Citation:

H. Kamerlingh Onnes & P. Ehrenfest, Simplyfied deduction of the formula from the theory of combinations which Planck uses as the basis of his radiation-theory, in: KNAW, Proceedings, 17 II, 1914, pp. 870-873
3. The temperature coefficient was small: 1.06—1.11 for 10°.
4. The velocity of the pinacone formation is greatly dependent on
   the alcohol; for instance, the methyl alcohol and the allyl
   alcohol were oxidised much more slowly than other primary
   and secondary alcohols.
5. The velocity of the pinacone formation is greatly dependent
   on the ketone, the benzophenone is attacked rapidly, most of
   the ketones as yet examined less rapidly, many not at all.
6. The ratio of these velocities in different alcohols is constant.
7. The active light of the ketone reduction is sure to be situated
   in the spectrum between 400 and 430 mμ and very probably
   in, or adjacent to, the rays 404.7 and 407.8 of the mercury
   quartz lamp.
8. The ratio of the velocities of the pinacone formation in sun­
   light and in mercury light is the same.
9. When two ketones are present simultaneously one of them
   absorbs a part of the rays required by the other ketone; this
   also appears when the light passes through a solution of the
   one ketone and falls on that of the other.
   Particularly in the case of the powerfully absorbing ketones
   the hindrances are stronger than was to be expected.

Delft, October 1914.

Physics. — "Simplified deduction of the formula from the theory
of combinations which PLANCK uses as the basis of his radiation­
theory." By Prof. P. EHRENFEIST and Prof. H. KAMERLINGH ONNES.
(Communicated in the meeting of Oct. 31, 1914).

We refer to the expression

\[ C^N_p = \frac{(N-1+P)!}{P!(N-1)!} \quad \ldots \quad (A) \]

which gives the number of ways in which \( N \) monochromatic reso­
nators \( R_1, R_2, \ldots R_N \) may be distributed over the various degrees
of energy, determined by the series of multiples \( 0, \varepsilon, 2\varepsilon \ldots \) of the
unit energy \( \varepsilon \), when the resonators together must each time contain
the given multiple \( Pe \). Two methods of distribution will be called
identical, and only then, when the first resonator in the one distri­
bution is at the same grade of energy as the same resonator in the
second and similarly the second, third, \ldots and the \( N \)th resonator
are each at the same energy-grades in the two distributions.

Taking a special example, we shall introduce a symbol for the
distribution. Let \( N = 4 \), and \( P = 7 \). One of the possible distributions
is the following: resonator $R_1$ has reached the energy-grade $4\varepsilon$ ($R_1$ contains the energy $4\varepsilon$), $R_2$, the grade $2\varepsilon$, $R_3$, the grade $0\varepsilon$ (contains no energy), $R_4$, the grade $\varepsilon$. Our symbol will, read from left to right, indicate the energy of $R_1$, $R_2$, $R_3$, $R_4$ in the distribution chosen, and particularly express, that the total energy is $7\varepsilon$. For this case the symbol will be:

$$\begin{array}{c}
\varepsilon, \varepsilon, \varepsilon, 0, \varepsilon, 0, 0, \varepsilon \\
\end{array}$$

or also more simply:

$$\begin{array}{c}
\varepsilon, \varepsilon, \varepsilon, 0, \varepsilon, 0, 0, \varepsilon \\
\end{array}$$

With general values of $N$ and $P$ the symbol will contain $P$ times the sign $\varepsilon$ and $(N-1)$ times the sign $0$. The question now is, how many different symbols for the distribution may be formed in the manner indicated above from the given number of $\varepsilon$ and $0$? The answer is:

$$\frac{(N-1+P)!}{P!(N-1)!}$$

Proof: first considering the $(N-1+P)$ elements $\varepsilon, \varepsilon, \varepsilon, 0, \ldots, 0$ as so many distinguishable entities, they may be arranged in different manners between the ends $\begin{array}{c}
\varepsilon, \varepsilon, \varepsilon, 0, \varepsilon, 0, 0, \varepsilon \\
\end{array}$. Next note, that each time of the combinations thus obtained give the same symbol for the distribution (and give the same energy-grade to each resonator), viz. all those combinations which are formed from each other by the permutation of the $P$ elements $\varepsilon$ or the $(N-1)$ elements $0$. The number of the different symbols for the distribution and that of the

1) We were led to the introduction of the $(N-1)$ partitions between the $N$ resonators, in trying to find an explanation of the form $(N-1)!$ in the denominator of (A) (compare note 1 on page 872). Planck proves, that the number of distributions must be equal to the number of all combinations with repetitions of $N$ elements of class $P$ and for the proof, that this number is given by the expression (A), he refers to the train of reasoning followed in treatises on combinations for this particular case. In these treatises the expression (A) is arrived at by the aid of the device of "transition from $n$ to $n+1"$, and this method taken as a whole does not give an insight into the origin of the final expression.

2) See appendix.
distributions themselves required is thus obtained by dividing (2) by (3) q. e. d. 1).

APPENDIX.

The contrast between Planck's hypothesis of the energy-grades and Einstein's hypothesis of energy-quanta.

The permutation of the elements $\varepsilon$ is a purely formal device, just as the permutation of the elements $\Theta$ is. More than once the analogous, equally formal device used by Planck, viz. distribution of $P$ energy-elements over $N$ resonators, has by a misunderstanding been given a physical interpretation, which is absolutely in conflict with Planck's radiation-formula and would lead to Wien's radiation formula.

As a matter of fact Planck's energy-elements were in that case almost entirely identified with Einstein's light-quanta and accordingly it was said, that the difference between Planck and Einstein consists here in that the latter assumes the existence of mutually independent energy-quanta also in empty space, the former only in the interior of matter, in the resonators. The confusion which underlies this view has been more than once pointed out 2). Einstein really considers $P$ similar quanta, existing independently of each other. He discusses for instance the case, that they distribute themselves irreversibly from a space of $N_1 \text{ cm}^3$ over a larger space of $N_2 \text{ cm}^3$ and he finds using Boltzmann's entropy-formula: $S = k \log W$, that this produces a gain of entropy 3):

$$ S - S_0 = k \log \left( \frac{N_2^P}{N_1^P} \right) \ldots \ldots \ldots \ldots (e) $$

1) It may be added, that the problem of the distribution of $N$ resonators over the energy-grades corresponds to the following: On a rod, whose length is a multiple $P\ell$ of a given length $\ell$, notches have been cut at distances $\varepsilon$, $2\varepsilon$, etc. from one of the ends. At each of the notches, and only there, the rod may be broken, the separate pieces may subsequently be joined together in arbitrary numbers and in arbitrary order, the rods thus obtained not being distinguishable from each other otherwise than by a possible difference in length. The question is, in how many different manners (comp. Appendix) the rod may be divided and the pieces distributed over a given number of boxes, to be distinguished from each other as the 1st, 2nd, ..., $N$th, when no box may contain more than one rod. If the boxes, which may be thought of as rectangular, are placed side by side in one line, they form together as it were an oblong drawer with $(N-1)$ partitions, formed of two walls each, (comp. the above symbol in its first form, from which the second form was derived by abstracting from the fact, that each multiple of $\varepsilon$ forms one whole each time), and these double partitions may be imagined to be mutually exchanged, the boxes themselves remaining where they are. The possibility of this exchange is indicated by the form of the symbol chosen.

As a further example corresponding to the symbol we may take a thread on which between $P$ beads of the same kind, $(N-1)$ beads of a different kind are strung, which divide the beads of the first kind in a 1st, 2nd, ..., $N$th group.
