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this experiment, not with garden soil for infection material, but by using the stomacal contents of such a case of stomacal sarcine. The "not cultivability" of DE BARY may mean the same as anaerobiosis, for it is well known how difficult it is, even at the present time, to cultivate anaerobics if the particulars of their life conditions are not exactly known.

For the rest I do not doubt of the precision of FALKENHEIM's[1] and MIGULA's[2] observations, wno have seen aerobic colonies of micrococci originate from stomacal sarcine. It is true that I for my part have not succeeded in confirming this observation with regard to the fermentation sarcine, but for other species of *Sarcina* I have, with certainty, stated the transition into micrococci, and with various anaerobics, although not belonging to the genus *Sarcina*, I have seen now and then colonies originate of facultative anaerobics, which in all other respects, corresponded to the obligative anaerobics used for the cultures. Therefore this modification also seems possible for some individuals of the fermentation sarcine. Accumulation or transfer experiments with stomacal contents will however only then give positive results, if these are used when still in fermentation; with long kept material nothing can be expected.

Already the older observers[3] as SCHLOSSBERGER (1847), SIMON (1849) and CRAMER (1858) have tried, although in vain, by a kind of accumulation experiments, to cultivate the stomacal sarcine, wherefore they prepared, as nutrient liquid, artificial gastric juice with different additions. Remarkable, and illustrating the biological views of those days, is the fact, that for the infection they did not use the stomacal contents themselves, but beer yeast, supposing, that the sarcine might originate from the yeast cells, which somewhat resemble it, and are always found in the stomach together with the sarcine itself.

Physics. — *"The motion of electrons in metallic bodies."* II. By Prof. H. A. LORENTZ.

(Communicated in the meeting of January 28, 1905).

§ 11. By a mode of reasoning similar to that used in the last §, we may deduce a formula for the intensity i of the current in a *closed* thermo-electric circuit. For this purpose we have only to suppose the ends P and Q, which consist, as has been said, of the

[1] Archiv f. experiment. Pathologie und Pharmacologie. Bd. 10, pg. 339, 1885.
[2] System der Bacteriën. Bd. 2, pg. 259, 1900.
[3] Cited from SURINGAR (l. c.).

40

same metal and are kept at the same temperature, to be brought in contact with each other. The potentials φ_P and φ_Q will then become equal, but the stream of electrons v will no longer be 0. We shall have on the contrary, denoting by Σ the normal section, which may slowly change from point to point, as has already been observed,

$$i = e\,v\Sigma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

Taking this into account and using (23), we get from (21) and (30)

$$\frac{d\varphi}{dx} = -\frac{1}{e}\frac{dV}{dx} - \frac{m}{e}\frac{d}{dx}\left(\frac{1}{h}\right) - \frac{m}{2eh}\frac{d\log A}{dx} - \frac{1}{\sigma\Sigma}i.$$

We shall integrate this along the circuit from P to Q. Since i has the same value everywhere and

$$\varphi_P = \varphi_Q \quad , \quad V_P = V_Q \quad , \quad h_P = h_Q,$$

we find

$$-\frac{m}{2e}\int_P^Q \frac{1}{h}\frac{d\log A}{dx}\,dx - i\int\frac{dx}{\sigma\Sigma} = 0.$$

Here, the first term is reduced to the form (34), if we integrate by parts. Hence, if we put

$$\int\frac{dx}{\sigma\Sigma} = R,$$

the result is

$$F = iR, \quad i = \frac{F}{R},$$

as was to be expected. Indeed, σ being the coefficient of conductivity, R is the resistance of the circuit.

§ 12. We shall now proceed to calculate the heat developed in a circuit in which there is an electric current i, or rather, supposing each element of the wire to be kept at a constant temperature by means of an external reservoir of heat, the amount of heat that is transferred to such a reservoir per unit time. Let us consider to this effect the part of the circuit lying between the sections whose positions are determined by x and $x + dx$ and let wdt be the work done, during the time dt, by the forces acting on the electrons in this element. $W\Sigma$ being the quantity of heat traversing a section per unit time, we may write

$$\frac{d}{dx}(W\Sigma)\,dx$$

for the difference between the quantities of heat leaving the element at one end and entering it at the other, and the production of heat is given by

$$q = w - \frac{d}{dx}(W\Sigma) dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

In order to determine w, we observe in the first place that the work done, during the time dt, by the force acting on a single electron is

$$m X \xi dt$$

and that, by the formula (1), the element Σdx contains

$$f(\xi, \eta, \zeta) \Sigma dx d\lambda$$

electrons having their velocity-points within the element $d\lambda$ of the diagram of velocities. Taking together the forces acting on all these particles, we find for their work

$$m X \Sigma dx dt . \xi f(\xi, \eta, \zeta) d\lambda ,$$

an expression that has yet to be integrated over the whole extent of the diagram. On account of (4), the result becomes

$$m X v \Sigma dx dt ,$$

so that, by (36)

$$w = \frac{miX}{e} dx .$$

Now, the value of X may be taken from (21). Substituting

$$v = \frac{i}{e\Sigma}$$

and using at the same time (23), we find

$$X = \frac{1}{2h} \frac{d \log A}{dx} + \frac{d}{dx}\left(\frac{1}{h}\right) + \frac{e i}{m\sigma\Sigma}, \quad . \quad . \quad . \quad (38)$$

so that

$$w = w_1 + w_2 ,$$

if we put

$$w_1 = \frac{mi}{e}\left[\frac{1}{2h} \frac{d \log A}{dx} + \frac{d}{dx}\left(\frac{1}{h}\right)\right] dx. \quad . \quad . \quad . \quad (39)$$

and

$$w_2 = \frac{i^2}{\sigma\Sigma} dx.$$

The expression (22) may likewise be transformed by introducing into it the value (38), or, what amounts to the same thing, the value of

$$2 h A X - \frac{dA}{dx},$$

that may be drawn from (21). One finds in this way

$$W = W_1 + W_2 ,$$

if

$$W_1 = \frac{2\pi m l A}{3h^4} \frac{dh}{dx} \quad \ldots \ldots \quad (40)$$

and

$$W_2 = \frac{m}{h} v = \frac{mi}{e \Sigma h}. \quad \ldots \ldots \quad (41)$$

§ 13. The expression (37) for the amount of heat produced in the element dx may now be divided into *three* parts.

The first of these

$$w_2 = \frac{i^2}{\sigma \Sigma} dx$$

corresponds to JOULE's law. Indeed $\frac{dx}{\sigma \Sigma}$ is the resistance of the part of the circuit extending from (x) to $(x + dx)$.

The second part

$$- \frac{d}{dx} (W_1 \Sigma) dx$$

is entirely independent of the current, as appears from (40). It may therefore be considered to be due to ordinary conduction of heat. This is confirmed by comparing it with what has been said in § 9.

It remains to consider the quantity of heat

$$q' = w_1 - \frac{d}{dx} (W_2 \Sigma) dx,$$

or, if (39) and (41) are taken into account,

$$q' = \frac{mi}{2eh} \frac{d \log A}{dx} dx.$$

This expression, proportional to the current and changing its sign if the latter is reversed, will lead us to formulae for the PELTIER-effect and the THOMSON-effect. Reduced to unit current, it becomes

$$q'_{i=1} = \frac{m}{2eh} \frac{d \log A}{dx} dx \quad . \quad \ldots \ldots \quad (42)$$

a. I shall suppose in the first place that, between two sections of the circuit, there is a gradual transition from the metal I to the metal II, the temperature and consequently h being the same throughout this part of the circuit. Then, reckoning x from the metal I towards II, and integrating (42), I find for the heat produced at the "place of contact" by a current of unit strength flowing from I towards II,

$$\frac{m}{2eh} \log \left(\frac{A_{II}}{A_I} \right) = \frac{2\alpha T}{3e} \log \left(\frac{A_{II}}{A_I} \right).$$

Hence, if we characterize the PELTIER-effect by the *absorption* of heat $\Pi_{I, II}$ taking place in this case,

$$\Pi_{I, II} = \frac{2 \alpha T}{3 e} \log \left(\frac{A_I}{A_{II}} \right)_I = \frac{2 \alpha T}{3 e} \log \left(\frac{N_I}{N_{II}} \right) \quad . \quad . \quad . \quad (43)$$

b. In the second place, substituting again for h the value (14), we shall apply (42) to a homogeneous part of the circuit. We have then to consider $\log A$ as a function of the temperature T, so that we may write

$$q'_{i=1} = \frac{2 \alpha T}{3 e} \frac{d \log A}{dT} dT$$

for the heat developed between two points kept at the temperatures T and $T + dT$, if a current of unit strength flows from the first point towards the latter. What KELVIN has called the "specific heat of electricity" (THOMSON-effect) is thus seen to be represented by

$$\mu = - \frac{2 \alpha T}{3 e} \frac{d \log A}{dT} \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

§ 14. An important feature of the above results is their agreement with those of the well known thermodynamic theory of thermo-electric currents. This theory leads to the relations

$$\mu_{II} - \mu_I = T \frac{d}{dT} \left(\frac{\Pi_{I, II}}{T} \right) \quad . \quad . \quad . \quad . \quad . \quad (45)$$

and

$$F = - \int_{T'}^{T''} \frac{\Pi_{I, II}}{T} dT, \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

in which μ_I and μ_{II} are the specific heats of electricity in the metals I and II, at the temperature T, whereas F denotes what we have calculated in § 10, viz. the electromotive force in a circuit composed of these metals and whose junctions are kept at the temperatures T' and T''', the force being reckoned positive if it tends to produce a current which flows from I towards II through the first junction.

The values (44), (43) and (35) are easily seen to satisfy the equations (45) and (46).

Instead of verifying this, we may as well infer directly from (42) that our results agree with what is required by the laws of thermo-dynamics. On account of the first of these we must have

$$\Sigma q'_{i=1} = - F$$

and by the second

$$\Sigma \frac{q'_{i=1}}{T} = 0,$$

the sums in these formulae relating to all elements of the closed circuit we have examined in § 11. Now, by (42), these formulae become

$$F = - \frac{m}{2e} \int_P^Q \frac{1}{h} \frac{d \log A}{dx} dx$$

and

$$\int_P^Q \frac{1}{hT} \frac{d \log A}{dx} dx = 0.$$

The first of these equations is identical with (34) and the second holds because hT has everywhere the same value.

It must also be noticed that the formula (35) implies the existence of a *thermo-electric series* and the well known law relating to it. This follows at once from the fact that the value (35) may be written as the difference of two integrals depending, for given temperatures of the junctions, the one on the properties of the first and the other on those of the second metal. Denoting by III a third metal, we may represent by $F_{I, II}$, $F_{II, III}$, $F_{III, I}$ the electromotive forces existing in circuits composed of the metals indicated by the indices, the junctions having in all these cases the temperatures T' and T''' and the positive direction being such that it leads through the junction at the first temperature from the metal indicated in the first towards that indicated in the second place. Then it is seen at once that

$$F_{I, II} + F_{II, III} + F_{III, I} = 0 \quad . \quad . \quad . \quad . \quad . \quad (47)$$

Strictly speaking there was no need to prove this, as it is a consequence of the thermodynamic equations and our results agree with these.

§ 15. In what precedes we have assumed a *single* kind of free electrons. Indeed, many observations on other classes of phenomena have shown the negative electrons to have a greater mobility than the positive ones, so that one feels inclined to ask in the first place to what extent the facts may be explained by a theory working with only *negative* free electrons.

Now, in examining this point, we have first of all to consider the absolute value of the electromotive force F. If we suppose the temperatures T'' and T''' to differ by one degree and if we neglect the

variability of N_I and N_{II} in so small an interval, we may write for (35)

$$F_{1°} = \frac{2\,\alpha}{3\,e} \, log \, \frac{N_{II}}{N_I},$$

$$log \, \frac{N_{II}}{N_I} = \frac{3\,e}{2\,\alpha} \cdot F_{1°}.$$

The value of the first factor on the righthand side may be taken from what, in § 9, we have deduced from the electrochemical aequivalent of hydrogen [1]). We found for $T = 291$

$$\frac{\alpha T}{e} = 38 \times 10^5,$$

so that

$$log \, \frac{N_{II}}{N_I} = 0,00011 \, F_{1°}.$$

In the case of bismuth and antimony, $F_{1°}$ amounts to 12000, corresponding to

$$log \, \frac{N_{II}}{N_I} = 1,32 \quad ; \quad \frac{N_{II}}{N_I} = 3,7$$

I see no difficulty in admitting this ratio between the number of free electrons in two metals wide apart from each other in the thermo-electric series [2]).

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[1]) The numbers of that § contain an error which, however, has no influence on the agreement that should be established by them. The value of $3\,p$ and that of $\frac{\alpha T}{e}$ deduced from the measurements of Jaeger and Diesselhorst are not 38 and 47, but

$$3\,p = 38 \times 10^5$$

and

$$\frac{\alpha T}{e} = 47 \times 10^5.$$

[2]) Let n be the mean value of $log \, \frac{N_{II}}{N_I}$ between the temperatures T' and T''. Then the equation (35) may be put in the form

$$Fe = \frac{2}{3} \, n \, \alpha \, (T'' - T').$$

This may be expressed as follows: The work done by the electromotive force in case one electron travels around the circuit is found if we multiply by $\frac{2}{3} n$ the increase of the mean kinetic energy of a gaseous molecule, due to an elevation of temperature from T' to T''.

The question now arises whether it will be possible to explain all observations in the domain of thermo-electricity by means of suitable assumptions concerning the number of free electrons. In order to form an opinion on this point, I shall suppose the PELTIER-effect to be known, at one definite temperature T_0, for all combinations of some standard metal with other metals and the THOMSON-effect to have been measured in all metals at all temperatures. Then, after having chosen arbitrarily the number N_0 of free electrons in the standard metal at T_0, we may deduce from (43) the corresponding values for the other conductors, and the equation (44) combined with (13) and (14), will serve to determine, for all metals, the value of N at any temperature we like. Now, the numbers obtained in this way, all of which contain N_0 as an indeterminate factor, will suffice to account for all other thermo-electric phenomena, at least if we take for granted that these phenomena obey the laws deduced from thermodynamics. Indeed, these laws leading to the relation

$$\Pi_{I, II} + \Pi_{II, III} + \Pi_{III, I} = 0,$$

similar to (47), the values of N we have assumed will account not only for the PELTIER-effect at the temperature T_0 for all metals combined with the standard metal, but also for the effect, at the same temperature, for any combination. Finally, we see from (45) that the value of $\Pi_{I, II}$ at any temperature may be found from that corresponding to T_0, if we know the THOMSON-effect for all intermediate temperatures and from (46) that the values of the electromotive force are determined by those of Π.

There is but one difficulty that might arise in this comparison of theory with experimental results; it might be that the assumptions we should have to make concerning the numbers N would prove incompatible with theoretical considerations of one kind or another about the causes which determine the number of free electrons.

As to the conductivities for heat and electricity, it would always be possible to obtain the right values from (24) and (27), provided only we make appropriate assumptions concerning the length l of the free path between two encounters [1]).

It must be noticed, however, that, whatever be the value of this length, the foregoing theory requires that the ratio $\dfrac{k}{\sigma}$ shall be the

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[1]) If the electric conductivity were inversely proportional to the absolute temperature, as it is approximately for some metals, and if we might neglect the variations of N, the formula (24) would require that l is inversely proportional to $\sqrt{T}$. I am unable to explain why N should vary in this way.

same for all metals. The rather large deviations from this law have led DRUDE to assume more than one kind of free electrons, an hypothesis we shall have to discuss in a sequel to this paper. For the moment I shall only observe that one reason for admitting the existence not only of negative but also of positive free electrons lies in the fact that the HALL-effect has not in all metals the same direction.

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(March 22, 1905).