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Physiology. — "The measurement of Chronaxia". By Prof. J. K. A. Wertheim Salomonson.

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Hoorweg's law for the stimulation of excitable tissues by means of electric currents states that the rate of excitation rapidly lessens during the flow of the current. His law may be mathematically expressed by

\[ y = \alpha \int_{0}^{\infty} \varphi(t) e^{-\beta t} dt \quad \ldots \ldots \ldots \quad (1) \]

if \( y \) represents the excitation caused by a current \( I = \varphi(t) \), \( \alpha \) and \( \beta \) being constants. Hoorweg calls \( \alpha \) the initial constant, \( \beta \) the extinction constant. The reciprocal of the latter \( \frac{1}{\beta} \) is generally called the time constant of the stimulated tissue. Lapicque introduced the word chronaxia to designate this time constant.

The measurement of the chronaxia is of the greatest importance if we wish to indicate the excitability by electricity of the animal tissue. According to Doumer we can calculate it from the results of three condenser discharges under definite conditions. Hoorweg proposed two condenser discharges and one stimulation with the direct constant current, which for a long time has been the only available method. Cluzet uses a series of discharges of condensers of different capacity and calculates the chronaxia from the one in which the smallest amount of electrical energy was needed to obtain a minimal contraction. Lapicque has published a method for directly measuring the chronaxia with the direct current, closing and opening the circuit with two rapidly working contact keys of an instrument called a chronaximeter. A similar method was used by Keith, Adrian and others. I worked out a method in which a calibrated induction coil was used.

In all these measurements we find not only the chronaxia, but also the rheobasis, a constant which received this name from Lapicque. It is the smallest direct current causing a minimal muscle twitch, and is equal to the quotient of Hoorweg's constants \( \frac{\beta}{\alpha} \).
When the rheobasis and the chronaxia are known we find from them
Hoorweg's initial constant $\alpha$, or its reciprocal value $\frac{1}{\alpha}$ as quantity constant.

It is generally much less important to know the rheobasis than
the chronaxia. The latter seems to be a true constant, whereas the
rheobasis depends on the surface of the electrodes, the place where
they are applied, the condition of the skin, and even on the duration
of the constant current, used in measuring it. With the same
electrodes I found it in one case to be 4.4 milliampere with the
condensor method, 6.1 milliampere with the induction coil-method,
against 2.6 milliampere as measured with the constant current, the
three measurements being made immediately one after the other.
But in all three cases the chronaxia came out as 0.00008 second.
It seems possible that also differences in the resistance of the body
might be responsible for the differences in the rheobasis.

The rheobasis is easily determined and is even a matter of routine
work in neurological praxis. But little attention if any is given to
the chronaxia. Only if the methods are simplified there may be
some chance of a more extensive use of this way of stating the
excitability of muscular tissue and motor nerves. Below I give two
simplified methods for directly measuring the chronaxia, either
by two condensor-discharges or by two measurements with an
ordinary non-graduated induction coil. In every case the chronaxia
is found without any calculation or with merely one division.

Condensor-method.

Muscles and nerves respond in the same way to discharges of
condensors of different capacity $C$ if the Voltage $V$ in each indi-
vidual case be

$$V = \frac{\beta}{\alpha} R + \frac{1}{\alpha C} \quad \ldots \ldots \ldots \ldots \ldots \ldots$$

in which $R$ is the resistance of the circuit, $\beta$ and $\alpha$ the above-
mentioned constants. This relation was found by Hoorweg. It may
be calculated from (1) by putting

$$q(t) = \frac{V}{R} e^{-t/R}$$

which is the well-known formula for a condensor discharge through
a non-conductive resistance. If the expression is integrated and we
take for $y$ a unity-effect $= 1$ we get formula (2).\n
We first make a measurement with a capacity $C$, and find a
voltage $V$ which first gives a minimal contraction. Next we take
another somewhat larger capacity \( C_s \) and without modifying \( V \) but by putting a resistance \( W \) into the circuit we again obtain a minimal contraction.

We then put:

\[
\beta R + \frac{1}{C_1} = \beta (R + W) + \frac{1}{C_s}
\]

or

\[
\frac{1}{\beta} = W \frac{C_1 C_s}{C_s - C_1} \quad \cdots \quad \cdots \quad (3)
\]

We may take for \( C_s \) any capacity larger than \( C_1 \). By taking

\( C_s = 2C_1 \) we get the extremely simple expression for \( \frac{1}{\beta} \):

\[
\frac{1}{\beta} = WC_1 \quad \cdots \quad \cdots \quad (4)
\]

or the chronaxia is the product of the resistance \( W \) and the capacity \( C_1 \).

If we can use a complete set of graduated condensers we might use a fixed resistance \( W \) and try to find the value of \( C_s \) giving a minimal contraction, using formula (3). But as a matter of fact we find the use of a calibrated rheostat more convenient, especially as it allows of the use of formula (4), which is simpler. In this case only two condensers of say 0.05 or 0.1 \( \mu F \) and a rheostat up to 10000 Ohm are necessary; whereas in the first method a set of condensers from 0.001—0.5 \( \mu F \) and a fixed resistance \( W \) of 1000 or 2000 Ohm would be required.

The actual measurement of the Voltage \( V \) is unnecessary.

**Induction coil method.**

The secondary discharge of a medical induction coil may be represented by

\[
L_{11} = q(t) = I_1 \frac{M}{L_{11}} e^{-\frac{R_{11}}{L_{11} t}} \quad \cdots \quad \cdots \quad (5)
\]

in which \( I_1 \) is the primary current, \( M \) the mutual induction coefficient, \( L_{11} \) the selfinduction and \( R_{11} \) the resistance of the secondary circuit.

Putting \( q(t) \) in HOOVERG'S formula (1) we get

\[
\eta = \alpha \frac{I_1}{L_{11}} \frac{M}{\int_0^\infty e^{-\left(\frac{R_{11}}{L_{11}} + \beta\right)t} \, dt} = \frac{\alpha I_1 M}{R_{11} + L_{11} \beta}
\]

and taking \( \eta = 1 \) we get:

\[
a \frac{1}{M} = R_{11} + \beta L_{11} \quad \cdots \quad \cdots \quad (6)
\]

We now make again two measurements of a minimal contraction. In the first one we insert a selfinduction \( \mathcal{L} \) into the secondary
circuit and by varying either the primary current strength or more conveniently the coil distance we finally get a minimal contraction. Now leaving the coil distance and the primary current unchanged we take away the added selfinduction $L$ and put in its place a rheostat from which so much resistance is unstopped till the minimal contraction appears again. Then we have

$$a I, M = (R_{11} + W) + \beta L_{11} = R_{11} + \beta (L_{11} + L)$$

or

$$\frac{1}{\beta} = \frac{L}{W}$$

(7)

giving the chronaxia $\beta$ as the quotient of a resistance into a selfinduction.

This method can still be applied in two different ways. We can either use a fixed selfinduction and a variable resistance or a fixed resistance and a variable selfinduction. The latter might even be provided with a scale graduated to 10000th parts of a second so as to make it a direct reading instrument. Or if we take 1000 ohms for the fixed resistance, the readings of the graduated scale of the induction variometer in Henry will give the value of the chronaxia in 1000th parts of a second.

The advantage of the induction coil method may be gathered from the fact that the combination of a rheostat and graduated selfinduction coil may be used with any medical coil and with any ordinary interrupter. The only restriction to be made is against the use of a secondary coil with less than 3000 windings. The formula used ceases to be accurate as soon as an extremely large resistance is inserted in the secondary circuit. With secondary coils of only a few hundred secondary windings this is already the case with a total secondary resistance of perhaps 1000 ohms. But with coils of more than 3000 windings the secondary resistance may be increased up to 15000 or 20000 Ohms without any appreciable error. The error in these cases is caused by the presence of capacity in the secondary coil, the influence of which is to lower the initial strength of the secondary discharge and to render the calculated chronaxia a trifle too small. But with coils of more than 3000 turns this error is considerably smaller than the error of the measurement itself, as was proved by a special physiological and oscillographic investigation.

Finally I wish to add that with the condenser or the induction coil it is possible to arrange for the use of an ungraduated but variable resistance and a special ohmmeter, which may be graduated for the chronaxia to be directly read on the scale.