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deviations do not exceed $\frac{1}{100}$, which corresponds to a difference of 0.004 ohms at the most.

The last measurements may also serve to control whether the results of the earlier ones were not fully accurate owing to the contact resistances. The deviations that I have just mentioned might arise from this source of error, but, as they go in different directions at $t = -78^{\circ}.6$ C. and $t = -193^{\circ}$ C. one is inclined to think that these deviations may originate in other errors too, for instance in variations of the temperature bath at $t = -78^{\circ}.6$ C. (solid carbonic acid and ether).

O. REICHENHEIM¹⁾ and J. KOENIGSBERGER²⁾ have examined pyrite from Val Giuf, Graubünden and have found a minimum of resistance at about $t = -10^{\circ}$ C. This pyrite has a specific resistance of 0.0240 at 0° C., thus eight times larger than mine. An explanation of this difference of the conductivity is given by J. KOENIGSBERGER³⁾.

My pyrite shows no minimum of resistance above -258° C. The resistance throughout the whole temperature interval follows the formula (1), which is the same, mathematically, as

$$\frac{1}{W} \frac{dW}{dt} = \text{const.}$$

It seems very probable that there does not exist any minimum below -258° C., but that the resistance at still lower temperatures approaches asymptotically to a limit value, as is the case in, for instance, not perfectly pure gold and platinum.

A. WESELY⁴⁾ has recently examined a pyrite crystal from the same place of origin, MalMBERGET, GELLIVARE. He found a still smaller specific resistance, $w_0 = 0.00247$ and a temperature coefficient at 0° C. of 0.00228.

Physics. — "*Investigation of the viscosity of gases at low temperatures. I. Hydrogen.*" By H. KAMERLINGH ONNES, C. DORSMAN and SOPHUS WEBER. Communication N^o. 134a from the Physical Laboratory at Leiden by H. KAMERLINGH ONNES.

§ 1. *Introduction.*⁵⁾ The investigation of the dependence of the viscosity of gases upon the temperature at densities near the normal,

¹⁾ O. REICHENHEIM, Inaug. Dissert. Freiburg 1906.

²⁾ J. KOENIGSBERGER, Jahrbuch der Rad. u. Elektr. 4, p. 169, 1907.

³⁾ J. KOENIGSBERGER, Phys. Zeitschr. 13, p. 282, 1912.

⁴⁾ A. WESELY, Phys. Zeitschr. 14, p. 78, 1913.

⁵⁾ This Comm. includes the paper on the same subject by KAMERLINGH ONNES and DORSMAN, which is referred to in Comm. Suppl. No. 25. (Sept. 1912) § 6, note 1.

is chiefly of importance for the knowledge of the mechanism of the impact of two molecules, or, more simply in monatomic gases, of two atoms. In the nature of the case it is desirable to extend this investigation with one substance over the largest possible range of reduced temperature. This gives a particular significance to very low temperatures and substances such as hydrogen, neon, and helium.

The pupils of DORN¹⁾ at Halle have made systematic researches into the viscosity of different gases. By these both absolute values and temperature coefficients have been determined, and they have gone as low as the temperature of liquid air.

In our researches we particularly wished to investigate hydrogen temperatures, while the viscosity apparatus was so arranged that it could be used without alteration for helium at helium temperatures.

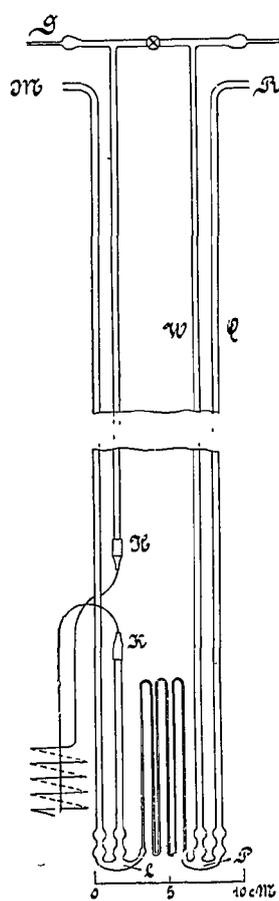


Fig. 1.

But it was natural for us to extend our investigation to the viscosity of our gases at less low temperatures. It then appeared that besides being of value for the confirmation of the above mentioned researches as far as the temperature of solid carbonic acid, it was also of value for the knowledge of viscosity in the field of the temperature of liquid air.

In the field of hydrogen temperatures we found the viscosity of hydrogen while flowing through a capillary tube dependent upon the mean pressure. From MAXWELL'S¹⁾ researches we know that the viscosity of gases at normal density is independent of the pressure, and WARBURG and VON BABO have shown in the investigation of carbon dioxide, that in dense vapours, it increases with the density. There is, therefore, every reason to further investigate the dependence of the viscosity upon the pressure in hydrogen vapour.

§ 2. *Method.* The measurements were made according to the transpiration method. This presents experimentally perhaps the greatest difficulties, but it seems to allow

¹⁾ There is a survey of these researches by K. SCHMITT. Ann. d. Phys. (30). p. 393, 1909.

²⁾ For the older literature see H. KAMERLINGH ONNES and W. H. KEESOM. Leiden Comm. Suppl. N^o. 23, page 86.

better than any other the fulfilment of the conditions which are assumed in the theoretical deduction.

The form which we choose (diagrammatically represented in fig. 1, compare further fig. 2) is distinguished by the following special features:

1. the pressure at both ends of the capillary tube through which the gas flows, can be kept constant as long as desired at any height.
2. the mean pressure and the difference of pressure are immediately measured at both ends of the capillary.
3. before it enters the capillary the gas flows through a copper tube (in our case 70 cm. long) where it acquires the desired temperature.

The calculation of the measurements got by the transpiration method was made by the formulas of O. E. MEYER and M. KNUDSEN¹⁾; for the amount of gas that passes through a capillary they give:

$$Q = \frac{\pi}{8} \cdot \frac{1}{\eta} (p_1 - p_2) \bar{p} \tau \frac{R^4}{L} \left(1 + \frac{4\zeta}{R} \right)$$

in which

$$\frac{\zeta}{\lambda} = 1.05 - \frac{1}{5 + \frac{\lambda}{R}} \quad \text{and} \quad \bar{p} \lambda = \sqrt{\frac{\pi}{8} \cdot \frac{1}{0.30967} \cdot \frac{\eta}{\rho_1}}$$

η = coefficient of viscosity,

R = radius of the capillary.

L = length of the capillary.

τ = time of flow.

$\bar{p} = \frac{p_1 + p_2}{2}$ = mean pressure.

p_1 = pressure at beginning of capillary.

p_2 = pressure at end of capillary.

Q = the quantity that has flowed through, measured by the product of volume and pressure, and corrected for the temperature of the capillary.

ζ = the gliding coefficient which is determined by the two last equations, in which ρ_1 is the density of the gas.

The units are those of the C. G. S. system.

§ 3. *Arrangement of the apparatus.* The manner in which the various quantities in these formulas were determined in the measurement, will easily be understood with the help of fig. 2.

¹⁾ M. KNUDSEN: Ann. d. Phys. 28, 1909. p. 75.

The pure gas ¹⁾ from a store cylinder is first let into a pipette *P*, in which it can be brought to a suitable pressure by means of mercury. By a high pressure regulating tap it is then conducted by a brass capillary to *A*, where the capillary forks. One branch leads to a mercury-water differential manometer, in which the level of the water is kept constant by the regulation of the tap *C*. At first we worked with oil manometers, which allow an easy adjustment at any pressure required. With these no constant values were found for the viscosity coefficient, which was perhaps in consequence of oil vapour solidifying on the capillary. On this account the oil manometers were replaced by mercury-water differential manometers.

The other branch of the capillary at *A* leads the gas through a steel capillary *EFG* to the viscosity apparatus in the cryostat.

Between *D* and *E* is a *U* tube of charcoal immersed in liquid air, by means of which the last possible traces of air would be kept back.

A vertical glass tube carried the gas further. To this was soldered the spiral copper capillary of about 70 cm. length, in which the above mentioned cooling of the gas took place, which had been shown to be indispensable. This terminated at *K*, from where the gas was carried to *L*. In *L*, which was a small reservoir, the tube divides into two branches viz. the capillary and the tube *LMN* to the mercury manometer *O*. *L* and *P* could be directly connected by a tube in parallel with the capillary and provided with a stop-cock. This was necessary during the exhaustion. The transition from the capillary tube into *L*, in which the gas may be considered as at rest, is very gradual. This is of importance for the correction of HAGENBACH, which can be omitted in these circumstances. From *S* a branch *TO* leads further to a second mercury manometer *V*, which registered the pressure *p* at the beginning of the capillary tube. Through the capillary tube, (about 65 cm. long, with a diameter of 0.122 mm.) the gas flowed into *P*. As at *L* a tube *PQR* leads from here to the other end of the mercury manometer *O*. By means of this manometer we could thus read the difference $p_1 - p_2$. Another tube *WMYZ* leads the gas from *P* to *Z*. *Z* is connected on one side to the mercury-water differential manometer *b*, and on the other side by *a* to *c*. At *a* there is a regulating tap, which enables us to keep the level of the manometer *b* at a constant height during the experiment. By doing this during the experiment we can keep $p_1 - p_2$

¹⁾ The gas was purified by passing over a spiral cooled by liquid hydrogen (Comm. N^o. 83). A trace of air was afterwards found in the gas, this may have been absorbed during the compression in spite of the precautions taken.

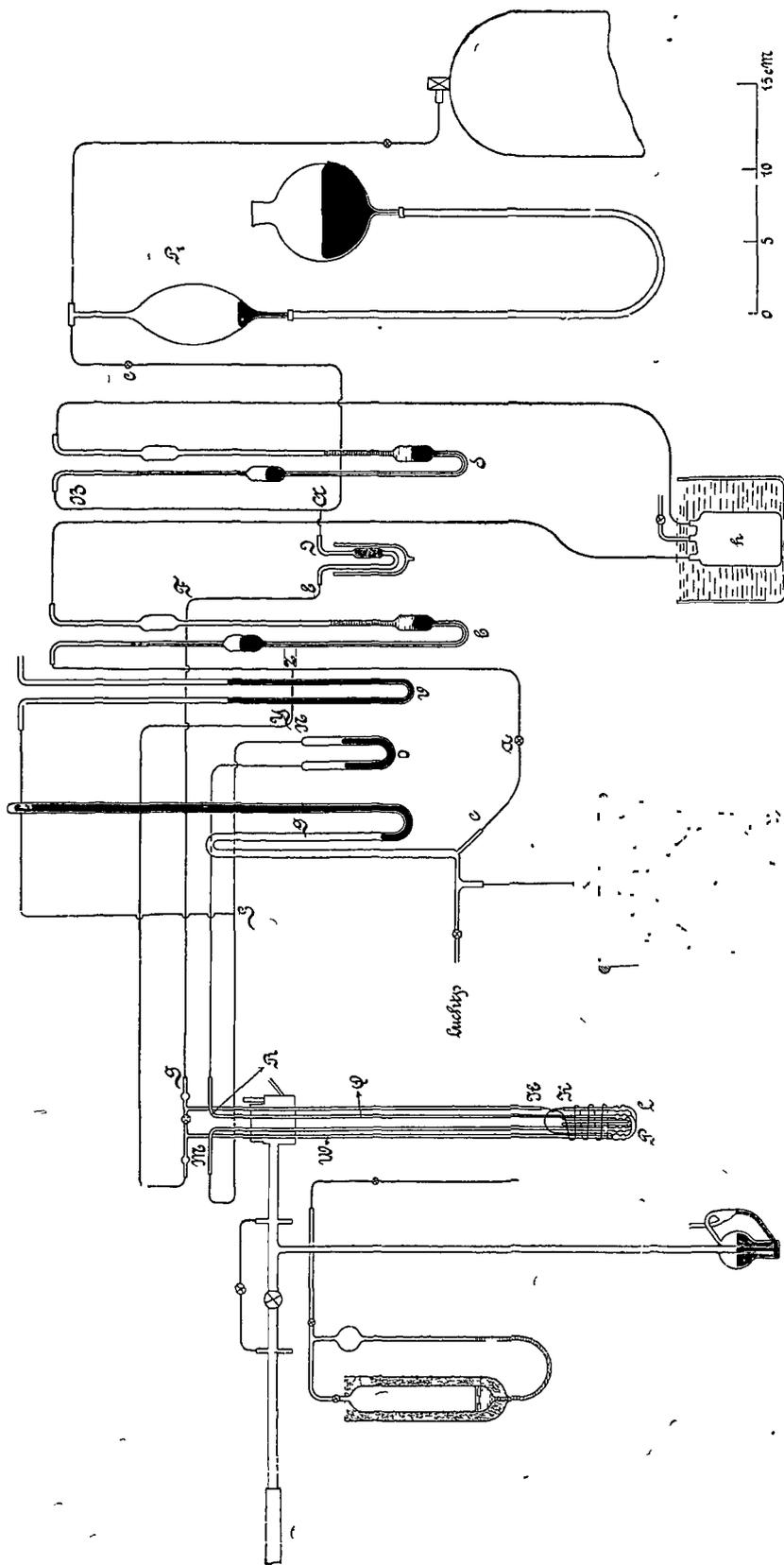


Fig. 2.

and p_1 steady, except for the small irregularities due to the regulation of the taps. The tube c is connected to a vessel e of about 4 l. placed in ice, intended for the determination of the volume. The gas pressure in this reservoir was determined before and after every experiment, by reading the manometer g . As can be seen, a small portion of this volume is not reduced to 0° C., it remains at about the temperature of the room. This portion is only about 15% of the whole volume.

The three manometers were read by a kathetometer, and were so placed that they could all be viewed by turning the kathetometer.

The temperature of the viscosity capillary was determined by a platinum resistance thermometer placed beside it, which was reduced to the hydrogen thermometer of the Laboratory by comparison with a standard resistance thermometer.

For the arrangement of the cryostat with stirrer see Com. N^o. 123. The measuring apparatus were immersed in a cryostat glass exactly like that of the helium cryostat. As we said in § 1, it is our intention to determine the viscosity of helium at helium temperature with the same apparatus. The cryostat glass was covered by a cap, which is like that of the helium cryostat, but simplified in an obvious way. In fig. 2 the cryostat glass with stirrer and thermometer are omitted.

§ 4. *Course of the experiments.* When the tightness of the apparatus had been properly tested and all found to be in order, the experiment was made in the following way. The volumenometer and the whole apparatus were pumped out and the tap a was closed. Then tap c was opened, and regulated so that the manometers b and d were at the desired height. When this was attained the experiment was begun, and simultaneously with the beginning of the regulation of tap a the knob of a chronometer was pressed. During the experiment, as already said, the taps a and c were so regulated that the differential manometers which acted as indicators, kept constant, at the same time the manometers O and V were read, and the small irregularities which at the most were 1% were noted as well as was possible. By the determination of a mean value we find from these readings the pressure difference, which existed between the extremities of the capillary tube during the experiment. If we reduce the readings of the manometer v by $\frac{p_1 - p_2}{2}$, we find the mean pressure \bar{p} .

The chronometer was compared with the standard clock of the laboratory immediately after pressing the knob. The latter served

as the actual time measurer. The end of the experiment was registered in the same way. At the same time tap a was closed. Then the pressure in the volumenometer was read, and herewith the necessary data were obtained.

The distribution of temperature in the cryostat during the evaporation of the bath, may be a source of error, as it affects the distribution of the density in the tubes leading to the capillary tube. These tubes and the time of flow were therefore so chosen that the errors which might arise from this were negligible.

The experiments were usually made at a mean pressure of about $\frac{1}{2}$ atmosphere.

RUCKES¹⁾ has proved that REYNOLDS' criterion applies also to gases. When we determine the critical velocity for our experiments at hydrogen temperatures, we find 3253 cm./sec. while the greatest which occurred in the experiments was 419 cm./sec.

§ 5. *Results.* In the first measurements made with hydrogen in the manner described above, the viscosity became higher and the higher, which can be explained by the hydrogen still containing some traces of air which froze in the capillary tube. On this account we introduced the tube with charcoal described above. The later determinations gave constant results.

The whole observational material is collected in Table I. The first column contains the temp. in Kelvin degrees, the second and third the difference of pressure and the mean pressure. These results, as already said, are calculated from a great number of observations, the deviations from the mean were about 2‰. The fourth column contains the time of flow in seconds, the fifth the increase of pressure in the volumenometer.

This increase of pressure combined with the volume, gives the amount of gas which flows through, and this must be reduced to mean pressure and temperature of the tube through which it flows. For this purpose the equation of state was used, which had been deduced from the measurements of KAMERLINGH ONNES and DE HAAS (Comm. N^o. 127) and KAMERLINGH ONNES and BRAAK (Comm. N^o. 97a).

The two first observations were used to calibrate the apparatus, in which we assumed with MARKOWSKI²⁾ $\eta_0 = 841.10^{-7}$, while for C in SUTHERLAND's formula 83 was taken.

By this means the values were determined which are given under

1) W. RUCKES Ann. d. Phys. 25, 1908 pag. 983.

2) H. MARKOWSKI loc. cit. and K. SCHMITT loc. cit.

T A B L E I.

$T^{\circ}\text{K.}$	$\Delta p_{\text{cm.Hg}}$	$\bar{p}_{\text{cm.Hg}}$	τ_{sec}	ΔP_{cmHg}	$\eta' \cdot 10^7$	$\eta \cdot 10^7$
293.90	11.455	41.83	12739	1.479	"	} 887.2
293.88	10.750	42.61	16814	1.866	"	
170.2	10.315	43.42	4755.5	1.282	616.8	609.2
170.2	10.310	43.43	6600.5	1.777	617.0	609.4
89.60	6.020	39.47	4760.0	1.999	399.4	392.1
89.65	5.545	40.39	3472.5	1.374	399.4	392.1
89.65	8.485	38.86	3045.0	1.773	399.8	392.5
70.9	6.010	39.48	2610.0	1.711	323.1	316.7
70.9	8.385	38.91	2301.5	2.056	326.2	319.8
70.9	8.300	38.92	1834.0	1.614	327.7	321.3
20.06	4.648	39.08	1264.2	6.565	114.1	111.5
20.04	4.651	39.16	1264.0	6.628	113.5	110.9
20.03	4.630	39.70	1265.5	6.694	113.5	110.9
20.04	3.945	20.40	1684.1	4.021	108.5	106.0
20.04	4.190	19.12	1576.3	3.799	107.0	103.5
20.04	4.580	20.71	1625.0	4.575	108.4	105.9
20.04	4.603	20.37	1357.6	3.787	108.2	105.7

$\eta' \cdot 10^7$. They were corrected for the change of R^2/L with the temperature and for the gliding. The corrected values are in column 7 under $\eta \cdot 10^7$.

From Table I we can immediately see the degree of accuracy that may be ascribed to the measurements with regard to accidental errors. As we have said before the determinations were usually made at a mean pressure of half an atmosphere. At and above oxygen temperatures a determination at one pressure is sufficient, at hydrogen temperatures this appeared to be no longer the case. Table I shows that there the viscosity changes with the density, and in the same direction as was found by WARBURG and BABO for carbon dioxide. Our differential manometers were not yet arranged, as we intend to do, for working with different mean pressures, and the apparatus was thus not very suitable for determining the influence of the pressure. In order to perform a few experiments with a different pressure, the two differential manometers were connected to an artificial atmosphere h , as can be seen in the plate.

For the further experiments which we intend to make (see § 1) concerning the dependence of the viscosity upon the density, it may appear that where it is a case of relative determinations only, the oscillation method is perhaps the most suitable.

TABLE II.			
T° K	$\eta_{\text{obs.}} \cdot 10^7$	$\eta_s \cdot 10^7$	$\eta_{\text{calc.}} \cdot 10^7$
457.3*	1212	1203	1207
373.6*	1046	1050	1052
293.95	—	887.2	886
287.6*	877	874	875
273.0*	844	843	843
261.2*	821	814	816
255.3*	802	800	803
233.2*	760	747	757
212.9*	710	697	709
194.4*	670	648	666
170.2	609.3	582	608
89.63	392.2	326	389
[78.2*	374.2	284	354]
70.87	319.3	257	329
20.04	105—111	58	137

In Table II our results are put together with those of MARKOWSKI and of KOPSCH¹⁾ (the last are marked with an asterisk). Fig. 3 shows that our measurements correspond well with the previous ones, KOPSCH's determination in liquid air forms an exception. The cause of this is probably an insufficient fore-cooling in KOPSCH's apparatus, as it is improbable that the density should have an important influence²⁾ here.

In column 3 under $\eta_s \cdot 10^7$ are given the values calculated by SUTHERLAND's formula with $\eta_0 = 841 \cdot 10^2$ and $C = 83$. The differences become very great at the lower temperatures, in liquid hydrogen more than 100 %.

KOPSCH has already pointed out that SUTHERLAND's formula no longer holds for hydrogen at the temperature of liquid air, and although the deviation which he found seems to be partly ascribable

¹⁾ H. MARKOWSKI. Ann. d. Phys. 14. 1904 pag. 742.

²⁾ Observations for *He* which will be treated in a following paper, show that there is no such dependence on the density.

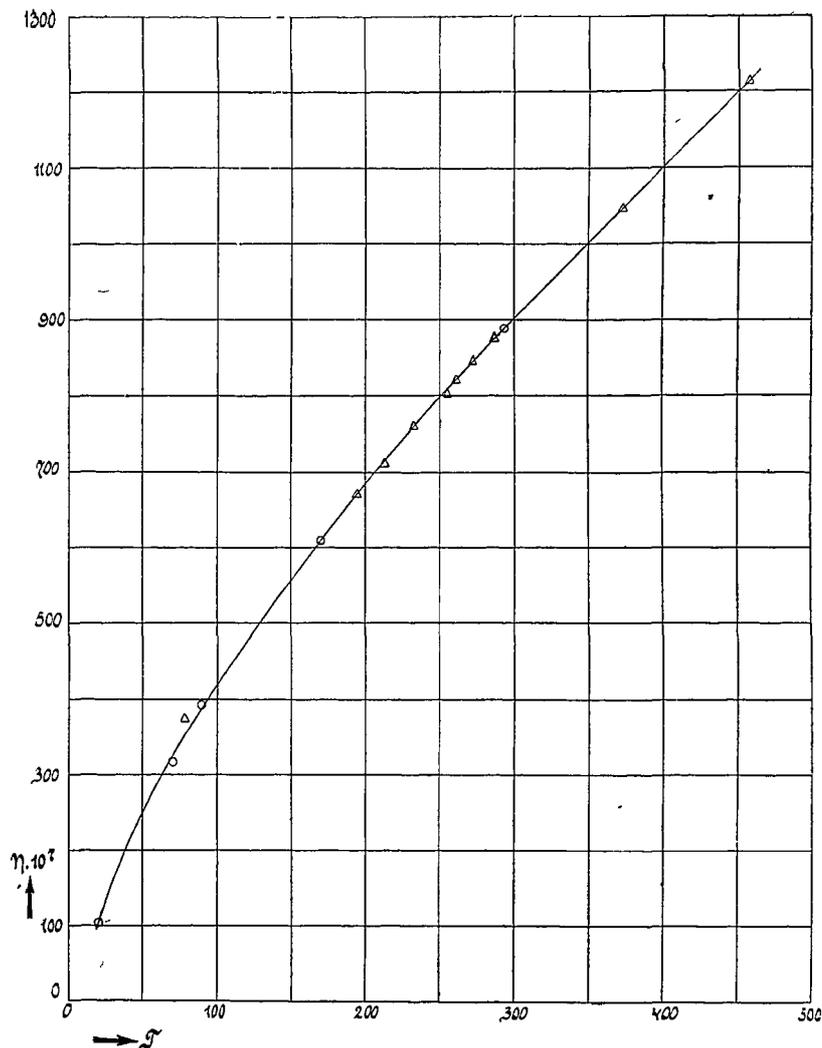


Fig. 3

to insufficient fore-cooling his conclusion remains correct, as the amount of the deviation of the observations from the formula even at the temperature of liquid air is larger than that of the deviation which is due to insufficient cooling.

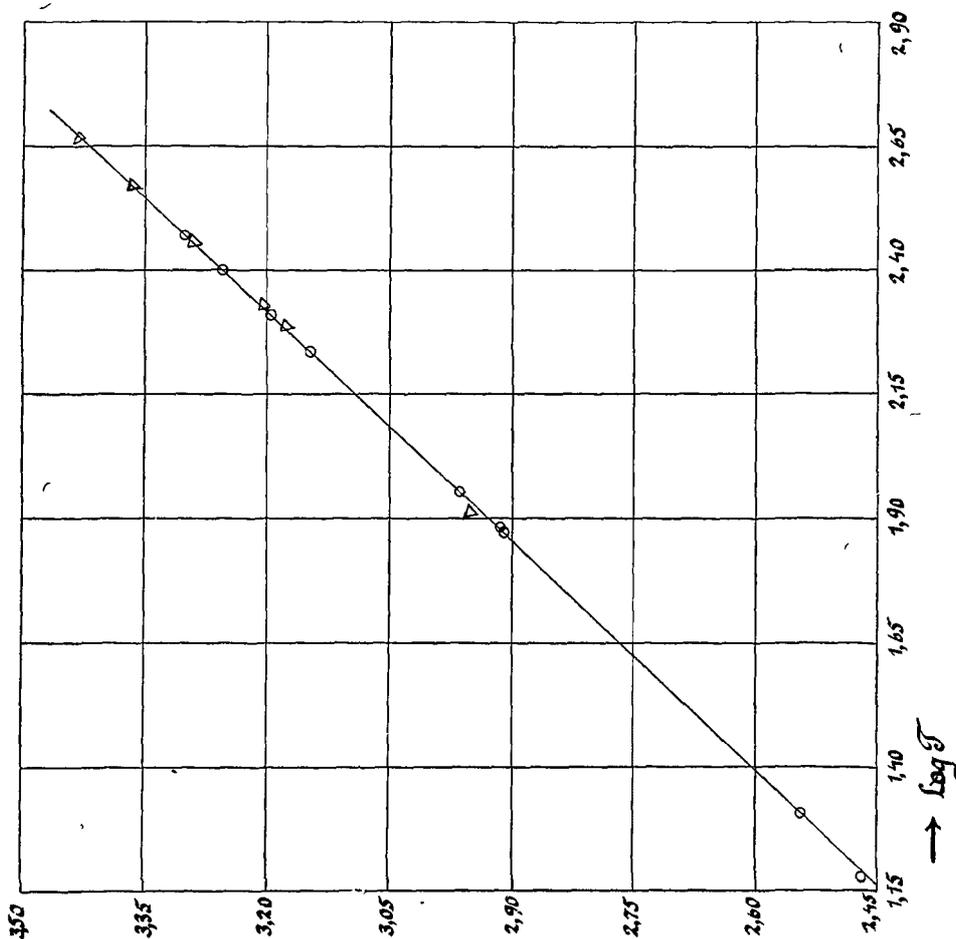
We endeavoured to find a simple relation between $\log \eta$ and $\log T$, which would correspond to the observations better than SUTHERLAND'S formula. Column 4 under $\eta_{calc} 10^7$ contains the values of η calculated by the formula

$$\eta = \eta_0 \left(\frac{T}{273} \right)^{0.695}$$

The correspondence is satisfactory as far as the temperature of reduced oxygen. We shall return to this question in the following paper about the viscosity of helium, in which we shall further deal with the change of the nucleus volume b_n with the temperature, as it follows from our experiments.

Physics. — “Investigation of the viscosity of gases at low temperatures. II. Helium”. By Prof. KAMERLINGH ONNES and SOPHUS WEBER. Communication N°. 134b from the physical Laboratory at Leiden by Prof. H. KAMERLINGH ONNES.

§ 1. *Results.* With the same apparatus as was used for the investigation of the viscosity of hydrogen¹⁾, a series of measure-



¹⁾ H. KAMERLINGH ONNES, C. DORSMAN and SOPHUS WEBER: Comm N°. 134a.

$\log(\eta \cdot 10^7)$
↑