produced a clear indication *that the double bonds of unsaturated fatty acids may indeed be points of attack for the degradation of these acids in the living organism, and that also in this mode of degradation normal saturated dicarboxylic acids are formed as intermediate products, or at any rate may be formed in certain cases.* It becomes constantly more apparent that the latter acids play a very important part in fat metabolism. A further investigation into the catabolism of the unsaturated fatty acids is no doubt highly necessary.

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Astronomy. — *On the foundations of the theory of relativity, with special reference to the theory of the expanding universe.* By W. DE SITTER.

(Communicated at the meeting of October 27, 1934).

The restricted theory of relativity was based on the postulate of invariance of the laws of nature for LORENTZ-transformations, and the line-element of space-time accordingly was taken to be

$$ds^2 = -dx_1^2 - dx_2^2 - dx_3^2 + c^2\,dt^2 \; . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

In the general theory of relativity the invariance is postulated for *all* transformations of coordinates, and the line-element (1) is replaced by the more general riemannian form

$$ds^2 = g_{\alpha\beta}\,dx_\alpha\,dx_\beta\,, \qquad (\alpha, \beta = 1 \ldots 4) \quad (2)$$

with the restriction, however, that it shall be of index $+1$, and therefore *locally* reducible to the galilean form (1). In other words the fundamental hypothesis is:

I. The laws of nature are expressible as equations between tensors in a continuous four-dimensional manyfold, the line-element (2) of which is reducible to the form

$$ds^2 = -a^2\,d\sigma^2 + v^2\,dt^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

a and v being real functions of the four variables x_1, x_2, x_3, x_4, and

$$d\sigma^2 = \gamma_{pq}\,dx_p\,dx_q \qquad (p, q = 1, 2, 3)$$

being a positive definite quadratic form of the differentials of the three variables x_1, x_2, x_3.

In the popular expositions of the theory too much stress has as a rule been laid on the *relativity* of space and time, which is a consequence of this general invariance. It has even been proposed to replace the principle of invariance by other principles expressing this relativity, such as the so-called "cosmological principle", which asserts that statistically the world pictures of two different observers must be the same. We have, however, no means of communicating with other observers, situated on faraway stars, or moving with excessive velocities, such as that of a β-particle, and the building of world structures on the supposed experiences of these fictitious observers is equivalent to the introduction in disguise of certain specific assumptions regarding the interpretation of our own observations. The task to remove this disguise, and to ascertain what exactly these assumptions are, is not always an easy one. It seems better to introduce our assumptions explicitly and in clear language.

The hypothesis I is the necessary foundation of an intelligent description of nature. The question whether the postulated four-dimensionality and continuity are properties of the outside world, or of our way of describing it, can be left out of consideration for the moment. [As to the continuity, I may remark that we are here considering macroscopic phenomena only.] Also the possibility can perhaps not be excluded a priori that a further development of the theory with a view to make it embrace more phenomena, may lead to the adoption of a non-riemannian line-element. But, however this may be, we are at present entitled to postulate our "hypothesis" I, and to consider it simply as a precept how to introduce coordinates into our description of the world. It implies, of course, the axiom that the laws of nature are the same always and everywhere. In particular, with a view to the observations on which our knowledge of the expansion of the universe is based, the mechanism that produces the H and K lines of ionised calcium in a distant nebula is the same as in our laboratory.

The extrapolation outside our immediate neighbourhood is based on the following assumptions or hypotheses:

II. The tracks of material particles and photons (world lines) are geodesics in the four-dimensional continuum.

This again is not really an assumption regarding the nature of these tracks, but a precept for the construction of the space-time geometry by which we represent them.

III. In the universe considered on a large scale there is in every region of sufficient extent a definite *mean* motion, the deviations of the actual motions of the individual material particles from this mean being unsystematic.

WEYL has been the first to point out that some such postulate regarding the coherency of matter is unavoidable. If we adopt our hypothesis III, we can at each point choose the variable x_4 along the tangent to the geodesic representing this mean motion, and call this the "time" t, and we can choose the other variables so that the "spaces" $t = constant$ are orthogonal to this time. It should be emphasized that the "cosmic time" thus defined is not introduced by the observer or the theoriser, but depends on the actual state of motion of the matter in the universe. Space and time are therefore not mathematical concepts, which can be chosen at will, but are determined by the physical status of matter and energy according to the precepts I and II.

On the other hand this definition of "cosmic time" is a statistical one, and it consequently is only determined within certain limits of uncertainty, depending on the spread of the individual motions relative to the mean. It shares this statistical character with most physical quantities, like the temperature of a gas, etc. The time used in our physical theories, or in the planetary motions, is the proper time of the earth, or of the sun. The deviation of these different times from each other is negligibly small, and, according to our hypothesis III, their deviation from the "cosmic time" is also small.

In the universe considered on a large scale our hypothesis III naturally suggests the further hypothesis of three-dimensional isotropy, or stated explicitly:

IV. In any three-dimensional space $t = constant$ the spacial directions around any point are physically indistinguishable.

This, of course, like III, is a statistical postulate, valid only for the average distribution of matter over a large region, but not for the particular motions of individual particles, which are expressly excluded from consideration.

It can be proved [1]) that, if the hypotheses I to IV are true, the line-element is reducible to the form

$$ds^2 = -R^2\, d\sigma_k^2 + c^2\, dt^2, \quad \ldots \ldots \ldots \quad (4)$$

where $d\sigma_k$ is the line-element of a three-dimensional space of constant curvature ($k = +1$, 0 or -1), c being a constant and R a function of the time t alone (which might be a constant). It is thus seen that isotropy implies homogeneity.

The three-dimensional line-element can be written in the form

$$d\sigma_k^2 = d\chi^2 + b^2\,(d\psi^2 + \sin^2\psi\, d\vartheta^2), \ldots \ldots \ldots \quad (5)$$

where $b = \sin\chi$, χ, $\sinh\chi$ for $k = +1$, 0, -1 respectively. An observer interpreting his observations does, however, not use the radius vector χ of the unit space, but a galilean system of coordinates for his particular position in space and time, of which the radius vector is given by

$$dr = R\, d\chi.$$

[1]) See e. g. ROBERTSON, *Proc. Nat. Acad. Sci. Washington*, XV, p. 822, 1929. It should be remarked that strictly speaking it is sufficient to make the hypothesis IV for *one* space $t = t_0$, and for *two* of its points.

This is not a complete differential, and to define the radius vector, or distance, outside our immediate neighbourhood the path of integration must be prescribed, or the distance must be otherwise defined[1]). The distance that can be astronomically measured by the parallax, the angular diameter or the intensity of light, is in the three cases

$$k=1 \quad , \quad k=0 \quad , \quad k=-1$$
$$r=R\sin\chi \qquad r=R\chi \qquad r=R\sinh\chi.$$

The function R of t is not known. We put

$$h=\frac{1}{R}\frac{dR}{c\,dt}. \qquad (6)$$

This h is the coefficient of expansion of the universe. It also is an unknown function of the time.

It follows from (4) that the relation between the observed and emitted wavelengths, λ and λ_0 respectively, is

$$\frac{\lambda-\lambda_0}{\lambda_0}=\frac{R-R_0}{R_0}, \qquad (7)$$

which for small displacements gives, by DOPPLER's principle, the velocity-distance relation

$$\frac{V}{c}\sim r\,h. \qquad (8)$$

From the observations of the spectra of spiral nebulae it follows that at the present moment h is positive, and its value is approximately

$$h=\tfrac{1}{2}\,.\,10^{-27}\,cm^{-1}.$$

At present this is all that these observations can teach us, but it seems not impossible that with the increase of optical power of our telescopes we shall ultimately be able to see so far into space, or, which comes to the same thing, so far back into time, that it may become possible to determine also the differential quotient dh/cdt and the curvature $\varepsilon=k/R^2$.

It should be remarked that the distance-velocity relation (8) is thus derived from the hypotheses I to IV only, without making use of either the field-equations, or the concept of "velocity" as a change of coordinates.

The field-equations

$$G_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}G+\lambda g_{\mu\nu}+\varkappa T_{\mu\nu}=0 \qquad (9)$$

express the manner in which the metric of the physical universe, i.e. the metric tensor $g_{\mu\nu}$, is determined by the matter, i.e. the material tensor $T_{\mu\nu}$. The equations (9) are equivalent to the law of gravitation, and thus "gravitation" may be defined as that property of matter by which it determines the metric of time-space. It only becomes of importance when

[1]) See *B. A. N.* VII, No. 261, p. 205, 1934.

we consider the universe on a small scale, concentrating our attention on the motions of individual particles instead of on the statistical average of a large number of particles. By analogy we may define "inertia" as that property of matter which is expressed by the hypotheses I to IV (especially II), and we can thus say that *the expansion of the universe is a direct consequence of the law of inertia.*

This statement is, however, only qualitatively correct. Quantitatively the numerical values of the coefficient of expansion and its differential quotient are connected with the density ϱ and pressure p of matter (and radiation) by means of the field-equations, which give

$$\left.\begin{aligned} h^2 &= \tfrac{1}{3}(\lambda + \varkappa\varrho) - \varepsilon \\ \frac{dh}{c\,dt} &= \varepsilon - \tfrac{1}{2}\varkappa(\varrho + p). \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (10)$$

WEYL, and after him EDDINGTON, have emphasized the necessity that there must be woven into the structure of the world a standard of length (and time) making possible the comparison of lengths at different points in space-time. According to MACH's principle this natural standard must be of material origin. It appears to me that it is provided by the equations of conservation of mass (energy) and momentum

$$div\ T_{\alpha\beta} = 0,$$

which give

$$\frac{d\varrho}{c\,dt} + \frac{3}{R}\frac{dR}{c\,dt}(\varrho + p) = 0 \quad \ldots\ldots\ldots \quad (11)$$

In the simplest case (which is, however, a very good approximation in the actual world) that the pressure is neglected, the integral is

$$\varkappa\varrho = \frac{3R_1}{R^3} \quad \ldots\ldots\ldots\ldots \quad (12)$$

If the universe is finite ($k = +1$), so that it has a total mass, this mass is $M = 3\pi^2 R_1/\varkappa$. In other cases the integral is not so simple, but it still contains the same constant of integration R_1, which is still a measure of the material contents of the universe, and, since it has the dimension of a length, it is the natural unit of length and time. In empty universes, containing no matter, R_1 is zero: there is no natural material unit of length, and none is required. Using EDDINGTON's suggestive metaphor [1]) we can say that R_1 is the "real Gulliver", and evidently no Gulliver is required when there are no islands for him to visit. The discussion of empty universes is not a physical problem, but an exercise in geometry.

It should be pointed out that the equation (11) is also independent of the field-equations (9), and depends only on the assumption that the line-element has the form (4).

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1) *The expanding universe,* chapter IV, Cambridge, 1933.