Mathematics. — *On the construction of simple perfect squared squares.* By C. J. BOUWKAMP. (Communicated by Prof. J. G. VAN DER CORPUT.)

(Communicated at the meeting of November 29, 1947.)

First of all, I have to apologize for having misled the readers of these Proceedings with regard to the construction of a simple perfect squared square as given in "paper A" by the authors of the preceding note. Indeed, my criticism of their method has proved wholly unjustifiable. Therefore, I have to withdraw part of section 8 of my paper III. Nevertheless, the discussion there given, upon appropriate changes, remains of value. To make the following independent of all previous results, it is worth while here to reproduce figs 12 and 13 of my paper III.

Consider the "rotor" network of fig. 12, with terminals A_1, A_2, A_3. Let its wires have unit conductance, and let currents $87a$, $87b$ leave the network at A_2, A_3, respectively. The current entering at A_1 must then be $87\ (a+b)$. The complete set of currents is uniquely determined, and is shown in fig. 12. The currents are integral linear combinations of a and b. Without lack of generality, we may suppose a and b to be integers, subject to $0 < b < a$.

This network is a generalization of the "polar" networks treated before, in so far that now more than two terminals are present. It corresponds to a *squared polygon* of angles $\pi/2$ and $3\pi/2$. For example, the rotor network of fig. 12, in action, corresponds to a squared polygon P the dimensions of which are shown in fig. 13.

The vertical left side of the polygon may be considered as the terminal A_1, and the remaining vertical boundaries at the right correspond to A_3, A_2. The current flows horizontally from left to right. The ingoing current $87(a+b)$ is equal to the left vertical side, the two outgoing currents $87a$, $87b$ are equal to the other vertical boundaries.

The typical corner elements C_1, C_2 (shaded in fig. 13) have sides $27a—8b$, $8a+35b$, respectively. It must be noted that the situation of fig. 13 is possible only if $27a—8b < 49\ (a—b)$ and $8a+35b < 87b$; thus $41/22 < a/b < 13/2$. Otherwise at least one of the corner elements is too large. If the inequality above is not fulfilled, it is impossible to draw in fig. 13 the rectangle R which is important in the further construction.

If the skew-symmetrical rotor network of fig. 12 is replaced by its reflection (leaving the currents at A_1, A_2, A_3 invariant), the new squared polygon P' will have the same shape as the old one P; this follows from the triad symmetry of the rotor. The set of currents in the reflected rotor are easily found from those in fig. 12. We could also have interchanged

a and b, without reflecting the rotor; we prefer, however, the former method, in order to have always $a > b$ in the following.

Let us now re-consider fig. 13. By varying the ratio a/b we change the shape of the polygon P. The rectangle R becomes a square if and only if

$$87b — (8a+35b) = 49(a—b) — (27a—8b);$$

that is, if $10a = 31b$. Therefore, let us take $a = 31$, $b = 10$. It is then easily verified that of the elements of the polygons P and P' no two are equal. Moreover, the polygon P' can be brought into such position with respect to P that it overlaps P in the latter's corner elements $C_1 = 757$,

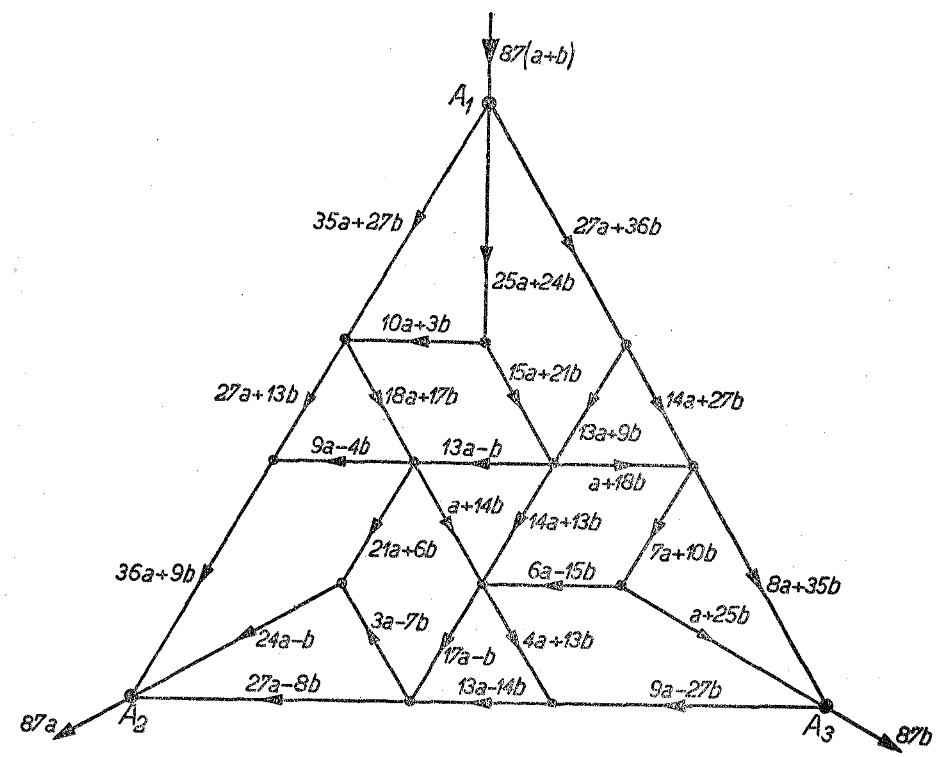

Fig. 12. Currents in a typical rotor network.

$C_2 = 598$. *Now, the important point is that these overlapped parts need not coincide with the corner elements of* P', which are of sides $27a—9b = 747$ and $9a+36b = 639$. Consequently, if the full-drawn parts of the boundaries of C_1 and C_2 are removed, the two systems of elements of P and P' fit together. The additional squares, 1901 and 1940, at the left-upper and the right-lower side, respectively, together with the square $R = 272$ at the middle then complete the square, which in fact is a *simple* perfect squared square of order 55. Upon turning the square over an angle π and codifying it, we obtain the solution given by the authors of the preceding note; the elements there distinguished by an asterisk are those of P'.

Attention may be drawn to a second solution, obtained when the roles of P and P' are interchanged. In that case the shape of P' is fixed by $a = 91$, $b = 10$ — in order that the corresponding rectangle R' shall be a square ($R' = 762$). The final result is a simple perfect squared square of code

(5739, 3555, 2860, 4022) (695, 2165) (2153, 2097) (1003, 3019)
(1152, 2016) (56, 1516, 525) (2209) (3433, 2306) (1677)
(1462, 3573) (749, 767) (81, 947, 649) (1307, 871, 762, 18) (866)
(1127, 1179) (2111) (109, 653) (436, 544) (1813) (2891, 1669)
(1617, 1305) (1197) (312, 2105, 85) (2020, 5562) (1222, 2376)
(4113) (583, 3542) (2959).

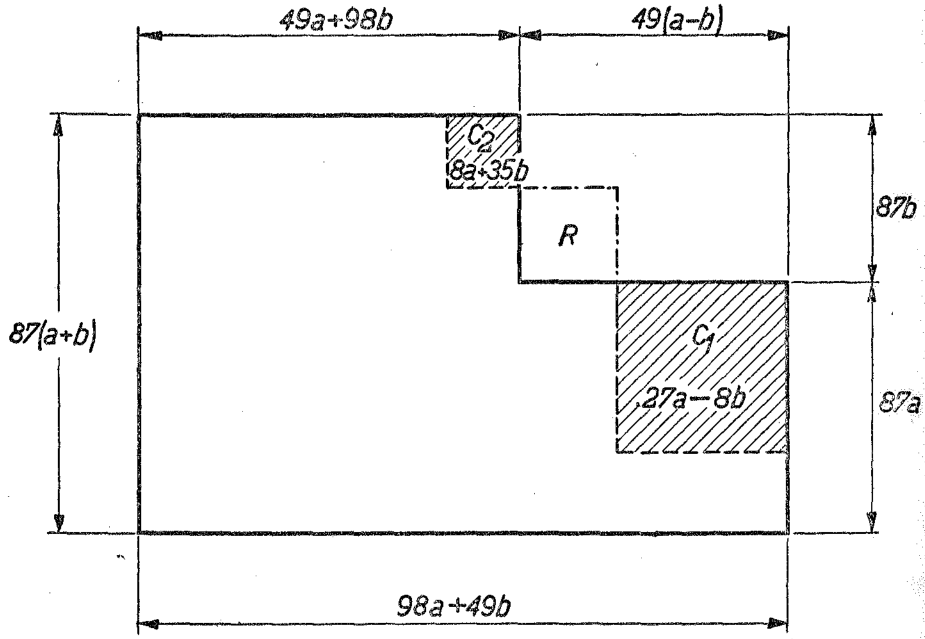

Fig. 13. Dimensions of the polygon corresponding to the rotor of fig. 12.

In conclusion, I am much indebted to Mr TUTTE and co-workers for calling attention to my incorrect interpretation of their construction. Moreover, Mr TUTTE kindly pointed out to me an error in my list of squared rectangles. The very last squaring of the list in my paper II ($C = 1176$) is unprimed as if it were perfect. Actually, it contains two equal elements of side 7. Consequently, there are not 214 but 213 simple perfect squared rectangles of order 13; this number is in complete agreement with Mr TUTTE's unpublished results, as he kindly informed me. In this connection table I of paper I should read as follows (of course, corresponding corrections have to be made in the text):

TABLE I.

Numbers of squared rectangles of different type, and of order less than 14. Only the trivial imperfections are excluded (the latter show equal elements lying aside, and thus belong to the compound type).

Type \ Order	9	10	11	12	13	Totaal
Simple, perfect	2	6	22	67	213	310
Simple, imperfect	1	0	0	9	34	44
Trivially compound, perfect	0	4	16	60	194	274
Non-trivially compound, perfect	0	0	0	0	1	1
Trivially comp., non-trivially imperfect	0	2	2	2	20	26
Non-triv. compound, non-triv. imperf.	0	0	0	0	2	2
Perfect	2	10	38	127	408	585
Non-trivially imperfect	1	2	2	11	56	72
Total	3	12	40	138	464	657

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