wave-lengths if Röntgen-rays are simple periodical disturbances:

for plate	A,	B,	C,	
$\lambda =$	0,16	0,05	0,12	$\mu\mu$.

Now that this supposition does certainly not hold, we shall have to consider these values as estimations of wave-lengths, which in the three different experiments have been more or less prominent in the curve of energy [1]) of the Röntgen-rays.

Mention ought to be made here, that, although not too much importance must be attached to the three values of λ as far as the absolute figures go, the difference they show is probably real and connected with the difference in hardness of the tubes. As was mentioned above the tubes used for plate B were distinguished by a considerable hardness from the others, which were relatively very soft.

Worth noticing is also the fact, that the values of λ found here are of the same order as those deduced from our former experiments.

Finally we wish to state emphatically that we continue to regard as the chief result of our investigations the proof they furnish that the Röntgen-rays ought to be considered as a phenomenon of radiation in the ether.

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Physics. — H. A. LORENTZ. "*The fundamental equations for electromagnetic phenomena in ponderable bodies, deduced from the theory of electrons.*"

§ 1. In framing a theory that seeks to explain all electromagnetic phenomena, in so far as they do not take place in free aether, by means of small charged particles, electrons, we have to start from two kinds of equations, one relating to the changes of state in the aether, the other determining the forces exerted by this medium on the electrons. To these formulae we have to add properly chosen assumptions concerning the electrons existing in dielectrics, conductors and magnetizable substances, and the forces with which the ponderable particles act on the electrons in these several cases.

In former applications of the theory I have restricted myself to

1) C. H. WIND. ll. cc.

the problem of the propagation of light in transparent substances, moving with a constant velocity. I shall now treat a more general case. I shall transform the original equations into a set of formulae, which, instead of quantities belonging to the individual electrons, contain only such as relate to the state of visible parts of the body and are therefore accessible to our observations. These formulae will hold for bodies of very different kinds, moving in any way we like.

The greater part of the results have already been established by POINCARÉ in the second edition of his *Electricité et Optique*. The mode of treatment is however rather different.

§ 2. With some exceptions, I shall use in the fundamental equations the same notation and the same units as on former occasions. The aether will again be supposed to remain at rest and to penetrate the charged particles; the equations of the electromagnetic field are therefore to be applied to the interior of the electrons, as well as to the spaces between them. We shall consider a distribution of the charges with a finite volume-density, whose value is a continuous function of the coordinates. If we speak of "electrons", we think of the charges as confined to certain small spaces, wholly separated from one another; however, in writing down our first equations, we may as well imagine a charge distributed over space in any arbitrary way.

We shall conceive the charges as being carried by "matter", though we might, if we chose, leave the latter out of consideration. We should then speak of the forces acting, not on charged matter, but on the charges themselves.

Let us call

ϱ the density of the charge,
$\mathfrak{v}$ the velocity of the charged matter,
$\mathfrak{d}$ the dielectric displacement, [1])
$\mathfrak{s}$ the current,
$\mathfrak{h}$ the magnetic force,
V the velocity of light.

Then we shall have

$$Div\, \mathfrak{d} = \varrho, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (I)$$

$$\frac{\partial \varrho}{\partial t} + Div\,(\varrho\, \mathfrak{v}) = 0\,, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (II)$$

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[1]) The dielectric displacement, the current and the magnetic force are here represented in small type, because we wish to keep in reserve large type for corresponding quantities which we shall have to introduce later on.

$$-\mathfrak{s} = \dot{\mathfrak{d}} + \varrho\,\mathfrak{v}, \quad \ldots\ldots\ldots \quad (1)$$

$$Rot\,\mathfrak{h} = 4\pi\mathfrak{s}, \quad \ldots\ldots\ldots \quad (III)$$

$$-4\pi V^2\,Rot\,\mathfrak{d} = \dot{\mathfrak{h}}, \quad \ldots\ldots\ldots \quad (IV)$$

$$Div\,\mathfrak{h} = 0, \quad \ldots\ldots\ldots \quad (V)$$

and the electric force $\mathfrak{f}$, i. e. the force acting on the charged matter per unit charge, will be given by

$$\mathfrak{f} = 4\pi V^2\,\mathfrak{d} + [\mathfrak{v}\,.\,\mathfrak{h}] \quad \ldots\ldots\ldots \quad (VI)$$

§ 3. If it were possible, by means of our observations, to penetrate into the molecular structure of a ponderable body, containing an immense number of charged particles, we should perceive within and between these an electromagnetic field, changing very rapidly and in most cases very irregularly from one point to another. This is the field to which the equations (I)—(V) must be applied, but it is not the field our observations reveal to us. Indeed, all observed phenomena depend on the mean state of things in spaces containing a very large number of particles; the proper mathematical expressions for such phenomena will therefore not contain the quantities themselves appearing in the formulae (I)—(V) but only their mean values. Of course, the dimensions of the space for which these values are to be taken, though very large as compared with the mutual distance of neighbouring particles, must at the same time be very much smaller than the distance over which one must travel in the body in order to observe a perceptible change in its state. We may express this by saying that the dimensions must be *physically infinitely small.*

Let P be any point in the body and σ a physically infinitely small closed surface of which it is the centre. Then we shall define the mean value at the point P of a scalar or vectorial quantity A by the equation

$$\overline{A} = \frac{1}{S}\int A\,d\tau, \quad \ldots\ldots\ldots \quad (2)$$

in which the integration has to be extended to all elements $d\tau$ of the space S, enclosed by σ. It is to be understood that, if we wish to calculate the mean value for different points P, P', the corresponding spaces S, S' are taken equal, of the same form and in the same position relatively to P, P'. The result $\overline{A}$ will depend on the coordinates of the point considered; however, the above mentioned rapid changes will have disappeared from it; it is only the slow changes from point to point, corresponding to the perceptible changes in the state of the body, that will have been preserved in the mean value.

It is easily seen that

$$\frac{\partial \overline{A}}{\partial x} = \overline{\frac{\partial A}{\partial x}}, \text{ etc.}, \qquad \frac{\partial \overline{A}}{\partial t} = \overline{\frac{\partial A}{\partial t}}.$$

Hence, if we take the mean values of every term in the equations (I)—(V) and (1), as we shall soon do, we may replace $\dot{\overline{\mathfrak{d}}}$ and $\overline{\mathfrak{h}}$ by $\overline{\dot{\mathfrak{d}}}$ and $\overline{\dot{\mathfrak{h}}}$, $\overline{Div\, \mathfrak{d}}$ by $Div\, \overline{\mathfrak{d}}$, etc.

§ 4. Before proceeding further, it is necessary to enter into some details concerning the charged particles we must suppose to exist in ponderable bodies.

Each of these particles calls forth in the surrounding aether a field, determined by the amount, the distribution and the motion of its charges, and it may be shown that, if x, y, z are the coordinates relatively to an origin O taken somewhere within the particle, and if the integrations are extended to the space occupied by it, the field, at distances that are large as compared with the dimensions of the particle, is determined by the values of the expressions

$$\int \varrho\, d\tau, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\int \varrho\, x\, d\tau, \qquad \int \varrho\, y\, d\tau, \qquad \int \varrho\, z\, d\tau, \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\int \varrho\, v_x\, d\tau, \qquad \int \varrho\, v_y\, d\tau, \qquad \int \varrho\, v_z\, d\tau, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\int \varrho\, v_x\, x\, d\tau, \qquad \int \varrho\, v_x\, y\, d\tau, \qquad \int \varrho\, v_x\, z\, d\tau, \text{ etc.}, \quad . \quad . \quad . \quad (6)$$

Now, we might conceive particles of such a nature that for each of them all these quantities had to be taken into consideration. For the sake of clearness, it will however be preferable to distinguish between different kinds of particles, the action of each of these kinds depending only on some of the integrals (3)—(6).

a. If the charge of a particle has the same algebraic sign in all its points, the actions corresponding to the integrals (3) and (5) will far surpass those that are due to (4) and (6); we may then leave out of consideration these latter integrals. Such particles, whose field is determined by their charge and their motion as a whole, may be called *conduction-electrons*. We shall imagine them to be crowded together at the surface of a charged conductor and to constitute by their motion the currents that may be generated in metallic wires.

b. In the second place, we shall consider particles having in one part of their volume or surface a positive and in another part an equal negative charge. In this case, for which a pair of equal and opposite electrons would be the most simple example, the surrounding field is due to (4) and (5). We shall say that a particle of this kind is *electrically polarized* and, denoting by r the vector drawn from the origin towards the element of volume $d\tau$, we shall call the vector

$$\int \varrho\, \mathfrak{r}\, d\tau = \mathfrak{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

the *electric moment* of the particle. In virtue of the supposition

$$\int \varrho\, d\tau = 0,$$

this vector is independent of the position of the origin of coordinates. From (7) we may infer immediately

$$\int \varrho\, x\, d\tau = \mathfrak{p}_x, \text{ etc.} \quad , \quad \int \varrho\, v_x\, d\tau = \dot{\mathfrak{p}}_x, \text{ etc.}$$

In all dielectrics, and perhaps in conductors as well, we must admit the existence of particles that may be electrically polarized. We shall refer to their charges by the name of *polarization-electrons.*

c. Finally, let there be a class of particles whose field is solely due to the expressions (6), the integrals (3), (4), (5) being all 0. If we suppose the values of

$$\int \varrho\, x^2\, d\tau, \quad \int \varrho\, x\, y\, d\tau, \quad \int \varrho\, x\, z\, d\tau, \quad \text{etc.}$$

not to vary in the course of time, we can express all the integrals (6) by means of the vector

$$\frac{1}{2}\int \varrho\, [\mathfrak{r} \,.\, v]\, d\tau = \mathfrak{m}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

i. e. of the vector whose components are

$$\mathfrak{m}_x = \frac{1}{2}\int \varrho\, (y\, v_z - z\, v_y)\, d\tau, \text{ etc.},$$

Indeed, we shall have

$$\int \varrho\, v_x\, x\, d\tau = 0, \quad \int \varrho\, v_x\, y\, d\tau = -\mathfrak{m}_z, \quad \int \varrho\, v_x\, z\, d\tau = +\mathfrak{m}_y, \text{ etc.} \quad (9)$$

The field produced by a particle satisfying the above conditions may be shown to be identical to the field due to a small magnet whose moment is $\mathfrak{m}$. For this reason, we shall speak of a *magnetized particle* and we shall call $\mathfrak{m}$ its *magnetic moment.*

According to the view here adopted, this moment is caused by rotating or circulating motions of the charges within the particle, similar to AMPÈRE's molecular electric currents. If, for the product ev of a charge e and its velocity, we introduce the name of "quantity of motion of the charge", the integral in (8) may be said to represent the moment about the origin O of the quantities of motion of all the charges present.

A very simple example is furnished by a spherical shell, rotating round a diameter, and enclosing an immovable, concentric sphere, the shell and the sphere having equal and opposite charges, uniformly distributed.

Whatever be the motion of the charges which call forth the moment $\mathfrak{m}$, we may properly apply to them the denomination of *magnetization-electrons*.

§ 5. In the determination of the mean values of the quantities in (I), (II) and (1), the following considerations and theorems will be found of use.

a. Consider a space containing an immense number of points Q, whose mutual distances are of the same order of magnitude as those between the particles of a ponderable body. Let N be the number of these points per unity of volume. If the density of the distribution gradually changes from point to point — in a similar way as may be the case with the observed density of a body — the value of N belonging to a point P is understood to be derived from the number of points Q lying within a physically infinitely small space of which P is the centre.

Draw equal and parallel vectors $QR = \mathfrak{r}$ from all the points Q, and consider a physically infinitely small plane $d\sigma$ whose normal, drawn towards one of its sides, is n. The question is to find the number of the vectors QR that are intersected by the plane, a number which we shall call positive if the ends of the vectors, and negative if their starting points lie on the side of $d\sigma$ indicated by n.

If N has the same value throughout the whole space, and if the points Q are *irregularly* distributed, like the molecules of a liquid or a gas, the number in question will be the same for all equal and parallel planes, whence it is easily found to be

$$N \mathfrak{r}_n \, d\sigma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The problem is somewhat less simple if the points Q have a regular geometric arrangement, such as those one considers in the theory of the structure of crystals. If, in this case, the length of the vectors QR is smaller than the mutual distance δ of neigh-

bouring points, it may come to pass that there are a certain number of intersections with one plane $d\sigma$ and none at all with another plane of the same direction. We shall meet this difficulty by irregularly undulating the element of surface, in such a way that the distances of its points from a plane $d\sigma$ are of the same order of magnitude as the distance δ, and that the direction of the normal is very near that of the normal n to this plane; so that the extent of the element and the normal to it may still be denoted by $d\sigma$ and n. It is clear that, if N is a constant, the number of intersections of the vectors $Q\,R$ with such an undulated element may again be said to depend only on its direction and magnitude, and that it may still be represented by the formula (10).

The same formula will hold in case the value of N should slowly change from point to point, provided we take for N the value belonging to the centre of gravity of the element.

b. Let us apply the above result to the elements $d\sigma$ of a closed surface σ. Let n_1 be the number of ends R, and n_2 the number of starting points Q lying within σ.

Supposing the normal n to be drawn in the outward direction, we may write for the difference of these numbers

$$n_1 - n_2 = -\int N v_n \, d\sigma, \quad \ldots\ldots\ldots \quad (11)$$

an expression, which of course can only be different from 0, if N changes from point to point.

c. Leaving the system of points, we pass to a set of innumerable equal particles, distributed over the space considered. Let q be a scalar quantity, whose values in the points $A_1, A_2, \ldots A_k$ of one of the particles are $q_1, q_2, \ldots q_k$, the position of these points and the values of q being the same in all particles, and these values being such that

$$q_1 + q_2 + \ldots + q_k = 0 \quad \ldots\ldots\ldots \quad (12)$$

We proceed to determine the sum Σq of the values q, belonging to all the points A that lie within the above mentioned closed surface σ. Of course, the particles lying completely within the surface will contribute nothing to this sum. Yet, it may be different from 0, because a certain number of particles are cut in two by the surface, so that only a part of the values $q_1, q_2, \ldots q_k$ belonging to each of these are to be taken into account.

Assume in each particle an origin O (having the same position in each) and regard this as composed of k points $O_1, O_2, \ldots O_k$. Attach to these the values $-q_1, -q_2, \ldots -q_k$. Then, in virtue

of (12), we may, without changing the sum Σq, include in it not only the points A, but likewise the points O. Now, if the vectors $O_1 A_1$, $O_2 A_2$, . . . $O_k A_k$ are denoted by $\mathfrak{r}_1$, $\mathfrak{r}_2$, . . . $\mathfrak{r}_k$, the part of Σq due to the points O_1 and A_1 will be

$$-\int N q_1 \mathfrak{r}_{1n} \, d\sigma,$$

as may easily be inferred from (11). There are similar expressions for the parts of the sum corresponding to O_2 and A_2, O_3 and A_3, etc. Hence, if we introduce for a single particle the vector

$$\mathfrak{q} = \Sigma q \mathfrak{r} \, . \, . \, . \, . \, . \, . \, . \, . \, . \, . \quad (13)$$

and if we put

$$N \mathfrak{q} = \mathfrak{Q}, \, . \, . \, . \, . \, . \, . \, . \, . \, . \, . \quad (14)$$

the final result will be

$$-\int \mathfrak{Q}_n \, d\sigma \, . \, . \, . \, . \, . \, . \, . \, . \, . \quad (15)$$

In this formula, the vector $\mathfrak{Q}$ is to be considered as a function of the coordinates because the number N may gradually change from one point to another (this §, *a*) and the vector $\mathfrak{q}$ may vary in a similar way. If now the surface σ is taken physically infinitely small, though of so large dimensions that it may be divided into elements, each of which is large in comparison with molecular dimensions, the expression (15) may, by a known theorem, be replaced by

$$-Div \, \mathfrak{Q} . S, \, . \, . \, . \, . \, . \, . \, . \, . \, . \quad (16)$$

S being the space within the surface σ.

d. It has been assumed till now that the quantity q occurs only in a limited number of points within each particle. By indefinitely increasing this number k, we obtain the case of a quantity q continuously distributed. We shall then write $q d\tau$ instead of q, and replace the sums by integrals. The condition (12) becomes

$$\int q \, d\tau = 0,$$

which we shall suppose to be fulfilled for each separate particle, the vector $\mathfrak{q}$ is now to be defined by the equation

$$\mathfrak{q} = \int q \mathfrak{r} \, d\tau, \, . \, . \, . \, . \, . \, . \, . \, . \, . \quad (17)$$

and the sum Σq, whose value we have calculated, becomes $\int q \, d\tau$, taken for the space enclosed bij σ. If we still understand by $\mathfrak{Q}$ the vector given by (14), the value of the integral for a physically infinitely small space will be

$$= -Div\ \mathfrak{Q}.\ S.$$

Now, according to the definition of mean values (§ 3), division of this by S will give the mean of q; hence

$$\overline{q} = -Div\ \mathfrak{Q}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

This result may even be extended to a state of the body, in which the distribution of the values of q is not the same in neighbouring particles. In this case we may again apply to each particle the formula (17), but $\mathfrak{Q}$ can no longer be calculated by (14). We have now to define this vector by

$$\mathfrak{Q} = \frac{1}{S} \Sigma\, \mathfrak{q}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

the sum being taken for all the particles that lie wholly in the space S, without attending to those that are cut in two by the surface. We may express this in words by saying that $\mathfrak{Q}$ is the sum of all the vectors $\mathfrak{q}$, reckoned per unit of volume.

e. The case still remains that a quantity q, given for every point, has such values that the integral $(q) = \int q d\tau$, taken for a single particle, is not 0. If this quantity were constant throughout the space occupied by a particle, it would be unnecessary to take into account those which are cut in two by the surface σ and we should have

$$\overline{q} = N(q).$$

The most general case may be reduced in the following way to this case and to those that have already been disposed of before. If q is distributed in some arbitrary manner, we begin by calculating for a single particle the mean value $q_1 = \frac{1}{s}(q)$, s being the volume of the particle, and we put in every point $q - q_1 = q_2$. We shall then have

$$\overline{q} = \overline{q_1} + \overline{q_2}.$$

The problem is therefore reduced to the determination of two mean values, one of which may be found by what has just been said, and the other by applying the formula (18).

§ 6. The mean value of each of the quantities ϱ and $\varrho\, \mathfrak{v}$ in the equations (I), (II) and (1) may be decomposed into three parts, belonging to the conduction-electrons, the polarization-electrons and the magnetization-electrons. In determining them, we shall suppose the ponderable matter to have a visible motion with velocity $\mathfrak{w}$, and we shall write $\mathfrak{v}$ for the velocity the charged matter may have in addition to this. We have therefore to replace $\mathfrak{v}$ by $\mathfrak{w} + \mathfrak{v}$, and to determine separately $\overline{\varrho\, \mathfrak{w}}$ and $\overline{\varrho\, \mathfrak{v}}$.

a. Conduction-electrons. The mean value of ϱ, in so far as it depends on these, may be called the (*measurable*) *density of electric charge;* we shall denote it by ϱ_1.

The mean value $\mathfrak{C}$ of $\varrho\,\mathfrak{w}$ may be represented by

$$\mathfrak{C} = \varrho_1\,\mathfrak{w}.$$

This is the *convection-current,* and the vector

$$\mathfrak{J} = \overline{\varrho\mathfrak{v}},$$

taken for the conduction-electrons, may fitly be called the *conduction-current.*

b. Polarization-electrons. Let the body contain innumerable particles electrically polarized, each having an electric moment $\mathfrak{p}$. The vector defined by the equation

$$\mathfrak{P} = \frac{1}{S}\,\Sigma\,\mathfrak{p}, \quad \ldots\ldots\ldots \quad (20)$$

where the sign Σ is to be understood in the same sense as in the formula (19), is the *electric moment for unit volume* or the *electric polarization* of the body. Replacing q by ϱ in the formulae of § 5, *d*, and taking into account (7), we find for the part of ϱ that is due to the polarization-electrons,

$$\varrho_2 = -\,Div\,\mathfrak{P}.$$

We may next remark that the visible velocity $\mathfrak{w}$ is practically the same in all points of a particle. Since, for the space occupied by it,

$$\int \varrho\,d\tau = 0,$$

we have likewise

$$\int \varrho\,\mathfrak{w}_x\,d\tau = \int \varrho\,\mathfrak{w}_y\,d\tau = \int \varrho\,\mathfrak{w}_z\,d\tau = 0,$$

so that the values of $\overline{\varrho\,\mathfrak{w}_x}$, $\overline{\varrho\,\mathfrak{w}_y}$, $\overline{\varrho\,\mathfrak{w}_z}$ may be found by means of (18). The result is

$$\overline{\varrho\,\mathfrak{w}_x} = -\,Div\,(\mathfrak{w}_x\,\mathfrak{P}), \text{ etc.} \quad \ldots\ldots \quad (21)$$

We have finally to determine $\overline{\varrho\,\mathfrak{v}}$. Now, the quantities $\varrho\mathfrak{v}_x$, $\varrho\mathfrak{v}_y$, $\varrho\mathfrak{v}_z$ are of the kind considered at the end of § 5, *e*. However, there are cases, especially if the velocities $\mathfrak{v}_x$, $\mathfrak{v}_y$, $\mathfrak{v}_z$ and the dimensions of the particles are sufficiently small, in which the parts of $\overline{\varrho\mathfrak{v}_x}$, $\overline{\varrho\mathfrak{v}_y}$, $\overline{\varrho\mathfrak{v}_z}$ corresponding to $\overline{q_2}$ of § 5, *e*, may be neglected. Confining ourselves to such cases, we shall determine $\overline{\varrho\mathfrak{v}}$ without taking into consideration the particles intersected by the surface σ.

For a single particle we may write

$$\int \varrho\, \mathfrak{v}\, d\tau = \frac{d\mathfrak{P}}{dt},$$

and for a physically infinitely small space, partaking of the visible motion

$$\int \varrho\, \mathfrak{v}\, d\tau = \frac{d}{dt} \Sigma\, \mathfrak{p}.$$

On account of (20) this is equal to

$$\frac{d}{dt}(\mathrm{S}\,\mathfrak{P}),$$

so that

$$\overline{\varrho\, \mathfrak{v}} = \frac{1}{\mathrm{S}} \int \varrho\, \mathfrak{v}\, d\tau = \frac{1}{\mathrm{S}} \frac{d}{dt}(\mathrm{S}\,\mathfrak{P}).$$

In performing the differentiation we must attend to the change of $\mathfrak{P}$ in a point that moves with the velocity $\mathfrak{w}$. If $\dot{\mathfrak{P}}$ relates to a fixed point of space, we have

$$\frac{d\mathfrak{P}}{dt} = \dot{\mathfrak{P}} + \mathfrak{w}_x \frac{\partial \mathfrak{P}}{\partial x} + \mathfrak{w}_y \frac{\partial \mathfrak{P}}{\partial y} + \mathfrak{w}_z \frac{\partial \mathfrak{P}}{\partial z},$$

and, since

$$\frac{d\mathrm{S}}{dt} = \mathrm{S}\,.\,Div\, \mathfrak{w}\,,$$

$$\overline{\varrho \mathfrak{v}} = \dot{\mathfrak{P}} + \mathfrak{w}_x \frac{\partial \mathfrak{P}}{\partial x} + \mathfrak{w}_y \frac{\partial \mathfrak{P}}{\partial y} + \mathfrak{w}_z \frac{\partial \mathfrak{P}}{\partial z} + \mathfrak{P}\, Div\, \mathfrak{w}.$$

Combining this with (21), we get for the mean value of the current corresponding to the motion of the polarization-electrons

$$\dot{\mathfrak{P}} + Rot\,[\mathfrak{P}\,.\,\mathfrak{w}].$$

c. *Magnetization-electrons.* If the body contains magnetized particles (§ 4, *c*), we have nothing to add to $\overline{\varrho}$ and $\overline{\varrho \mathfrak{w}}$. There will however be a new part of $\overline{\varrho \mathfrak{v}}$. We can calculate it by applying (18), because the quantities (5) vanish for every particle.

Let us first replace, in the formulae of § 5 *d*, q by $\varrho \mathfrak{v}_x$. We then find

$$q_\iota = 0, \qquad q_\eta = -\mathfrak{m}_z, \qquad q_\zeta = +\mathfrak{m}_\eta,$$

and, if we denote by $\mathfrak{M}$ the *magnetic moment for unit volume* or the *magnetization*, a vector that is to be defined in a similar way as $\mathfrak{P}$,

$$\Omega_\iota = 0, \qquad \Omega_\eta = -\mathfrak{M}_z, \qquad \Omega_\zeta = +\mathfrak{M}_\eta.$$

Finally, by (18),

$$\overline{\varrho \mathfrak{v}_\iota} = -\frac{\partial \mathfrak{M}_z}{\partial y} - \frac{\partial \mathfrak{M}_\eta}{\partial z},$$

with similar expressions for $\varrho \mathfrak{v}_\eta$ and $\varrho \mathfrak{v}_\zeta$.

The mean value of the current, in so far as it is due to the magnetization-electrons, is therefore

$$Rot\ \mathfrak{M}.$$

It may be called the *current equivalent to the magnetization.*

§ 7. It remains to take together the different parts of the second member of (1). Putting

$$\mathfrak{D} = \overline{\mathfrak{d}} + \mathfrak{P}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

$$\mathfrak{B} = \dot{\mathfrak{D}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

and

$$\mathfrak{N} = Rot\,[\mathfrak{P} \,.\, \mathfrak{w}], \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

we have

$$\overline{\mathfrak{s}} = \mathfrak{B} + \mathfrak{J} + \mathfrak{C} + \mathfrak{N} + Rot\,\mathfrak{M}.$$

Now, we might understand by the *current in the ponderable body* the whole of this vector. Conformly to general usage we shall however exclude from it the last term. We therefore define the current as the vector

$$\mathfrak{S} = \mathfrak{B} + \mathfrak{J} + \mathfrak{C} + \mathfrak{N}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

so that

$$\overline{\mathfrak{s}} = \mathfrak{S} + Rot\,\mathfrak{M} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

We may call $\mathfrak{D}$ the *dielectric displacement in the ponderable body*, and $\mathfrak{B}$ the *displacement-current.* As to the total current $\mathfrak{S}$, the formula (25) shows that it is composed of the displacement-current, the conduction-current $\mathfrak{J}$, the convection-current $\mathfrak{C}$ and the fourth vector $\mathfrak{N}$, for which POINCARÉ has proposed the name of *Röntgen-current*, because its electromagnetic effects have been observed in a well-known experiment of RONTGEN.

§ 8. We shall now write down the equations that arise from (I)—(V) and (1) if every term is replaced by its mean value. In order to obtain these formulae in a usual form, we shall put

$$\overline{\mathfrak{h}} = \mathfrak{B}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

$$\overline{\mathfrak{h}} - 4\pi\mathfrak{M} = \mathfrak{H}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

$$4\pi V^2 \overline{\mathfrak{d}} = \mathfrak{E}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

these quantities being the *magnetic induction*, the *magnetic force in the ponderable body* and the *electric force in the body.*

Beginning with the equation (I), and writing ϱ instead of ϱ_1 for the (measured) density of electric charge, we find

$$Div\,\overline{\mathfrak{d}} = \varrho - Div\,\mathfrak{P},$$

whence

$$Div\,\mathfrak{D} = \varrho \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (I')$$

We may further deduce from (1), taking into account (I) and (II),

$$Div\, \mathfrak{s} = 0,$$

and consequently $$Div\, \overline{\mathfrak{s}} = 0.$$

Now the expression $Rot\, \mathfrak{M}$ we have found for the current that is equivalent to the magnetization, shows immediately that the distribution of this current, taken by itself, is solenoïdal. We conclude from this that

$$Div\, \mathfrak{S} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (II')$$

From (III) we may deduce, if we introduce the value (26),

$$Rot\, \mathfrak{B} = 4\pi\, \mathfrak{S} + 4\pi\, Rot\, \mathfrak{M},$$

or, taking into account the relation

$$\mathfrak{B} = \mathfrak{H} + 4\pi\, \mathfrak{M}$$

which results from (27) and (28),

$$Rot\, \mathfrak{H} = 4\pi\, \mathfrak{S} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (III')$$

Finally we find by (IV)

$$Rot\, \mathfrak{E} = -\dot{\mathfrak{H}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (IV')$$

and by (V)

$$Div\, \mathfrak{B} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (V')$$

We have thus been led back to the equations of the electromagnetic field in a form that has long been known. In this form we may use them without even thinking of the individual electrons. As soon however as we seek to penetrate into the mecanism producing the phenomena, we must keep in mind the definitions that have been given of the different quantities appearing in the equations and the manner in which they are connected with the distribution and the motion of the elementary electric charges. The formulae (27) and (28) e. g. show the precise meaning that is to be attached in the theory of electrons to the terms "magnetic force" and "magnetic induction".

The equations (I')—(V') may be applied to all bodies indifferently. It is otherwise with the formulae expressing the relation between $\mathfrak{S}$ (or $\mathfrak{D}$) and $\mathfrak{E}$, and that between $\mathfrak{B}$ (or $\mathfrak{M}$) and $\mathfrak{H}$ [1]); the form of these depends entirely on the particular properties of the bodies considered. I shall not here discuss these more special formulae; in order to deduce them from the theory of electrons it is necessary to consider the forces acting on the electrons in a conductor, the "molecular motion" of these particles and the circumstances which determine the electric and magnetic moments of a single molecule or atom.

[1]) See Voigt, Electronenhypothese und Theorie des Magnetismus. Nachr. d. Ges. d. Wiss. zu Göttingen, 1901, Heft. 3.