$$\frac{d\tilde{\omega}}{dt} = +\ 2''.28 \quad ; \quad \frac{d\Omega}{dt} = -\ 2''.22$$

Perturbations caused by the second ellipsoid.

I find:

$$\frac{\partial^2 \Omega}{\partial x^2} = \frac{\partial^2 \Omega}{\partial y^2} = -2E_2 ; \frac{\partial^2 \Omega}{\partial x \partial y} = 0;$$

$$\frac{\partial^2 \Omega}{\partial x \partial z} = -\ 2\,(a^2-c^2)\,E_3 sin\ \Phi\ sin\ J; \quad \frac{\partial^2 \Omega}{\partial y \partial z} = 2(a^2-c^2)\,E_3\ cos\ \Phi\ sin J;$$

$$\frac{\partial^2 \Omega}{\partial z^2} = -\ 2E_2 - 2(a^2-c^2)E_3$$

from which follows:

$$\frac{R}{k^2 \pi q a^2 c} = \frac{1}{2}\frac{a_1'^2}{a_1^2}\,[-2E_2 a_1^2 - 3E_2 a_1^2 e^2 - E_3(a^2-c^2)a_1^2\ sin^2\ i$$
$$+\ 2(a^2-c^2)\ a_1^2\ E_3 sin\ J\ sin\ i\ cos\ (\Omega - \Phi)].$$

Although the term a^2-c^2 is not small, yet it is allowed to omit the periodic term.

I get $E_2 = 0.684$, $E_3 = 2.445$ from which follows taking as unit of time the century:

$$\frac{d\tilde{\omega}}{dt} = -\ 0''.16 \quad ; \quad \frac{d\Omega}{dt} = -\ 0''\,28$$

Thus both ellipsoids together give:

$$\frac{d\tilde{\omega}}{dt} = +\ 2''.12 \quad ; \quad \frac{d\Omega}{dt} = +\ 2''\,50;$$

both insensible amounts.

Astronomy. — "*Remarks on Mr.* WOLTJER'*s paper concerning* SEELIGER'*s hypothesis.*" By Prof. W. DE SITTER.

(Communicated in the meeting of April 24, 1914).

SEELIGER'S explanation of NEWCOMB'S anomalies in the secular motions of the four inner planets consists of three parts, viz:

a. The attraction of an ellipsoid entirely within the orbit of Mercury The light reflected by this ellipsoid is, on account of the neighbourhood of the sun, invisible to us.

b. The attraction of an ellipsoid which incloses the earth's orbit. The light reflected by this ellipsoid appears to us as the zodiacal light.

c. A rotation of the empirical system of co-ordinates with reference

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to the "Inertialsystem". This rotation is equivalent with a correction to the constant of precession. The value of this constant which is implied in NEWCOMB's anomalies is that used in his first fundamental catalogue (Astr. Papers Vol I). In "The Observatory" for July 1913 I have shown that this constant requires a correction of $+1''.24$ (per century). Consequently, of SEELIGER's rotation r only the part $r_1 = r - 1''.24$ can be considered as a real rotation.

The position of the equatorial plane of the ellipsoid a was determined by SEELIGER from the equations of condition: he found it not much different from the sun's equator. For the ellipsoid b the sun's equator was adopted as the equatorial plane.

It is important to consider the part which is contributed by each of the three hypotheses towards the explanation of the anomalies. By the way in which SEELIGER has published his results this is very easy. It then appears that the ellipsoid a is practically only necessary for the explanation of the anomaly in the motion of the perihelion of Mercury, and has very little influence on the other elements. Similarly the ellipsoid b affects almost exclusively the node of Venus. The rotation r of course has the same effect on all perihelia and nodes. In the following Table are given NEWCOMB's anomalies together with the residuals which are left unexplained by SEELIGER's hypothesis. In addition to SEELIGER's residuals I also give residuals which are derived: *A*. by rejecting the rotation r_1[1]), and *C*. by omitting the second ellipsoid. The constants implied in the three sets of residuals are thus

SEELIGER	$q_1 = 2.18 \times 10^{-11}$	$q_2 = 0.31 \times 10^{-14}$	$r_1 = +4''.61$
A	2.42	0.93	0
C	2.03	0	$+6.85$,

where q_1 and q_2 are the densities of the two ellipsoids expressed in the sun's density as unit.

SEELIGER did not compute the value of $\dfrac{di}{dt}$ for the earth. The residual given in the table is derived from the preceding paper by Mr. WOLTJER.

From the table it appears that the residuals C are quite as satisfactory as those of SEELIGER. Consequently the ellipsoid b is *not* a

[1]) The residuals *A* have already been given in the above quoted paper in "The Observatory". The density q_2 is there erroneously given as 0.37 instead of 0.93 (the correction to SEELIGER's value having been taken as 0.2 times this value, instead of 2.0). I have used the figures as published by SEELIGER. The small deviations found by Mr. WOLTJER are of no importance.

necessary part of the explanation. Of the residuals A on the other hand there are, amongst the 10 quantities which were considered

		Mercury	Venus	Earth	Mars
$\frac{de}{dt}$	NEWCOMB	$-0''.88 \pm 0''.50$	$+0''.21 \pm 0''.31$	$+0''.02 \pm 0''.10$	$+0''.29 \pm 0''.27$
$e\frac{d\tilde{\omega}}{dt}$	NEWCOMB	$+8.48 \pm 0.43$	-0.05 ± 0.25	$+0.10 \pm 0.13$	$+0.75 \pm 0.35$
	SEELIGER	-0.01	-0.10	$+0.03$	$+0.16$
	A	0.00	-0.05	$+0.18$	$+0.52$
	C	-0.02	-0.12	-0.04	0.00
$\sin i \frac{d\Omega}{dt}$	NEWCOMB	$+0.61 \pm 0.52$	$+0.60 \pm 0.17$	. . . . . .	$+0.03 \pm 0.22$
	SEELIGER	-0.04	$+0.02$	. . . . . .	-0.20
	A	$+0.55$	$+0.01$	. . . . . .	-0.11
	C	-0.31	$+0.05$	. . . . . .	-0.24
$\frac{di}{dt}$	NEWCOMB	$+0.38 \pm 0.80$	$+0.38 \pm 0.33$	-0.22 ± 0.27	-0.01 ± 0.20
	SEELIGER	-0.14	$+0.21$	$(+0.28)$	$+0.01$
	A	-0.12	$+0.17$	$+1.18$	$+0.05$
	C	-0.15	$+0.23$	-0.17	-0.01

by SEELIGER, 3 residuals exceeding their mean error. This in itself would not be sufficient to condemn the hypothesis, but the residual for the secular variation of the inclination of the ecliptic ($+1''.18$) is entirely inadmissible. We conclude therefore that the rotation r_1 is a vital part of the explanation.

The great influence of the ellipsoid b on the ecliptic is, of course, due to the large inclination of its equator. If this equator was e.g. supposed to coincide with the invariable plane of the solar system, instead of with the sun's equator, this influence would be much smaller. It is impossible to decide a priori whether it will be found possible so to adjust the position of the equator and the density of this ellipsoid that it has the desired effect on the node of Venus without appreciably affecting the earth's orbit.

The motion of the node of the earth's orbit is the planetary precession. Calling this λ, we have, for $t = t_0$

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$$\Delta\lambda \,.\, \sin \varepsilon = \frac{dp}{dt},$$

where p is the quantity so called by Mr. WOLTJER. We thus find for the three hypotheses

SEELIGER	$\Delta\lambda = + 0''.47$
A	$+ 1\ .13$
C	$+ 0\ .15$

NEWCOMB did not include a deviation between observation and theory for this quantity. At the time of the publication of the "Astronomical Constants" (1895) it was of course entirely correct to consider a determination of the planetary precession from observations as impossible. Since that time however very accurate investigations of the precession have been executed by NEWCOMB himself (Astr. Papers, Vol. VIII) and by Boss (Astr. Journal, Vol. XVI, Nrs. 612 and 614). Now the precession in right-ascension depends on the planetary precession, but that in declination does not. We have

$$m = l \cos \varepsilon - \lambda$$
$$n = l \sin \varepsilon$$

l being the lunisolar precession.

NEWCOMB determined l from the right-ascensions and the declinations separately, and found a large difference in the results. If this were interpreted as a correction to the planetary precession, we should find

$$\Delta\lambda = + 0''.47.$$

Boss determined m and n separately, the latter both from right-ascensions and from declinations. From his results I find (applying the correction of the equinox $\Delta e = + 0'.30$, adopted by both Boss and NEWCOMB):

$$\Delta\lambda = + 0''.85 \pm 0''.22$$

The mean error does not contain the uncertainty of the correction Δe. Its true value probably is about $= + 0''.25$. The mean error of the value of $\Delta\lambda$ derived from NEWCOMB's work is difficult to estimate; we may assume it to be equal to that of Boss. The mean of the two determinations would then be

$$\Delta\lambda = + 0''.66 \pm 0''.18$$ [1].

[1]) Also L. STRUVE (A. N. Vol. 159, page 383) finds a difference in the same sense. Neglecting the systematic correction e, I find from his results

$$\Delta\lambda = + 0''.93 \pm 0''.30\,.$$

The m. e. again is too small as it does not contain the effect of the uncertainty of the correction e.

Now it is certainly very remarkable that this correction is of the same sign and the same order of magnitude as the planetary precession derived from the attraction of SEELIGER's ellipsoids. It must however be kept in mind that it is very well possible to explain the discrepancy between the determinations of the constant of precession from right-ascensions and from declinations (or from m and from n) by the hypothesis of systematic proper motions of the stars. Thus HOUGH and HALM (M. N. Vol. LXX page 586) have from the hypothesis of unequal distribution of the stars over the two streams derived a systematic difference which is equivalent (for NEWCOMB)[1]) to a correction

$$\Delta \lambda = + 0''\,56.$$

As the effect of the attraction of SEELIGER's ellipsoids on the motion of the moon Mr. WOLTJER finds a secular motion of both the perigee and the node. Both of these are due chiefly to the inner ellipsoïd and are thus not much altered if SEELIGER's hypothesis is replaced by either of the hypotheses A or C. We find

SEELIGER	$\frac{d\tilde{\omega}}{dt} = + 2''.11$	$\frac{d\Omega}{dt} = - 2''.50$
A	$+ 2\,.04$	$- 3\,.30$
C	$+ 2\;10$	$- 2\,.06$

All these quantities are well within the limits of uncertainty of the observed values.

Chemistry. — *"The application of the theory of allotropy to electromotive equilibria."* II. By Dr. A SMITS and Dr. A. H. W. ATEN. (A preliminary communication). (Communicated by Prof. J. D. VAN DER WAALS).

(Communicated in the meeting of April 24, 1914).

1. In the first communication [2]) under the above title it has been demonstrated that the theory of allotropy applied to the electromotive equilibrium between metal and electrolyte, teaches that a metal that exhibits the phenomenon of allotropy and is therefore built up of different kinds of molecules immersed in an electrolyte, will emit different kind of ions.

The different kinds of ions assumed by the theory of allotropy, need not be per se different in size, as was remarked before. They

[1]) For STRUVE's stars the correction would be $+0''.77$. For Boss the corresponding computation has of course not been executed by HOUGH and HALM.

[2]) These Proc. Dec. 27, 1913, XVI. p. 699.