F} \cdot \frac{dx}{\frac{\partial f}{\partial y}},$$

where $J(x, y, z)$ denotes the Jacobian of f and of the pencil of the curves R .

Meanwhile it is clear that when a reducible integral W presents itself the curves R are not yet uniquely determined. The lower limit of the integral W is still arbitrary and what is said of the squares of thetas with the argument W is also applicable to the squares of thetas with the argument $W + \alpha$. Hence the functions R can be replaced by

$$\frac{S_1}{S_4} = \bar{p}(W + \alpha) - \varepsilon_1, \quad \frac{S_2}{S_4} = \bar{p}(W + \alpha) - \varepsilon_2, \quad \frac{S_3}{S_4} = \bar{p}(W + \alpha) - \varepsilon_3,$$

which functions can be expressed rationally in R_1, R_2, R_3, R_4 and F . Evidently the constant α may be regulated in such a way that one of the curves S , e.g. S_1 , touches f in a given point z', y' , and by the prescription of this point the remaining $r-1$ points of contact of S_1 are completely determined. Thus we infer that the existence of an elliptic integral W implies an involutory grouping of the points of the curve f in such a way, that the r points of any group may be regarded as the points of contact of some curve S .

The fact that the system of the curves R depends on an arbitrary parameter is important when we consider hyperelliptic curves. For then in the equation of the curve f one of the coordinates, say y , occurs only in the second power and the rational functions contain no power of y higher than the first. So it is always possible to choose the constant α in such a manner that in the ratio $S_1 : S_4$, and then also in the two others $S_2 : S_4$ and $S_3 : S_4$, the term containing y is wanting. In other words: in the case of an hyperelliptic curve admitting a reducible integral we can suppose beforehand that each of the curves R , by means of which the reduction has been effected, breaks up into a group of r right lines, drawn through the multiple point of the curve.

In the preceding the existence of the funtions R proved to be a necessary consequence of the reducibility of one of the integrals of the first kind; conversely, if the existence of the functions R is established at least one of the integrals is reducible.

For in this supposition the elliptic integral

$$\int \frac{d \frac{R_1}{R_4}}{\sqrt{\frac{R_1}{R_4} \cdot \frac{R_2}{R_4} \cdot \frac{R_3}{R_4}}}$$

is an integral of the first kind belonging to the curve, because it can assume the form

$$\int \frac{J(x, y, z)}{F} \cdot \frac{dx}{\frac{\partial f}{\partial y}}.$$

As for reducible non-hyperelliptic integrals the case $p = 3$, $r = 2$ has been treated by SOPHIE KOWALEVSKI. The curve f is here the general quartic upon which, r being equal to 2, an involutory correspondence one to one exists. It can be shown that in this case the curve can be transformed into itself by a reciprocal projective transformation of the plane. Consequently four double tangents of the curve pass through the centre of the transformation, so that its equation can always be thrown into the form

$$f = xy(ax + by)(cx + dy) - K^2 = 0.$$

Evidently the four double tangents passing through the origin can be identified with the curves R . For each of these tangents touches f in two points, together they belong to a pencil and the eight points of contact lie on the conic K .

Accordingly we get for the elliptic integral

$$\frac{\bar{p}W - \varepsilon_1}{\frac{x}{d}} = \frac{\bar{p}W - \varepsilon_2}{\frac{y}{c}} = \frac{\bar{p}W - \varepsilon_3}{\frac{ax + by}{cx + dy}} = \frac{1}{cx + dy},$$

and the integral itself is

$$W = \int \frac{\partial K}{\frac{\partial z}{\partial y}} dx.$$

The simplest case of the reducible hyperelliptic integral, $p = 2$, $r = 2$, was already known to LEGENDRE. Again the curve f is of the fourth order, but it has now a node from which six tangents can be drawn to the curve. If there is a reducible integral each of the first three functions R is made up by a pair of these tangents, as fourth function R_4 we must take one of the two double elements, counted twice, of the involution which is now necessarily formed by the three pairs of tangents.

The equation of the curve being

$$f = xy^2 - (x - 1)(x - kl)(x - k)(x - l) = 0$$

we get in this way

$$R_1 = x, R_2 = (x - 1)(x - kl), R_3 = (x - k)(x - l), R_4 = (x \pm \sqrt{kl})^2$$

and the reducible integral is

$$W = \int \frac{(x \mp \sqrt{kl}) dx}{xy} .$$

In accordance with what resulted from the theorem of WEIERSTRASS for the case $p = 2$ two independent reducible integrals are obtained.

Also the case $p = 2$, $r = 3$ has been considered from various sides. As before the integral is relative to a nodal quartic f the equation of which we take in the form

$$(x - a_1)(x - c_1)y^2 = (x - a_2)(x - a_3)(x - c_2)(x - c_3) .$$

BURKHARDT¹⁾ has pointed out the invariant relation existing between the binary cubics $(x - a_1)(x - a_2)(x - a_3)$ and $(x - c_1)(x - c_2)(x - c_3)$ when one of the integrals is reducible. Previously GOURSAT²⁾ had treated a more or less particular case of the reducibility and finally BURNSIDE³⁾ indicated in connection with his more general researches a remarkable form which the reducible integral can always assume.

After the deduction of some of their results a few remarks will be added.

The curves R , each of which breaks up into three right lines, must

¹⁾ *Math. Annal.*, Vol. 36, 1890, p. 410

²⁾ *Compt. Rendus*, 100, 1885, p. 622.

³⁾ *Proc. Lond. Math. Soc.*, 23, 1892, p. 173.

be required to touch — or at least to intersect in two coincident points — the curve f three times. From the node six tangents, the inflexional tangents included, can be drawn to the curve, three of which $(x - a_1), (x - a_2), (x - a_3)$ make up together the curve R_4 ; by joining to each of the remaining tangents $(x - c_1), (x - c_2), (x - c_3)$ a line through the node $(x - b_1), (x - b_2), (x - b_3)$, counted twice, the four functions thus obtained

$$R_1 = (x - c_1)(x - b_1)^2, \quad R_2 = (x - c_2)(x - b_2)^2,$$

$$R_3 = (x - c_3)(x - b_3)^2, \quad R_4 = (x - a_1)(x - a_2)(x - a_3)$$

indeed satisfy all the demands, if only we take care to choose the quantities b in such a way that the four functions are in involution. With the aid of this condition we can eliminate the b 's, after which still one relation remains between the a 's and the c 's. Hence of a reducible integral five branch-points out of six may be chosen arbitrarily.

Reducible integrals of the kind considered here are easy to construct if we observe that the four binary cubics belong to the system of first polars of a binary biquadratic α_x^4 . Among these polars there are four of the form $(x - c)(x - b)^2$, having a double point, so besides R_1, R_2, R_3 also $(x - c_4)(x - b_4)^3$ belongs to the system and the four quantities b are at once recognised as the roots of the Hessian Δ_x^4 . Also c_1, c_2, c_3, c_4 are the roots of a covariant, found by the following consideration. The four points y , whose first polars $\alpha_y \alpha_x^3$ contain a double point b , are the roots of the covariant¹⁾ $3i\Delta_x^4 - 2j\alpha_x^4$, where i and j denote the two invariants of α_x^4 . The result of the elimination of y between this covariant and $\alpha_x \alpha_y^3$, which result is of the 12th order in x and of the 8th order in the coefficients α , must be a covariant having the quantities b for double roots and the quantities c for single roots. After division by the square of Δ_x^4 a covariant of the 4th order in the coefficients α will remain, necessarily of the form $\lambda i\Delta_x^4 + \mu j\alpha_x^4$, the roots of which are c_1, c_2, c_3, c_4 . To determine the coefficients λ and μ , we consider the special case

$$\alpha_x^4 = \frac{2i}{3j} x^2 (x^2 - 1), \quad \Delta_x^4 = -\frac{2}{3} (2x^4 + 1).$$

In this case α_x^4 has a double point, the four values of c are

¹⁾ CLEBSCH-LINDEMANN. *Vorlesungen*, I, p 231.

evidently 0 and ∞ , each taken twice, and, as

$$i \Delta_x^4 + 2j \alpha_x^4 = -2ix^2,$$

we must take $\lambda = 1$ and $\mu = 2$. So in general the quantities c are determined by the equation

$$i \Delta_x^4 + 2j \alpha_x^4 = 0.$$

As soon as we regard as known one root c_4 of this equation we can construct a reducible integral. For then we can put

$$\alpha_z \alpha_x^3 = (x - a_1)(x - a_2)(x - a_3),$$

$$\frac{1}{x - c_4} (i \Delta_x^4 + 2j \alpha_x^4) = (x - c_1)(x - c_2)(x - c_3),$$

whence it follows that

$$R_4 \frac{dR_1}{dx} - R_1 \frac{dR_4}{dx} = \varrho \Delta_x^4,$$

$$\sqrt{R_1 R_2 R_3 R_4} = \frac{\Delta_x^4}{x - b_4} \sqrt{\alpha_z \alpha_x^3 \cdot \frac{1}{x - c_4} (i \Delta_x^4 + 2j \alpha_x^4)},$$

so that

$$W = \int \frac{(x - b_4) dx}{\sqrt{\alpha_z \alpha_x^3 \cdot \frac{1}{x - c_4} (i \Delta_x^4 + 2j \alpha_x^4)}}$$

is an integral of the first kind relative to the quartic f , which is transformed by the substitution

$$\begin{aligned} \frac{\bar{p} W - \varepsilon_1}{\varrho_1 (x - c_1)(x - b_1)^2} &= \frac{\bar{p} W - \varepsilon_2}{\varrho_2 (x - c_2)(x - b_2)^2} = \\ &= \frac{\bar{p} W - \varepsilon_3}{\varrho_3 (x - c_3)(x - b_3)^2} = \frac{1}{\varrho_4 (x - a_1)(x - a_2)(x - a_3)} \end{aligned}$$

into an elliptic integral with the variable pW .

It is now clear that in this way it is possible to construct the integral out of five assigned branch-points. If the five branch-points a_1, a_2, a_3, c_1, c_2 ,

are given, a binary biquadratic α_x^4 must be sought which has a given cubic A_x^3 with the roots a_1, a_2, a_3 among its first polars. So there is an identical equation of the form

$$\alpha_z \alpha_x^3 = q A_x^3,$$

from which we deduce

$$\frac{z \alpha_0 + \alpha_1}{A_0} = \frac{z \alpha_1 + \alpha_2}{A_1} = \frac{z \alpha_2 + \alpha_3}{A_2} = \frac{z \alpha_3 + \alpha_4}{A_3}.$$

In the first place these equations determine z , moreover for the coefficients of α_x^4 two relations remain, expressed by any two of the four following equations

$$6 A_2 \Delta_0 - 12 A_1 \Delta_1 + 6 A_0 \Delta_2 = A_0 i,$$

$$2 A_3 \Delta_0 - 6 A_1 \Delta_2 + 4 A_0 \Delta_3 = A_1 i,$$

$$4 A_3 \Delta_1 - 6 A_2 \Delta_2 + 2 A_0 \Delta_4 = A_2 i,$$

$$6 A_3 \Delta_2 - 12 A_2 \Delta_3 + 6 A_1 \Delta_4 = A_3 i.$$

There are still two other conditions for α_x^4 , namely that the fourth and the fifth of the given quantities are roots of the covariant $i \Delta_x^4 + 2 j \alpha_x^4$. By the four conditions together the quartic α_x^4 is entirely determined, and thereby the sixth branch-point c_3 of the integral.

We obtain the example treated by GOURSAT by putting

$$A_1 = 0, \quad \alpha_1 = 0, \quad \alpha_2 = 0.$$

Then we have

$$\Delta_0 = 0, \quad \Delta_1 = \alpha_0 \alpha_3, \quad \Delta_2 = \frac{1}{3} \alpha_0 \alpha_4, \quad \Delta_3 = 0, \quad \Delta_4 = -2 \alpha_3^2,$$

$$i = 2 \alpha_0 \alpha_4, \quad j = -6 \alpha_3^2 \alpha_0,$$

the coefficients α satisfying the equation

$$A_3 \alpha_0 \alpha_3 - A_2 \alpha_0 \alpha_4 - A_0 \alpha_3^3 = 0.$$

Lastly,

$$i \Delta_x^4 + 2 j \alpha_x^4 = 4 \alpha_0 (3 \alpha_3 x + \alpha_4) (-\alpha_0 \alpha_3 x^3 + \alpha_0 \alpha_4 x^2 - 4 \alpha_3^2),$$

therefore c_1, c_2, c_3 are the roots of the cubic

$$C_0 x^3 + 3 C_1 x^2 + C_3 = -\alpha_0 \alpha_3 x^3 + \alpha_0 \alpha_4 x^2 - 4 \alpha_3^2,$$

and c_4 , associated with the root $b_4 = \infty$ of the Hessian Δ_x^4 , is determined by $3\alpha_4 x + \alpha_4 = 0$.

Hence the integral

$$W = \int \frac{dx}{\sqrt{(A_0 x^3 + 3 A_2 x + A_3)(C_0 x^3 + 3 C_1 x^2 + C_3)}}$$

proves to be reducible under the condition that the coefficients A and C obey the relation

$$A_0 C_3 = 4 A_3 C_0 + 12 A_2 C_1.$$

The formulae of transformation will be found to be

$$\begin{aligned} \frac{\bar{p} W - \varepsilon_1}{\varrho_1 (x - c_1)(x - b_1)^2} &= \frac{\bar{p} W - \varepsilon_2}{\varrho_2 (x - c_2)(x - b_2)^2} = \frac{\bar{p} W - \varepsilon_3}{\varrho_3 (x - c_3)(x - b_3)^2} = \\ &= \frac{1}{A_0 x^3 + 3 A_2 x + A_3}, \end{aligned}$$

from which we can also deduce

$$\lambda \bar{p} W + \mu = \frac{C_0 x - C_1}{A_0 x^3 + 3 A_2 x + A_3}.$$

We obtain the second reducible integral by regarding $C_0 x^3 + 3 C_1 x + C_3$ as first polar of the biquadratic α_4^4 . In this supposition the Hessian Δ_x^4 will admit the root $b_4 = 0$, the polar $(x - c_4)(x - b_4)^2$ takes the form $x^2(A_2 x - A_3)$ and the required integral itself is

$$W = \int \frac{x dx}{\sqrt{(A_0 x^3 + 3 A_2 x + A_3)(C_0 x^3 + 3 C_1 x^2 + C_3)}}.$$

The reduction of the integral is obtained by putting

$$\lambda \bar{p} W + \mu = \frac{x^2 (A_2 x - A_3)}{C_0 x^3 + 3 C_1 x^2 + C_3}.$$

Another means of constructing reducible integrals is founded on a peculiar form which can always be given to the invariant relation between the six branch-points.

From the identical equation

$$(x-a_1)(x-a_2)(x-a_3) = \lambda_1(x-c_1)(x-b_1)^2 + \lambda_2(x-c_2)(x-b_2)^2$$

we derive by the successive substitutions $x = a_1, a_2, a_3$

$$\frac{(a_1-c_1)(a_1-b_1)^2}{(a_1-c_2)(a_1-b_2)^2} = \frac{(a_2-c_1)(a_2-b_1)^2}{(a_2-c_2)(a_2-b_2)^2} = \frac{(a_3-c_1)(a_3-b_1)^2}{(a_3-c_2)(a_3-b_2)^2},$$

or

$$\frac{(a_1-b_1)\sqrt{\frac{a_1-c_1}{a_1-c_2}}}{a_1-b_2} = \frac{(a_2-b_1)\sqrt{\frac{a_2-c_1}{a_2-c_2}}}{a_2-b_2} = \frac{(a_3-b_1)\sqrt{\frac{a_3-c_1}{a_3-c_2}}}{a_3-b_2}.$$

After multiplying numerator and denominator of these three fractions successively by (a_2-a_3) , (a_3-a_1) and (a_1-a_2) , addition gives

$$0 = (a_2-a_3)(a_1-b_1)\sqrt{\frac{a_1-c_1}{a_1-c_2}} + (a_3-a_1)(a_2-b_1)\sqrt{\frac{a_2-c_1}{a_2-c_2}} + (a_1-a_2)(a_3-b_1)\sqrt{\frac{a_3-c_1}{a_3-c_2}}.$$

Similarly by interchanging c_2 and c_3 we obtain

$$0 = (a_2-a_3)(a_1-b_1)\sqrt{\frac{a_1-c_1}{a_1-c_3}} + (a_3-a_1)(a_2-b_1)\sqrt{\frac{a_2-c_1}{a_2-c_3}} + (a_1-a_2)(a_3-b_1)\sqrt{\frac{a_3-c_1}{a_3-c_3}},$$

and having moreover the identity

$$0 = (a_2-a_3)(a_1-b_1) + (a_3-a_1)(a_2-b_1) + (a_1-a_2)(a_3-b_1),$$

the quantities (a_1-b_1) , (a_2-b_1) and (a_3-b_1) can be eliminated and the invariant relation between the branch-points of a reducible integral takes the form

$$0 = \begin{vmatrix} \frac{1}{\sqrt{a_1 - c_1}} & \frac{1}{\sqrt{a_2 - c_1}} & \frac{1}{\sqrt{a_3 - c_1}} \\ \frac{1}{\sqrt{a_1 - c_2}} & \frac{1}{\sqrt{a_2 - c_2}} & \frac{1}{\sqrt{a_3 - c_2}} \\ \frac{1}{\sqrt{a_1 - c_3}} & \frac{1}{\sqrt{a_2 - c_3}} & \frac{1}{\sqrt{a_3 - c_3}} \end{vmatrix},$$

the arguments of the nine surds being determined save as to a multiple of π . The symmetry between the a 's and the c 's shows, that, as soon as there exists the involution we started with, a similar involution can be found by interchanging the c 's and the a 's, so that reducible integrals always present themselves in pairs.

BURNSIDE stated incidentally that the curve

$$xy^2 = (x-1)(x-sn^2 u) \left(x-sn^2(u + \frac{2K}{3}) \right) \left(x-sn^2(u - \frac{2K}{3}) \right)$$

admits reducible integrals. The form given here to the invariant relation between the six branch-points readily provides a proof for this assertion.

In order to obtain this proof we introduce elliptic arguments instead of the a 's and the c 's by putting

$$\begin{aligned} a_1 &= e_1, & c_1 &= pu_1 = p(u + v_1), \\ a_2 &= e_2, & c_2 &= pu_2 = p(u + v_2), \\ a_3 &= e_3, & c_3 &= pu_3 = p(u + v_3). \end{aligned}$$

So the invariant relation becomes

$$0 = \begin{vmatrix} \frac{\sigma u_1}{\sigma_1 u_1} & \frac{\sigma u_2}{\sigma_1 u_2} & \frac{\sigma u_3}{\sigma_1 u_3} \\ \frac{\sigma u_1}{\sigma_2 u_1} & \frac{\sigma u_2}{\sigma_2 u_2} & \frac{\sigma u_3}{\sigma_2 u_3} \\ \frac{\sigma u_1}{\sigma_3 u_1} & \frac{\sigma u_2}{\sigma_3 u_2} & \frac{\sigma u_3}{\sigma_3 u_3} \end{vmatrix}.$$

As a function of u the determinant is doubly periodic; manifestly

it has several zeros, e.g. $u = -v_1$, hence the equation is identically satisfied, if we choose v_1, v_2, v_3 in such a way that the function has no poles. There might be nine poles, since each of the denominators can be made to vanish. Consequently we have to find out whether it is possible to make the corresponding residues equal to zero. If we take e.g. the pole corresponding to $\sigma_\alpha u_1 = 0$, that is if we take $u = -v_1 + \omega_\alpha$, the residue is proportional to

$$\frac{\sigma_\gamma u_2 \sigma_\beta u_3 - \sigma_\beta u_2 \sigma_\gamma u_3}{\sigma_\gamma u_2 \sigma_\beta u_2 \sigma_\gamma u_3 \sigma_\beta u_3}.$$

The above expression however can be written

$$\frac{2(e_2 - e_3) \sigma \frac{u_2 + u_3}{2} \sigma \frac{u_2 - u_3}{2} \sigma_\alpha \frac{u_2 + u_3}{2} \sigma_\alpha \frac{u_2 - u_3}{2}}{\sigma_\gamma u_2 \sigma_\beta u_2 \sigma_\gamma u_3 \sigma_\beta u_3},$$

and this is zero, if only for $u = -v_1 + \omega_\alpha$ we have at the same time $\frac{1}{2}(u_2 + u_3) \equiv \omega_\alpha$.

This takes place if we have

$$\frac{1}{2}(v_2 + v_3 - 2v_1) \equiv 0,$$

and the residues of all nine poles will vanish if moreover we have simultaneously

$$\frac{1}{2}(v_3 + v_1 - 2v_2) \equiv 0,$$

$$\frac{1}{2}(v_1 + v_2 - 2v_3) \equiv 0.$$

As solution of these equations can be taken

$$v_1 = \frac{4\Omega_1}{3}, \quad v_2 = \frac{4\Omega_2}{3}, \quad v_3 = \frac{4\Omega_3}{3},$$

where $2\Omega_1, 2\Omega_2, 2\Omega_3$ denote periods connected by the relation

$$\Omega_1 + \Omega_2 + \Omega_3 = 0.$$

So we find the reducible integral

$$w = \int \frac{(x-b_4) dx}{\sqrt{(4x^3 - g_2 x - g_3)(x-p(u+\frac{4\Omega_1}{3}))(x-p(u+\frac{4\Omega_2}{3}))(x-p(u+\frac{4\Omega_3}{3}))}}.$$

Every reducible integral, with five assigned branch-points a_1, a_2, a_3, c_1, c_2 , can be brought into this form by a linear substitution. For by means of the equations

$$\frac{a_1-a_2}{a_1-a_3} : \frac{c_1-a_2}{c_1-a_3} = \frac{e_1-e_2}{e_1-e_3} : \frac{p(u + \frac{4\Omega_1}{3}) - e_2}{p(u + \frac{4\Omega_1}{3}) - e_3},$$

$$\frac{a_1-a_2}{a_1-a_3} : \frac{c_2-a_2}{c_2-a_3} = \frac{e_1-e_2}{e_1-e_3} : \frac{p(u + \frac{4\Omega_2}{3}) - e_2}{p(u + \frac{4\Omega_2}{3}) - e_3}$$

we can determine the ratios $e_1 : e_2 : e_3$ and the argument u .

A somewhat laborious calculation gives the values of the quantities b and the formulae of substitution.

Putting

$$p(u_2-u_3) = p(u_3-u_1) = p(u_1-u_2) = pv,$$

the following results are obtained

$$b_1 = -2pv + p'v \cdot \frac{pu_2 - pu_3}{p'u_2 + p'u_3}, \quad b_2 = -2pv + p'v \cdot \frac{pu_3 - pu_1}{p'u_3 + p'u_1},$$

$$b_3 = -2pv + p'v \cdot \frac{pu_1 - pu_2}{p'u_1 + p'u_2},$$

$$b_4 = \begin{vmatrix} b_1 + 2pu_1 & b_2 + 2pu_2 & b_3 + 2pu_3 \\ b_1pu_1 & b_2pu_2 & b_3pu_3 \\ 1 & 1 & 1 \end{vmatrix} : \begin{vmatrix} pu_1 & pu_2 & pu_3 \\ b_1 & b_2 & b_3 \\ 1 & 1 & 1 \end{vmatrix}.$$

Furthermore the formulae of substitution are

$$\bar{p}W - \varepsilon_1 = (pu_2 + pu_3 + pv) \frac{(x-pu_1)(x-b_1)^2}{4x^3 - g_2x - g_3},$$

$$\bar{p}W - \varepsilon_2 = (pu_3 + pu_1 + pv) \frac{(x-pu_2)(x-b_2)^2}{4x^3 - g_2x - g_3},$$

$$\bar{p}W - \varepsilon_3 = (pu_1 + pu_2 + pv) \frac{(x-pu_3)(x-b_3)^2}{4x^3 - g_2x - g_3}$$

so that the roots of the \bar{p} -function are determined by

$$\varepsilon_2 - \varepsilon_3 = pu_2 - pu_3, \quad \varepsilon_3 - \varepsilon_1 = pu_3 - pu_1, \quad \varepsilon_1 - \varepsilon_2 = pu_1 - pu_2.$$

By interchanging the a 's and the c 's we can obtain the second reducible integral.

Proceeding to consider reducible hyperelliptic integrals of deficiency $p=3$, it is clear that by the same methods also these can be constructed without great difficulty. If the curve f is given by the equation

$$(x-a_1)(x-a_2)(x-a_3)y^2 = (x-a_4)(x-a_5) \dots (x-a_8),$$

that is: if the curve f is a quintic with a triple point, from which eight tangents can be drawn, each of the curves R_1, R_2, R_3, R_4 in the case $r=2$ is to be made up by a couple of these tangents. The twofold condition for the reducibility expresses that these four pairs of tangents are in involution and it is easily verified that when $Ax^2 + Bx + C$ defines the double elements of the involution, the reducible integral will have the form

$$\int \frac{Ax^2 + Bx + C}{(x-a_1)(x-a_2)(x-a_3)y} dx.$$

The investigation of the next case $p=3, r=3$, is closely allied to that of the case $p=2, r=3$ treated before.

Suppose the equation of the curve to be

$$(x-a_1)(x-a_2)(x-a_3)y^2 = (x-a'_1)(x-a'_2)(x-a'_3)(x-c_1)(x-c_2).$$

We regard the product of three tangents $(x-a_1)(x-a_2)(x-a_3)$ as the curve R_1 , in the same way we join the second three $(x-a'_1)(x-a'_2)(x-a'_3)$ to a curve R_2 ; the third curve R_3 will be the product of the next tangent $(x-c_1)$ and of a line through the node $(x-b_2)$, counted twice, similarly the curve R_4 is furnished by the tangent $(x-c_2)$ and the double line $(x-b_2)$.

The twofold condition for the reducibility is that the four binary cubics

$$R_1 = (x-a_1)(x-a_2)(x-a_3),$$

$$R_2 = (x-a'_1)(x-a'_2)(x-a'_3),$$

$$R_3 = (x-c_1)(x-b_1)^2,$$

$$R_4 = (x-c_2)(x-b_2)^2$$

are in involution. This will evidently take place when c_1 and c_2 are two roots of the covariant $i \Delta_x^4 + 2j \alpha_x^4$, where α_x^4 is the biquadratic which has R_1 and R_2 as first polars. The quantities b_1 and b_2 are two roots of Δ_x^4 ; if we call the other two b_3 and b_4 ,

$$W = \int \frac{(x-b_3)(x-b_4) dx}{\sqrt{\alpha_z \alpha_x^3 \cdot \alpha_t \alpha_x^3 \cdot \frac{1}{(x-c_1)(x-c_2)} (i \Delta_x^4 + 2j \alpha_x^4)}}$$

is a reducible integral,

As an example we may point to the following case

$$\alpha_x^4 = k_1^2 x^4 + 4k_1 x^3 + 6x^2 + 4k_2 x + k_2^2,$$

$$i = 2(k_1 k_2 - 1)(k_1 k_2 - 3), j = -6(k_1 k_2 - 1)^2,$$

$$\Delta_x^4 = 2(k_1 k_2 - 1)(2k_1 x^3 + (k_1 k_2 + 3)x^2 + 2k_2 x),$$

$$\alpha_z \alpha_x^3 = k_1^2 x^3 + 3k_1 x^2 + 3x + k_2, \quad \alpha_t \alpha_x^3 = k_1 x^3 + 3x^2 + 3k_2 x + k_2^2,$$

$$i \Delta_x^4 + 2j \alpha_x^4 = q(3x + k_2)(xk_1 + 3)(k_1 x^2 + (3 - k_1 k_2)x + k_2).$$

The Hessian Δ_x^4 has two special roots $b_3 = 0, b_4 = \infty$, the corresponding quantities c_3 and c_4 are given by $3x + k_2 = 0, xk_1 + 3 = 0$, c_1 and c_2 are the roots of $k_1 x^2 + (3 - k_1 k_2)x + k_2 = 0$ and the resulting reducible integral is

$$W = \int \frac{x dx}{\sqrt{(k_1^2 x^3 + 3k_1 x^2 + 3x + k_2)(k_1 x^3 + 3x^2 + 3k_2 x + k_2^2)(k_1 x^2 + (3 - k_1 k_2)x + k_2)}},$$

where under the radical sign we may replace each of the two cubic factors by any one of their linear combinations. The reduction of the integral will be effected by the substitution

$$\frac{A \bar{p} W + B}{C \bar{p} W + D} = \xi = \frac{x^3 k_1 + 3x^2}{3x + k_2}.$$

The case $p = 4, r = 3$ evidently allows quite a similar treatment. Seven branch-points of the integral can be assigned arbitrarily; the triple condition determining the three remaining branch-points is again readily obtained by the consideration of the cubic involution.

Chemistry. — "On melting points in systems of optic isomers."

By Prof. H. W. BAKHUIS ROOZEBOOM.

In order to test the views on melting points of mixtures of optic isomers, as communicated in the Proceedings of the Meeting of Feb. 25th 1899, page 312, Mr. ADRIANI has now in the first place examined two examples in which the certainty existed of the inactive substance being a racemic compound.

The first example is the dimethylic ethereal salt of tartaric acid. Here the *d*- and *l*-form have a lower melting point than the racemic compound.

The second example is the dimethylic ethereal salt of diacetyl tartaric acid, where the *d*- and *l*-form have a higher melting point than the racemic compound.

For the present only the melting points of mixtures of *d*- and *r*-forms have been examined in all their proportions. Consequently we could only get the right half of fig. 3 and 4 (vide page 312).

We found :

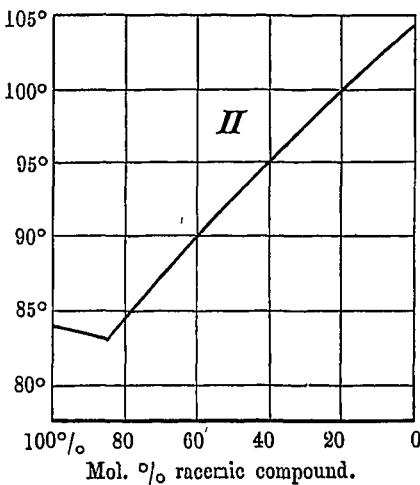
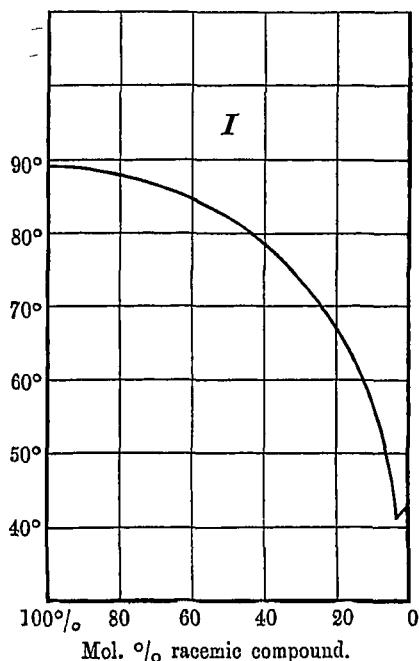
I. Dimethylic ethereal salt of tartaric acid.

Mol. percent racemic compound in the mixture.	Final melting point.
0	43°3
1.96	41.7
3.02	41.6
4.78	45.0
6.59	50.6
9.17	57.0
20	66.8
40	78.7
50	81.8
60	84.2
70	85.9
80	87.3
90	88.5
100	89.4

II. *Dimethylic ethereal salt of diacetyl tartaric acid.*

Mol. percent of racemic compound in the mixture.	Final melting point.
0	104°3
20	99.8
40	95.8
60	90.3
70	87.4
80	84.6
90	83.4
100	83.8

If we represent the numbers graphically, we get the following figures.



In their general form they correspond entirely with the right halves of fig. 3 and 4, page 312. In fig. I we remark however that the melting line for the racemic compound extends very consi-

derably so as to run as far as the concentration 3% before meeting the melting line of the *d*-substance. This is connected with the flat course of the melting line near the melting point of the racemic compound, which points to a great measure of dissociation in the two active components in the liquid state.

In fig. II it is the melting line of the racemic compound that, owing to the lower melting point, has the smallest extension (final point near 86%) but this too has a very flat form near the melting point.

Physics. — “*On the deduction of the characteristic equation.*” By Prof. J. D. VAN DER WAALS — discussion with Prof. BOLTZMANN.

In the proceedings of the former meeting Prof. BOLTZMANN has inserted a communication, accompanied by a letter. In this letter Prof. BOLTZMANN has expressed the wish that his communication would give rise to a discussion. As the result for Prof. BOLTZMANN’s values of the coefficients of $\frac{b}{v}$, $\frac{b^2}{v^2}$ etc., which appear in the characteristic equation, differs from that which I have obtained, I consider the invitation to discussion as also addressed to me. And though as a rule I prefer to leave the discussion between different results, obtained in two different ways, to the gradual development and extension of our views, which is brought about in course of time, I will not abstain from complying with his request, in the hope that this discussion may be „useful to science.”

I am perfectly aware of the difficulties attending this discussion. Prof. BOLTZMANN’s „Vorlesungen” form a logical coherent whole, and the different results agree so well, that we may be sure that they contain a perfectly correct solution of the problem, as Prof. BOLTZMANN conceives it. On the other hand I am convinced, that also my solution, leaving same unsolved questions and some minor points out of account, gives a true explanation of the problem, as I see it. So I pass by the question whether in equation (11) p. 403

the factor of $2Gb$ is to be put as $1 - \frac{17}{32} \frac{b}{v} + \beta \frac{b^2}{v^2}$ or whether β must possibly be diminished by $\frac{2357}{18440}$, as this is of minor interest.

As our results differ so much, we cannot but have considered two different problems, and to get some certainty on this point, I

have looked more closely into one of Prof. BOLTZMANN's equations, viz. the equation l. c. pag. 169

$$\left(v_g - 2b + \frac{17}{16} \frac{b^2}{v_g} \right) e^{\frac{4hma}{v_g}} = \left(v_f - 2b + \frac{17}{16} \frac{b^2}{v_f} \right) e^{\frac{4hma}{v_f}}$$

which equation is to be applied, when two phases are in equilibrium.

It is well known from thermodynamics that when two phases of a simple substance coexist, besides p and T , the thermodynamic potential must also be equal. The preceding equation of Prof. BOLTZMANN must therefore be the kinetic interpretation of the thermodynamic relation

$$\mu_g = \mu_f$$

or

$$\epsilon_g - T\eta_g + pv_g = \epsilon_f - T\eta_f + pv_f .$$

In a paper (Verslag der Vergadering 26 Januari '95, also published Arch. Neerl., T. XXX), entitled "de kinetische beteekenis van de thermodynamische potentiaal" I have given the kinetic interpretation of this thermodynamic relation. For a fuller discussion I refer to this paper, and when I compare this result with the equation of Prof. BOLTZMANN for that equilibrium, it is evident that we have a different conception of the problem. I do not mean to say, that our results do not agree at a first approximation. But they are not identical.

In the first place there is the following difference. According to Prof. BOLTZMANN the energy required to remove one molecule from the liquid phasis is simply the work required to overcome the cohesion; according to me that work must be diminished by the quantity which I have called the work of the thermic pressure. In the second place Prof. BOLTZMANN subtracts from the specific volume a quantity twice as large as the quantity which I think must be subtracted from it.

In my opinion there is no doubt that also the work of the thermic pressure must be taken into account, if we ascribe real dimensions to the molecules. If a molecule leaves a phasis, not only the increase of its potential energy is to be taken into account, but we must also pay attention to the fact that the quantity from which the molecule has escaped, has diminished, that the surface of that quantity has contracted, and that therefore a quantity of work has

been performed, equal to the product of the thermic pressure and the volume which the molecule occupied in the phasis.

And if a priori we should not be convinced of this, it would appear as soon as the significance of $\mu = \varepsilon - T\eta + pv$ is examined.

The quantity, which must be equal for the two phases, we may denote

$$-\eta + \frac{\varepsilon + pv}{T}$$

or according to the notation of Mr. GIBBS:

$$-\eta + \frac{\chi}{T},$$

in which χ is the function which Mr. GIBBS has called „heat function for constant pressure” (Equilibrium of heterogeneous substances, p. 148).

To compare it with the equation of Prof. BOLTZMANN we put

$$-\eta + \frac{pv - \varepsilon}{T} + \frac{2\varepsilon}{T} \dots \dots \dots \quad (\text{V})$$

$$\text{If } \varepsilon = -\frac{a}{v}, \quad pv - \varepsilon = v \left(p + \frac{a}{v^2} \right).$$

The quantity $p + \frac{a}{v^2}$, the sum of the external and molecular pressure we call “thermic pressure.”

If we put $p + \frac{a}{v^2} = rTF$, not inquiring for the present into the form of F , the above formula becomes

$$-\eta + rFv + \frac{2\varepsilon}{T} = -\eta + \frac{rFvT + 2\varepsilon}{T}.$$

Even in this form we see that the quantity $rFTv$, which may be considered as work of the thermic pressure, is of the same nature as the quantity 2ε .

This is still more evident, if we put $F = \frac{1}{v-b}$ and substitute $rT + rT\frac{b}{v-b}$ for $r\frac{v}{v-b}T$.

We may namely diminish the quantity which is to be the same for both phases, by an arbitrary constant or function of the temperature, and we put then

$$-\eta + \frac{rTFb - \frac{2a}{v}}{T}$$

If

$$p + \frac{a}{v^2} = \frac{rT}{v-b}, \quad \text{then} \quad \left(\frac{d\eta}{dv}\right)_T = \frac{r}{v-b}$$

and

$$\eta = r \left\{ \log(v-b) + \int \frac{db}{v-b} \right\}$$

and the quantity which is to be equal for the two phases, may be given under this form :

$$\log(v-b) - \frac{rT \{ Fb - \int Fdb \} - \frac{2a}{v}}{rT}.$$

In my communication to which I have referred before, I have deduced this formula, starting from the idea that there exists a thermic pressure, which expels the molecule and is counteracted by the molecular attraction.

We may write the quantity which must be the same in the two phases in this way :

$$\eta - v \frac{\partial \eta}{\partial v} - \frac{2\varepsilon}{T}.$$

By comparing it with the formula of Prof. BOLTZMANN, we find that he substitutes $r \log \{ v - 2b \text{ etc.} \} + C$ for

$$\eta - v \frac{\partial \eta}{\partial v}.$$

If the formula under the symbol \log . was perfectly accurate, Prof. BOLTZMANN should be able to prove, that the entropy could

be calculated perfectly accurately from

$$\eta - v \frac{\partial \eta}{\partial v} = r \log(v - 2b \text{ etc.})$$

Approximatively there is equality, but I have not succeeded in proving that the entropy of Prof. BOLTZMANN is really identical with the quantity η of this relation.

These considerations have suggested the question to me, whether the problem, the accurate solution of which Prof. BOLTZMANN gives, might be the following: In what way do a great number of moving material points arrange themselves, when they are subjected to an attractive force, which leads to a surface pressure $\frac{a}{v^2}$, when they cannot come nearer to each other than at a certain distance (diameter of a molecule). If they are really material points, the work of the thermic pressure does not exist — and Prof. BOLTZMANN's equation for coexisting phases might be defended. But then the real problem, how do molecules with dimensions arrange themselves, remains unsolved.

But then there would be no reason to be astonished, that there is no perfect agreement in the results. I am sooner astonished that the agreement is as great as it is.

For a full discussion it would of course be necessary to compare also other equations of Prof. BOLTZMANN with those I have deduced. Only if this were done it would be possible to make clear the difference in the nature of our views. This would be specially desirable for the calculation in first approximation of the influence of the molecular dimensions on the value of the pressure. I.c. p. 7 etc. Here too, the result at which we arrive is only equal at a first approximation, without the results being identical.

And the question presents itself, whether the problem which is treated in BOLTZMANN's „Vorlesungen”, might be formulated still more accurately than has been done above, by saying: A great number of moving material points move in spaces which are diminished by eight times the molecular volume — a conception, according to which the material points would move in a space, which would lie outside the distance spheres, which they themselves form.

It is indeed remarkable that Prof. BOLTZMANN succeeds in preventing that in this way he would find an influence on the pressure which is twice too large. By assuming a perfectly flat wall, and by using this wall as a means to eliminate the distance spheres

on one side, he gets in first approximation the same result as I had obtained from the beginning. But only in first approximation.

All this is the cause that I still prefer the simple direct deduction of the pressure, as I have given it. (Proc. of the meeting Nov. 1898) — even though there remain questions whose solution I have tried to find already for some time, but in which I have not yet succeeded.

Physics. — „*The galvano-magnetic and thermo-magnetic phenomena in bismuth.*” (Second communication continued). By Dr. E. VAN EVERDINGEN JR. (Communication N°. 48 (continued) from the Physical Laboratory at Leiden, by Prof. H. KAMERLINGH ONNES).

6. In § 6 of my first communication on this subject¹⁾ I mentioned that in my opinion RIECKE wrongly ascribes the *entire* conduction of heat to the motion of charged particles, and that for the theory we might perhaps derive some profit from such suppositions as I have introduced in the theory of the HALL-effect in electrolytes.²⁾ It certainly is remarkable, that, starting from the supposition, that the current of heat in metals is a *pure* current of energy, *not* accompanied by a current of ponderable substance, we for all transverse galvano-magnetic and thermo-magnetic phenomena can offer at least a qualitative explanation. For, we have only to represent the matter in the following way:

In the *electric current* the positive and negative charged particles, which for the sake of brevity we shall call ions, move with different velocities, while also their numbers may differ.

In the magnetic field the electro-magnetic force causes transverse displacements and consequently charges which increase until the currents of positive and negative electricity have reached the same value. This equalisation may be due to the arising difference of potential (this is the HALL-effect) as well as to currents of diffusion of the ions (If the latter are accompanied by a perceptible difference in the number of ions per unit of volume in different places, the difference of potential to be observed at the HALL-electrodes might be

¹⁾ Proceedings Royal Acad. Amsterdam June 25th 1898 p. 72. Comm. Phys. Lab. Leiden N°. 42 p. 18.

²⁾ Proceedings Royal Acad. Amsterdam May 25th 1898 p. 27. Comm. Phys. Lab. Leiden N°. 41.

modified by an electromotive contact force). After the condition has become stationary, there will still exist a transverse current of ions, and so on one side free ions must be captivated, a process by which heat is generated, and on the other side new ions must be freed, which is accompanied by loss of heat; in this way a transverse difference of temperature (*galvano-magnetic difference of temperature*) arises; together with this a displacement of unelectrified particles must be brought about.

So far as the free ions are concerned, we must imagine the *current of heat* (now that no displacement of substance may be assumed) to be such, that although across a surface perpendicular to the direction of the current of heat an equal number of ions goes in both directions per unit of time, yet the ions have on an average a greater velocity during a displacement in the direction of the current of heat than they have during a displacement in the reverse direction. The electromagnetic force may then cause transverse displacements, which have opposite directions for positive and for negative ions. In the same way as with the electric current there will arise in general a difference of potential (*transverse thermo-magnetic effect*) and a difference of temperature (*rotation of isothermals*). The answer to the question in how far these suppositions might lead to a quantitative explanation must be left to further investigations.

7. It is important to inquire whether molecular theories of thermo-electricity enable us to determine the sign of the variation in the magnetic field of the thermo-electric difference, for instance between bismuth and copper, only by means of the hypothesis mentioned in the previous communication¹⁾: viz. that the longitudinal phenomena would be explained by a variation of the number of free ions, due to the magnetic field. This however appears not to be the case, as new suppositions must still be introduced.

As an instance I choose the formula for the electromotive contact force, lately published by RIECKE²⁾, which can be put down in the form:

$$V_B - V_A = C \left(\log \frac{P_A}{N_A^2} - \log \frac{P_B}{N_B^2} \right)$$

where the quantities with *A* relate say to copper, and those with

¹⁾ Proceedings Royal Acad. Amsterdam. March 25th. 1899, p. 416 Comm. Phys. Lab. Leiden No. 48 p. 20.

²⁾ Wied. Ann. 66, p. 554, 1898.

B to bismuth, and in which V represents the potential, C a positive function of temperature, P and N numbers of positive and negative ions per unit of volume.

The electric conductivity for bismuth has been represented by RIECKE by a formula of the form

$$\gamma = P_B c_p + N_B c_n$$

in which c_p and c_n are again functions of temperature. In the magnetic field the conductivity decreases. The simplest supposition is now that P_B and N_B decrease proportionally; a thing which would be self-evident if P_B were equal to N_B , as in electrolytes.

Let us further suppose, which is also very probable, that only the quantities relating to bismuth vary in the magnetic field, then a proportional decrease of P_B and N_B appears to lead to a variation of the difference $V_B - V_A$ with a negative amount, and so to an increase of the thermo-electric difference between bismuth and copper, which is negative outside the magnetic field. If on the contrary we should suppose that the variation of N_B is small as compared with that of P_B , then the variation of $V_B - V_A$ would become positive and the thermo-electric difference between bismuth and copper would decrease.

The observations show that the variation of this thermo-electric difference in the magnetic field corresponds to an increase, and so, in consequence of the proportionality of increase of resistance and longitudinal effect found by me, we should have to prefer the former of the two suppositions.

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